MSMF GATE CENTRE

Subject: Control Systems

State Space Analysis and Controller

Time: 30 min **Marks= 15**

1. Number of state variables of a system are equal to

   (a) number of poles of a system
   (b) order of a system
   (c) number of independent storage elements of a system
   (d) all the above are true

2. State space representation of a system is

   (a) Unique
   (b) represented in only two ways
   (c) represented in only three ways
   (d) none of the above

3. State space representation of a system is

$$\frac{dX(t)}{dt} = \begin{bmatrix} -2 & 0 \\ 0 & -3 \end{bmatrix} X(t) + \begin{bmatrix} 1 \\ -1 \end{bmatrix} U(t)$$

$$Y(t) = \begin{bmatrix} 1 & 1 \end{bmatrix} X(t)$$

   The above representation is called as

   (a) diagonal canonical form
   (b) jordon canonical form
   (c) controllable canonical form
   (d) observable canonical form

4. Lag controller improves

   (a) stability
   (b) transient response
   (c) steady state response
   (d) none of the above

5. PD controller adds ____ to the open loop TF

   (a) pole
   (b) zero
   (c) pole and zero
   (d) zero at the origin

6. State space representation of a system is given by

$$\frac{dX(t)}{dt} = \begin{bmatrix} -2 & 1 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -3 \end{bmatrix} X(t) + \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} U(t)$$

the characteristic equation of a system is

(a) $(s^2 + 2s + 3)(s + 2)$
(b) $(s^2 + 2s + 3^2)$
(c) $(s + 2)^2 (s + 3)$
(d) $(s + 3)^2 (s + 2)$

7. $\frac{dX(t)}{dt} = AX(t) + BU(t)$ -state equation

$Y(t) = CX(t)$ --- Output equation

The TF $\frac{Y(S)}{U(S)}$ is

(a) $C[SI - A]^{-1} B + D$
(b) $C[SI - A]^{-1} B$
(c) $B[SI - A]^{-1} D + C$
(d) $C[SI - A]^{-1} B + C$

8. The State space representation system is

$$\frac{dX(t)}{dt} = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix} X(t) + \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} U(t)$$

$Y(t) = [1\ 1] X(t)$

the state transition matrix is

(a) $\begin{bmatrix} e^{-t} & 0 \\ 0 & e^{-2t} \end{bmatrix}$ (b) $\begin{bmatrix} e^{t} & 0 \\ 0 & e^{-2t} \end{bmatrix}$

(c) $\begin{bmatrix} e^{t} & 0 \\ 0 & e^{2t} \end{bmatrix}$ (d) $\begin{bmatrix} e^{-t} & 0 \\ 0 & e^{2t} \end{bmatrix}$

9. The SSR is

$$\dot{X}(t) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -2 & -3 & -4 \end{bmatrix} X(t) + \begin{bmatrix} 0 \\ 0 \\ 3 \end{bmatrix} U(t)$$

$Y(t) = [6\ \ 5\ \ 1]\ X(t)$. The TF $\frac{Y(S)}{U(S)}$ is

(a) $\frac{S^2 + 5S + 6}{S^3 + 4S^2 + 3S + 2}$
(b) $\frac{3S^2 + 5S + 6}{S^3 + 4S^2 + 3S + 2}$
(c) $\frac{3S^2 + 15S + 18}{S^3 + 4S^2 + 3S + 2}$
(d) None

10. The TF of a certain 2nd order system is $\frac{S+1}{S^2 + 4S + 5}$. The system is

(a) controllable & observable
(b) controllable but not observable
(c) not controllable, but observable
(d) neither controllable nor observable

11. The lag-lead controller is

(a) BPF (b) BRF
(c) LPF (d) HPF

Linked data and Common data questions

Linked data questions 12 and 13

The $TF = \dfrac{1+TS}{1+\alpha TS}$ is a lag controller

α & T >0

12. The range of α is

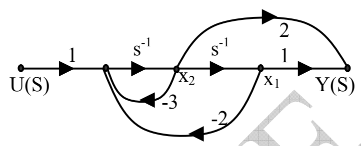
(a) $\alpha < 1$
(b) $1 < \alpha < \infty$
(c) $\alpha > 10$
(d) $\alpha < 10$

13. the value of ϕ_m is

(a) $\phi_m = \sin^{-1}\dfrac{\alpha-1}{\alpha+1}$
(b) $\varphi_m = \sin^{-1}\dfrac{1-\alpha}{1+\alpha}$
(c) $\phi_m = \cos^{-1}\dfrac{\alpha-1}{\alpha+1}$
(d) $\phi_m = \cos^{-1}\dfrac{1-\alpha}{1+\alpha}$

