

MSMF GATE CENTRE

Subject: Control Systems
Time response analysis

Time: 30 min

Marks= 15

- 1) TF = $\frac{20}{(S+1)(S+10)}$ The approximated TF is
- (a) $\frac{20}{(S+1)}$ (b) ∞
(c) $\frac{2}{(S+10)}$ (d) $\frac{2}{(S+1)}$
- 2) TF = $\frac{20}{(S+1)(S-10)}$
The steady state value to an impulse input is
- (a) 20 (b) -2
(c) ∞ (d) 0
- 3) If the transfer function of a control system is $T(s) = \frac{50}{(s^2 + 4)^2(s + 2)^2(s + 4)}$
initial value to a step input is
- (a) 50 (b) 0
(c) ∞ (d) none of the above
- 4) Dominant pole is a
- (a) fast pole
(b) slow pole
(c) insignificant pole
(d) a pole with large real part
- 5) Open loop TF of an unity feedback system is $\frac{20}{S^2(S+1)}$ the steady state error to a step input is
- (a) 0 (b) 2
(c) 1 (d) none of the above
- 6) TF = $\frac{80}{(S+4)(S+10)}$ the peak overshoot to a step input is
- (a) Mp (b) 2Mp
(c) 4Mp (d) none of the above
- where $M_p = e^{-\left(\frac{\zeta\pi}{\sqrt{1-\zeta^2}}\right)}$
- 7) Impulse response of a system is $(4e^{-2t} + 6e^{-20t})u(t)$, the approximate settling time to reach 95% of response to a step input is
- (a) 1.5sec (b) 3sec
(c) 6sec (d) 10sec
- 8) CLTF = $\frac{10(s+2)}{s^2 + s + 1}$
With respect to the above system consider the following statements
1. Steady state error to a step input is non zero.

2. In the steady state, response contains damped sinusoidal oscillations with damping ratio 0.5
3. steady state value to a step input is 20.
- of the above the true statement(s) is/are

- (a) 1,2 (b) 2,3
(c) 1 (d) 3,1

- 9) The velocity error coefficient of a system is 10, now the type of system is increased by one, then the steady state error to a parabolic input is

- a) 0 (b) 0.1
c) 20 (d) none

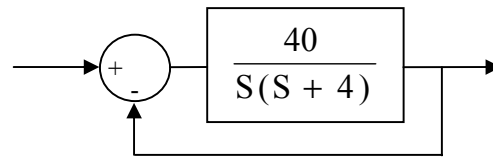
- 10) The position error coefficient of a system is 10, now the type of system is increased by one, then the steady state error to a ramp input is

- a) 0 (b) 0.1
c) 20 (d) none

- 11) Choose the correct statement from the following

- (a) Time response is decided by slow and fast poles
- (b) Transient response is decided by fast poles
- (c) Steady state response is decided by fast poles
- (d) Steady state response is decided by dominant poles

Linked data questions 12 and 13



- 12) The damping ratio of the above system, if a step of two units is applied is

- (a) 0.32 (b) 0.56
(c) 1.34 (d) 1.5

- 13) The peak overshoot of the above system to a step of two is

- (a) 25% (b) 62%
(c) 50% (d) 31%

Common data questions 14 and 15

1) $TF = \frac{2000}{(S+20)(S+200)}$

2) $TF = \frac{200}{(S+10)(S+2)^2}$

3) $TF = \frac{40(s+4)}{(S+20)(S+1)}$

4) $TF = \frac{200(S+1)}{(S-1)(S+50)}$

- 14) Among the above the fastest system is

- (a) 1 (b) 2
(c) 3 (d) 4

- 15) The steady state gain of the slowest system is

- (a) 0.5 (b) 5
(c) 8 (d) -4

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(Solutions)

1. **Ans: (d)**

Sol:

$$TF = \frac{20}{(S+1)(S+10)}$$

$$\text{Steady state gain} = \frac{20}{(1)(10)} = 2$$

Without change in steady state gain
neglecting the fast pole, $S = -10$

$$\text{the TF is } \frac{2}{2+1}$$

$$\text{Now also steady state gain} = \frac{2}{(1)} = 2$$

2. **Ans: (c)**

Sol:

$$TF = \frac{20}{(S+1)(S-10)} \text{ unstable system}$$

\therefore For any input output is ' ∞ '

3. **Ans: (b)**

Sol:

$$C(s) = \frac{50}{(s^2+4)^2(s+2)^2(s+4)} R(s) \Big|_{R(s)=\frac{1}{s}(\text{step input})}$$

$$\text{Initial value} = C(0) = \lim_{s \rightarrow \infty} Lt$$

$$S \frac{50}{(s^2+4)^2(s+2)^2(s+4)} \frac{1}{s} = 0$$

4. **Ans: (b)**

Sol:

