MSMF GATE CENTRE

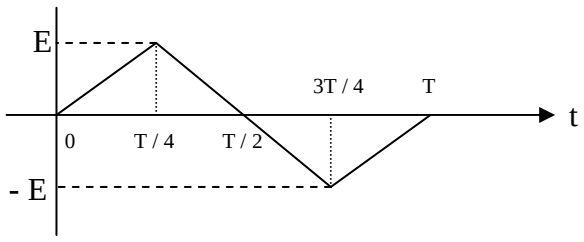
Subject: Signals & Systems

Fourier series & Transform

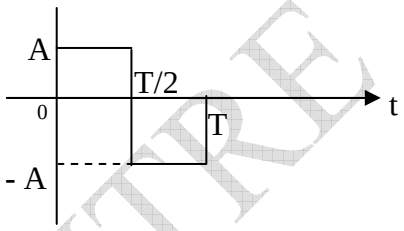
Time: 30 min **Marks= 15**

01. Which of the following periodic waveforms will have only odd harmonics of sinusoidal waveforms ?

1.



2.



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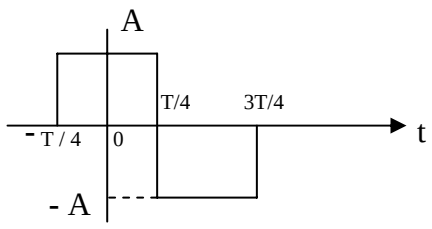

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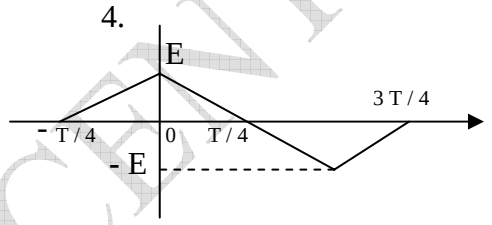


Select the correct answer using the codes given below **:**
Codes :

a) 1 and 2 b) 1 and 3 c) 1 and 4 d) 2 and 4

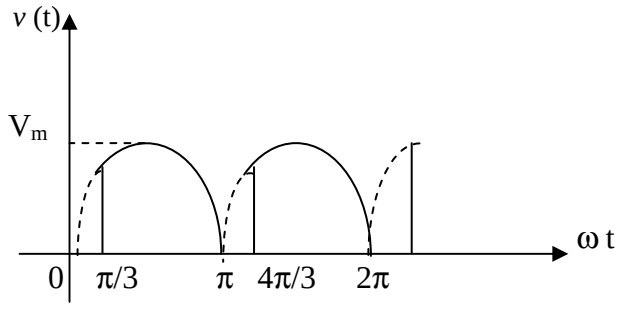
02. The average value of the delayed full – wave rectified sine wave as shown in the given figure is

a) $\frac{V_m}{\pi}$

b) $\frac{2\ V_m}{\pi}$

c) $\frac{3\ V_m}{2\ \pi}$

d) $\frac{V_m}{2\pi}$



03. A system is represented by the transfer function $\frac{10}{(s+1)(s+2)}$ The dc gain of this system is

a) 1 b) 2 c) 5 d) 10

04. Frequency response of the function T (s) = (s + 1) / (s + 2) exhibits a maximum phase at a frequency (in radian / sec)

a) 0 b) $1 / \sqrt{2}$ c) $\sqrt{2}$ d) ∞

05. Fourier transform F (j ω) of an arbitrary real signal has the property,

a) $F(j\omega) = F(-j\omega)$ b) $F(j\omega) = -F(-j\omega)$

c) $F(j\omega) = F^*(-j\omega)$ d) $F(j\omega) = -F^*(-j\omega)$

06. Match List I (Properties) with List II (Characteristics of the trigonometric form) in regard to Fourier series of periodic f (t) and select the correct answer using the codes given below the lists**:**

List I		**List II**
A.	$f(t) + f(-t) = 0$	1. Even harmonics can exist
B.	$f(t) - f(-t) = 0$	2. Odd harmonics can exist
C.	$f(t) + f(t - T/2) = 0$	3. The dc and cosine terms can exist
D.	$f(t) + f(t - T/2) = 0$ & $f(t) = f(-t)$	4. sine terms can exist 5. cosine terms of odd harmonics can exist

Codes :

	A	B	C	D
(a)	4	5	3	1
(b)	3	4	1	2
(c)	5	4	2	3
(d)	4	3	2	5

07. Given that the Fourier transform of f(t) is F (j ω), which of the following pairs of functions of time and the corresponding Fourier transforms are correctly matched ?