Common data questions Q14 and Q15

The state diagram of a system is given below



SSR is $\dot{X}(t) = AX(t) + BU(t)$;

$Y(t) = CX(t)$, where $X(t) = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

14. The matrix 'A' is

(a) $\begin{bmatrix} 0 & 1 \\ -3 & -2 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$

(c) $\begin{bmatrix} -2 & 1 \\ 0 & -3 \end{bmatrix}$ (d) None

15. The eigen values of matrix 'A'

(a) 2,3 (b)-1,- 2
(c) -2,-3 (d) 1,2

**** THE END ****

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Subject: Control Systems

State Space Analysis and Controller

(Solutions)

1. **Ans:(d)**
   Sol:
   No. of state variables = order of the system = poles of the system = independent storage
   elements.

2. **Ans: (d)**
   Sol:
   State space representation of a system is not unique i.e. it may be represented in many
   ways.

3. **Ans: (a)**
   Sol:
   The matrix 'A' is in diagonal form
   ∴Representation is called on as Diagonal canonical form.

4. **Ans: (c)**
   Sol:
   Lag improves steady state response, since a pole is added very near to the origin.

5. **Ans: (b)**
   Sol:
   PD controller $G(S) = K_P + K_DS$
   $$S = \frac{-K_P}{K_D}$$
   A zero is added to the OLTF.
   (All controller poles/zeros are added to the OLTF only)

6. **Ans: (c)**
   Sol:
   Matrix 'A' diagonal elements are poles
   ∴ S = -2, -2 & -3 are the poles
   Hence the characteristic equation
   $(S+2)^2(S+3) = 0$

7. **Ans: (b)**
   Sol:
   State & $\dot{X} = AX + BU$
   $Y = CX$
   Solving for $\frac{Y(S)}{U(S)}$ gives the TF
   $TF = C[SI-A]^{-1}B$

8. **Ans: (a)**
   Sol:
   $$A = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix}$$

$$(SI-A) = \begin{bmatrix} S+1 & 0 \\ 0 & S+2 \end{bmatrix}$$

$$(SI-A)^{-1} = \begin{bmatrix} \frac{1}{S+1} & 0 \\ 0 & \frac{1}{S+2} \end{bmatrix}$$

State transition matrix = $e^{At} = L^{-1}[(SI-A)^{-1}]$

$$= L^{-1}\begin{bmatrix} \frac{1}{S+1} & 0 \\ 0 & \frac{1}{S+2} \end{bmatrix}$$

$$e^{At} = \begin{bmatrix} e^{-t} & 0 \\ 0 & e^{-2t} \end{bmatrix}$$

9. **Ans: (c)**
Sol:
The state space representation (SSR), is in controllable canonical form (CCF)
∴ TF can be written directly from the SSR

$$TF = \frac{3[S^2+5S+6]}{S^3+4S^2+3S+2}$$

10. **Ans: (a)**
Sol:

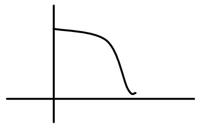
$$TF = \frac{S+1}{(S^2+4S+5)}$$

There is no pole – zero Cancellation in the TF

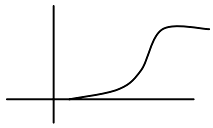
∴ System is completely controllable & observable.

11. **Ans: (b)**
Sol:

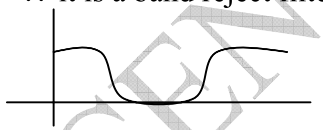
Lag – LPF



Lead – HPF



Combining Lag- Lead is
∴ it is a band reject filter



12. **Ans: (b)**
Sol:

$$\angle \text{Lag TF} = \angle\frac{1+TS}{1+\alpha TS} < 0 \text{ (-Ve)}$$

∴ 'α' should be greater than 1
$1 < \alpha < \infty$

13. **Ans: (b)**
Sol:

$$\phi_m = Sin^{-1}\left(\frac{1-\alpha}{1+\alpha}\right)_{\alpha>1} = -Ve$$

14. Ans: (b)
Sol:

From the state diagram it is clear that

the 1st integrator input is $\dot{X}_2$

2nd integrator input is $\dot{X}_1$

$$\therefore \dot{X}_1 = X_2$$

$$\dot{X}_2 = -2X_1 - 3X_2 + U(t)$$

State equation

$$\dot{X}(t) =$$

$$\begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} X(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} U(t)$$

$$\therefore A = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$$

15. Ans: (b)

Sol:

Characteristic equation is $S^2 + 3S + 2 = 0$

$(S + 1)(S + 2) = 0$

$S = -1$ & $S = -2$ are the eigen value of matrix 'A'