

11. Given $2nu[n]$ Z.T $X(Z)$, then time signal corresponding to $S(Z) = X(-Z)$ is _____
 (a) $(-2)^n u[n]$ (b) $(-2)^n u[-n]$ (c) $2^{-n} u[n]$ (d) $2^{-n} u[-n]$

12. Given $X(Z) = \frac{Z^{10}}{\left(Z - \frac{1}{2}\right)\left(Z - \frac{3}{2}\right)^{10}\left(Z + \frac{3}{2}\right)^2\left(Z + \frac{5}{2}\right)\left(Z + \frac{7}{2}\right)}$. This system is stable.

Then $x[n]$ at $n = -8$ is _____

- (a) $\frac{1}{24}$ (b) $\frac{1}{48}$ (c) $\frac{1}{96}$ (d) $\frac{1}{12}$

13. The initial & final values of $x[n]$ if $X(z) = \frac{2Z\left(Z - \frac{5}{12}\right)}{\left(Z - \frac{1}{2}\right)\left(Z - \frac{1}{3}\right)}$, $|Z| > \frac{1}{2}$ is _____ respectively

- (a) 0 & 2 (b) 2 & 0 (c) 0 & 1 (d) 1 & 0

14. Let the impulse response of a discrete-time system be given by $h(n) = \frac{1}{\pi n} \sin\left(\frac{\pi n}{4}\right)$.

Then the output for an excitation of $\frac{1}{\pi n} \sin\left(\frac{\pi n}{8}\right)$ is _____

- (a) $\frac{1}{\pi n} \sin\left(\frac{\pi n}{4}\right)$ (b) $\frac{1}{\pi n} \cos\left(\frac{\pi n}{4}\right)$ (c) $\frac{1}{\pi n} \sin\left(\frac{\pi n}{8}\right)$ (d) $\frac{1}{\pi n} \cos\left(\frac{\pi n}{8}\right)$

15. Given $X(Z) = \log\left(\frac{1}{1 - az^{-1}}\right)$, $|z| > |a|$ then inverse Z.T is _____

- (a) $\frac{a^n}{n} u[n-1]$ (b) $\frac{a^{n+1}}{n} u[n-1]$ (c) $\frac{a^{n-1}}{n} u[n]$ (d) $\frac{a^{n-1}}{n} u[n-1]$

** THE END **

MSMF GATE CENTRE

Subject: Signals & Systems
DTFT & Z Transform

SOLUTIONS

01. Ans: (d)

$$\text{Hint: } x[n] = b^n u[n] + \left(\frac{1}{b}\right)^n u[-n-1]$$

$$\begin{array}{cc} \downarrow & \downarrow \\ |z| > |b| & |z| < \left|\frac{1}{b}\right| \end{array}$$

$$\therefore \text{Common ROC } |b| < |z| < \left|\frac{1}{b}\right|$$

02. Ans: (d)

Hint: All FIR filters will have linear phase response.

03. Ans: (b)

Hint: Z^{-3} represents minimum number of delay elements are 3.

04. Ans: (c)

05. Ans: (b)

$$\text{Hint: } n x[n] \stackrel{\text{Z.T}}{\leftrightarrow} \frac{-Zd}{dz} X(z)$$

06. Ans: (c)

$$\text{Hint: } H_1(Z) = \frac{Z^2 + 1.5Z - 1}{Z^2}$$

$$H_2(z) = Z^2 + 1.5z - 1$$

07. Ans: (a)

Hint: All the poles lie inside unit circle.

08. Ans: (b)

$$\text{Hint: } h[n] = a^n u[n], |a| < 1$$

$$H(z) = \frac{1}{1 - az^{-1}}, |z| > |a| \text{ where } |a| < 1.$$

\therefore Pole a is located between $z = 0$ and $z = 1$.

09. Ans: (d)

10. Ans: (a)

$$\text{Hint: } h[n] = \frac{1}{4}\delta[n-1] + \frac{1}{2}\delta[n] + \frac{1}{4}\delta[n+1]$$

$$H(Z) = \frac{1}{4}z^{-1}[1 + 2z + z^2]$$

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LPF

11. Ans: (a)

$$\text{Hint: } a^n x[n] \xrightarrow{Z.T} X\left(\frac{Z}{a}\right)$$

12. Ans: (c)

$$\text{Hint: } x[n] = \frac{1}{2\pi j} \oint X(Z)Z^{n-1}dZ$$

$$x[-8] = \Sigma [\text{residues of } X(Z)Z^{-9} \text{ inside } C]$$

$$= \frac{Z}{\left(Z - \frac{1}{2}\right)\left(Z - \frac{3}{2}\right)^{10}\left(Z + \frac{3}{2}\right)^2\left(Z + \frac{5}{2}\right)\left(Z + \frac{7}{2}\right)} \Big|_{Z = \frac{1}{2}}$$

13. Ans: (b)

$$\text{Hint: Initial value } x[0] = \lim_{Z \rightarrow \infty} X(Z)$$

$$\text{Final value } x[\infty] = \lim_{Z \rightarrow 1} (1 - Z^{-1})X(Z)$$

14. Ans: (c)

15. Ans: (a)

$$\text{Hint: } \log(1-r) = -\sum_{n=1}^{\infty} \frac{r^n}{n} \quad |r| < 1$$

$$X(Z) = -\log(1-aZ^{-1}), |Z| > |a|$$

$$= \sum_{n=1}^{\infty} \frac{1}{n} (aZ^{-1})^n = \sum_{n=1}^{\infty} \frac{1}{n} a^n Z^{-n}$$