1. $f(t + 2)$ ….. $e^{j2\omega} F(j\omega)$
2. $f(-0.5\,t)$ ….. $2F(-2j\omega)$
3. $\int_{-\infty}^{t} f(t)\,dt$ …… $F(j\omega)\left(\frac{1}{j\omega} + \pi\,\delta(\omega)\right)$

Select the correct answer using the codes given below :

a) 1 and 2 b) 1 and 3 c) 2 and 3 d) 1, 2 and 3

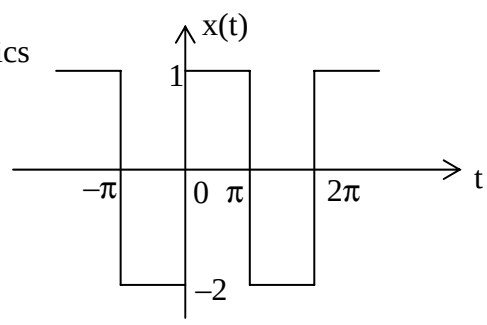
8. The periodic signal shown in figure contains _____

(a) d.c. & sine terms only for odd harmonics

(b) d.c. & cosine terms

(c) d.c. & sine terms

(d) None of these



9. A harmonic signal $x(t) = 3\sin(4t+20^o) - 4\cos(12t - 40^o)$. The phase of III harmonic is ___

(a) 180^o (b) 40^o (c) 140^o (d) None of these

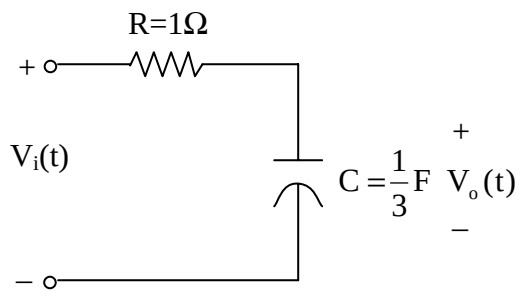
10. A continuous-time periodic signal x(t) is real valued & has a fundamental period T = 8 with Fourier series coefficient a_k.The non zero F.S. coefficients for x(t) are

$a_1 = a_{-1} = 2$, $a_3 = a_{-3} = 4j$. Then x(t) is TFS form is _____

(a) $4\cos\left(\frac{\pi t}{4}\right)+8\cos\left(\frac{3\pi t}{4}\right)$ (b) $4\cos\left(\frac{\pi t}{4}\right)+8\cos\left(\frac{3\pi t}{4}+\frac{\pi}{2}\right)$

(c) $4\cos\left(\frac{\pi t}{8}\right)+8\cos\left(\frac{3\pi t}{8}\right)$ (d) None

11 A periodic voltage waveform $V_i(t) = 4 - \sum_{n=-\infty}^{\infty} \frac{10}{n}\sin(3nt)$ volts is applied to RC circuit shown in figure the amplitude of first harmonic is _____

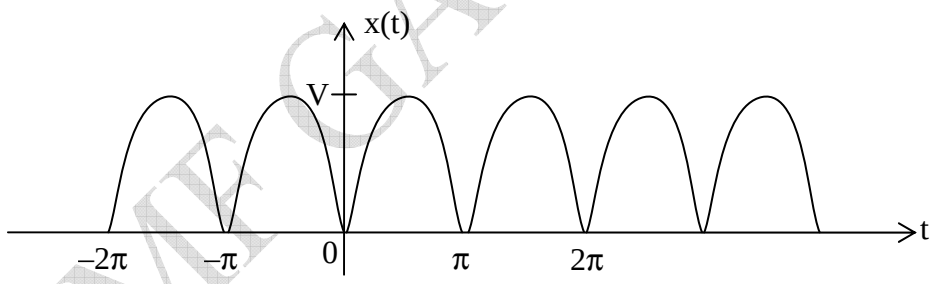


(a) 7.07 (b) 2.2361 (c) 4 (d) 1.054

12. Given the periodic signal $x(t) = \sum_{\substack{n=-\infty \\ (n\ even)}}^{+\infty} u(t-3n) - u(t-3n-1)$. The d.c. component of this signal is ____

(a) $\frac{1}{6}$ (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) None

13. The exponential F.S. coefficient for the periodic signal shown in figure is ___ ($x(t) \rightarrow C_n$)



(a) $-\frac{2V}{\pi(4n^2-1)}$ (n even) (b) $\frac{2}{\pi(1-4n^2)}$ (n even)

(c) $\frac{V}{\pi(4n^2+1)}$ (n even) (d) $\frac{V}{\pi(1-4n^2)}$ (n even)

14 The EFS of a signal is $x(t) = \sum_{n=-\infty}^{+\infty} \frac{1}{1+j\pi n} e^{\frac{j3\pi t}{2}}$. The fundamental period is _____

(a) $3\pi/2$ (b) 4/3 (c) 3/4 (d) None

15. Consider a C.T.LTI system with frequency response $H(\omega) = \begin{cases} 1; |\omega| \le 250 \\ 0; |\omega| > 250 \end{cases}$.