Dominant poles are nearer to the imaginary axis and dominates the transient response. Their time constant is large hence they are called as slow poles.

$$\text{E.g. } TF = \frac{10}{(s+1)(s+10)}$$

$$s = -1, T = 1 \text{ sec (slow pole)}$$

$$s = -10, T = 0.1 \text{ sec (fast pole)}$$

5. **Ans: (d)**

Sol:

$$\text{OLTF} = \frac{20}{s^2(s+1)}$$

Characteristics equation is

$$S^3 + S^2 + 20 = 0$$

Unstable system as there is a missing coefficient

∴ Steady state error is '∞' or unbounded

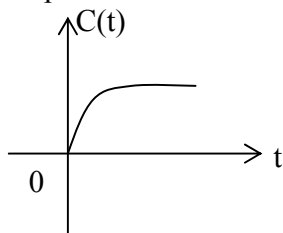
6. **Ans: (d)**

Sol:

$$\text{TF} = \frac{80}{(s+4)(s+10)}$$

System is over damped, as the poles are real & unequal

∴ Overshoot does not exist
Response is shown below



7. **Ans: (a)**

Sol:

$$\text{IR} = (e^{-2t} + 6e^{-20t})\mu(t)$$

Poles are -2 & -20

Settling time is determined by the slow pole, $S = -2$, Time constant is $\frac{1}{2}$ sec

To reach 95% system takes $3T = 3 \times \frac{1}{2}$
= 1.5 sec

8. **Ans: (d)**

Sol:

$$\text{Sol. OLTF} = \frac{10(s+2)}{s^2+5s+1}$$

$$C(\infty) = \frac{10(2)}{(1)} = 20$$

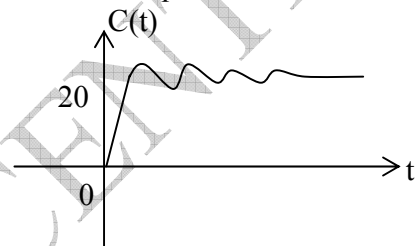
Input = 1 (step)

∴ Error = 1 - 20 = -19

Error is non zero

& steady state value = 20

In the steady state there are no oscillations in the response



9. **Ans: (d)**

Sol:

It depends on the OLTF

$$\text{Let OLTF} = \frac{10(s+1)}{s} \quad \& \quad H(s) = 1$$

$$K_v = 10$$

And the type increased by one

$$\text{OLTF} = \frac{10(s+1)}{s^2} \quad \& \quad H(s) = 1$$

Now $K_a = 10$ error = 0.1

$$\text{but if OLTF} = \frac{10}{s(s+1)} \quad \& \quad H(s) = 1$$

$$K_v = 10$$

And if the type increased by one

$$\text{OLTF} = \frac{10}{s^2(s+1)} \quad \& \quad H(s) = 1$$

System becomes unstable error can't be determined.

∴ It can't be decided

10. **Ans: (b)**

Sol:

$$\text{If OLTF} = \frac{10}{(s+1)} \text{ \& } H(s) = 1$$

$$K_p = 10$$

If the type is increased by one

$$\text{OLTF} = \frac{10}{s(s+1)} \text{ \& } H(1) = 1$$

$$K_v = 10$$

$$\text{Error} = \frac{1}{10} = 0.1$$

11. **Ans: (a)**

Sol:

Time response = Transient + steady state response, transient response is decided by slow poles and steady state response is decided by slow & fast poles

12. **Ans: (a)**

Sol:

$$\text{OLTF} = \frac{40}{S(S+4)}$$

Characteristics equation is

$$S^2 + 4S + 40 = 0$$

$$\omega_n^2 = 40$$

$$\omega_n = \sqrt{40}$$

$$2\xi\omega_n = 4$$

$$\xi = \frac{4}{2\sqrt{40}} = 0.32$$

13. **Ans: (d)**

Sol:

$$M_p = 2 e^{-\left(\frac{\xi \pi}{\sqrt{1-\xi^2}}\right)} = 0.62$$

$$\therefore \% M_p = 31\%$$

14. **Ans: (a)**

Sol:

$$TF = \frac{2000}{(s+20)(s+200)}$$

The poles are very far from the $j\omega$ axis, therefore system is faster

$$\therefore (\text{Time constant is short } \frac{1}{20} \text{ \& } \frac{1}{200})$$

15. **Ans: (c)**

Sol:

$$T_F = \frac{40(s+4)}{(s+20)(s+1)}$$

Pole $S = -1$ is very nearest

\therefore System is slowest

Steady state gain or DC gain

$$\Big|_{s=0} = \frac{(40)(4)}{(20)(1)} = 8$$