MSMF GATE CENTRE

Subject: Signals & Systems

DFT & FFT

Time: 30 min **Marks= 15**

(Note: D.F.T = discrete Fourier transform, F.T.T = fast Fourier transform)

1. The D.F.T. of a 4 point sequence is x[n] = {1, 0, -1, 0} is ________
(A) {2, 0, 0, 2} (B) {0, 2, 0, 2} (C) {2, 2, 0, 0} (D) {1, 2, 1, 2}

2. The inverse D.F.T of X[k] = {1+2j, j, -1+2j, -j} is________
(A) {1, 0, 0, –1} (B) {j, 1, 0, j} (C) {j, 0, j, 1} (D) {1, –1, 1, –1}

3. Let DFT of a signal x[n] is X[k] = {1, j, -1, -j} then the D.F.T of $x[(2-n)]_4$ is _______
(A) {1, j, 1, –j} (B) {1, –1, 1, j} (C) {–j, 1, j, –1} (D) {1, 0, 1, 0}

4. A 4 point real sequence x[n] has DFT X[k] = {1, j, 1, -j}then the DFT of $(-1)^n x[n]$ is___
(A) {1, –j, i, j} (B) {1, 1, j, –j} (C) {j, 1, 1, –j} (D) {–j, j, 1, –1}

5. Let the DFT of x[n] is X[k], then the DFT of Re {x[n]} is _________
(A) $\frac{X[k]+\left(X^*[-k]\right)_N}{2}$ (B) 2X[k]
(C) X[–k] (D) $\frac{X[-k]+j\left(X^*[k]\right)_N}{2}$

LINKED ANSWER QUESTIONS(6 & 7)

2 finite length sequences h & x have the DFTs:
X = [1, –2,1, –2] & H = [1, j, 1, –j]
Let y be a 4 point sequence circular convolution of h & x

6. Then y[0] is _______
(A) 0.5 (B) –3 (C) – 0.25 (D) – 0.75

7. D.F.T of $h([n+2])_4$ is _______
(A) [1, –j, 1, j] (B) [j, 1, –1, j] (C) [1, j, 1, –j] (D) [1, 1, –j, j]

8. The DFT of $x[n] = \begin{cases} \mathbf{1};\ \text{even } n & \mathbf{0} \le n \le N-\mathbf{1} \\ \mathbf{0};\ \text{odd } n & \mathbf{0} \le n \le N-\mathbf{1} \end{cases}$ is ________

(A) $X[k] = \begin{cases} N/2;\ K = 0\ \&\ N/2 \\ 0;\quad \text{otherwise} \end{cases}$

(B) $X[k] = W_N^{\ n}$

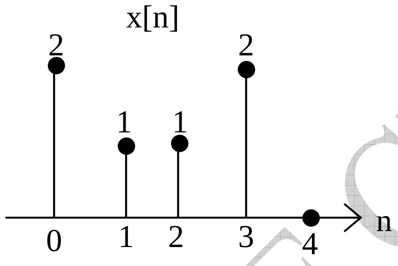
(C) $X[k] = N/2$ for all k

(D) $X[k] = \dfrac{1-W_N^{\ kN}}{1+W_N^{\ k}}$

9. Consider a finite length sequence x[n] = {1, 1, 1, 1, 1, 1} & its Z-transform is X(Z). If $X_1[k] = X(Z)\big|_{Z=e^{j(2\pi/4)k},\ k=0,1,2,3}$, then the inverse DFT of X_1 [k] is _______

(A) {1, 1, 2, 2} (B) {2, 2, 1, 1} (C) {1, 2, 1, 2} (D){2, 1,1, 1, 1, 2}

10. Consider the Signal x[n] shown in figure with its 5 point DFT X[k] Then the inverse DFT of $Y[k] = W_5^{-2k} X[k]$ is _______



(A) {1, 2, 0, 2, 1} (B) {2, 1, 0, 1, 2} (C) {0, 1, 2, 2, 1} (D) {1, 2, 2, 1}

11. In the implementation of N point FFT algorithm, the additions required are ___ per butterfly.

(A) 2 (B) 2^N (c) $N/2$ (d) 1

12. Let x[n] is having DFT X[k] = {1, j2, 4, –j2}then the DFT of the Zero – interpolated signal x[n/2] is ______

(A) {1, j2, 4, –22, 1, j2, 4, –j2}

(B) {1, j2,– j2, 4, 4, 1, 2, –j2}

(C) {1, j2, 4, –2j, 1, j2, 4, –j2}

(D) Same as X[k]

13. A Signal x(t) = 4 cos(100πt) is sampled at twice the Nyquist rate of 3 full periods. Then its DFT X[k] is_______

(A) X[3] = X[9] = 24

(B) X[1] = X[7] = 24

(C) X[0] = X[11] = 24

(D) X[k] = 24 ∀ k

14.A 3 sec Signal is sampled at f_s = 100 Hz. The maximum spectral spacing is to be 0.25 Hz. The number of samples required if we use radix-2 FFT algorithm are______

(A) 25 (B) 400 (C) 256 (D) 512

15. The DFT of a real signal is {1, A, –1, B, 0, –j2, C, –1 +j}. Then the value of C is_____

(A) –1 (B) j2 (C) –1 –j (D) 0

*** * THE END * ***

MSMF GATE CENTRE

Subject: Signals & Systems

DFT & FFT

SOLUTIONS

01. Ans: (b)

02. Ans: (c)

03. Ans: (a)

04. Ans: (a)

05. Ans: (a)

06. Ans: (d)

07. Ans: (a)

08. Ans: (a)

09. Ans: (b)

10. Ans: (a)

11. Ans: (a)

12. Ans: (c)

13. Ans: (a)
HINT: Sampling frequency is 200 HZ

Digital frequency $F = \frac{50}{200} = \frac{1}{4} = \frac{3}{12} = \frac{K}{N}$

N = 12 for 3 full periods.

$\therefore$ 2 non Zero values occurs at k = 3 & N-K = 9

14. Ans: (d)

HINT: For DFT $N = \frac{100}{0.25} = 400 \Rightarrow$ The next higher power of 2 is 512

15. Ans: (a)
HINT: Use $X[k] = X^*[N-K]$