When the input to this system is a signal x(t) with fundamental period $\pi/7$ & F.S. coefficient a_K & $y(t) \rightarrow x(t)$. For what values of K it is guaranteed that $a_K = 0$?

(a) $|K| \ge 18$ (b) $|K| > 19$ (c) $|K| > 17$ (d) $|K| \ge 17$

** THE END **

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SOLUTIONS

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01. Ans: (a)

Hint: Only for odd and halfwave symmetry signal b_n exists for odd 'n'
for even and half wave symmetry signals a_n exixts for odd n.

02. Ans: (c)

Hint: $V_{avg} = \frac{1}{T}\int_0^T f(t)dt = \frac{1}{\pi}\int_{\frac{\pi}{3}}^{\pi} \sin \omega t \, d\omega t$

03. Ans: (c)

Hint: For DC gain put S = 0 or ω = 0

04. Ans: (c)

Hint: $T(s) = \frac{1+s\tau}{1+\alpha s\tau}$

Max phase occurs at $\omega_m = \frac{1}{\tau\sqrt{\alpha}}$

05. Ans: (c)

Hint: F(ω) = F*(-ω) → Property of conjugate symmetry (or) even conjugate

06. Ans: (d)

Hint: A. f(t) = -f(-t) ⇒ oddsignal ; 'b_n' exists ; sine terms present
B. f(t) = f(-t) ⇒ even signal ; a_0 and a_n exists ; DC & cosine terms exists.
C. f(t) = -f(t-T/2) ⇒ Half wave symmetry ; a_n, b_n exists for 'n' odd.
D. f(t) = f(-t) & f(t) = -f(t-T/2) ⇒ even & half wave symmetry ; 'a_n' exists for odd 'n'; cosine terms of odd Harmonics can exists.

07. Ans: (d)

Hint: 1. $x(t-t_0) \leftrightarrow e^{-j\omega t_0}\, x(\omega)$

2. $x(at) \leftrightarrow \frac{1}{|a|}x(\omega/a)$

3. $\int_{-\infty}^{t} x(\tau)d\tau \leftrightarrow \frac{x(\omega)}{j\omega} + \pi x(0)\delta(\omega)$

08. Ans: (a)

Hint: The given signal contains –Ve dc of '1' and odd symmetry.

09. Ans: (c)

Hint: The –Ve real amplitude of IIIrd harmonic gives ± 180^0 phase
∴ Total phase =180^0 phase + actual given phase

10. **Ans:** (b)

Hint: For T = 8 $\rightarrow$ $\omega_0 = \frac{\pi}{4}$

From given F.S coefficients $a_3 = a_{-3} = 4j$

Here 'j' represents $\frac{\pi}{2}$ phase addition at $3\omega_0$

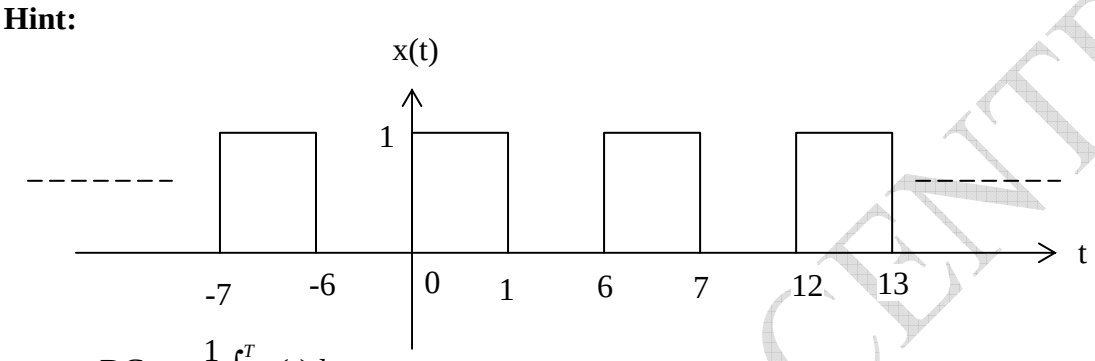
11. **Ans:** (a)

Hint: $V_o(t) = \frac{1}{1 + j\omega RC} \left(4 - \sum_{n=-\infty}^{\infty} \frac{10}{n} \sin(3nt) \right)$

1^{ST} harmonic amplitude for n = 1 is $\frac{10}{\sqrt{1 + \omega^2 R^2 C^2}}$

12. **Ans:** (a)

Hint:



DC = $\frac{1}{T}\int_0^T x(t)dt$

$= \frac{1}{6}\int_0^1 1dt = \frac{1}{6}$

13. **Ans:** (a)

Hint: F.S Coefficients $c_n = \frac{1}{T}\int_0^t x(t)\, e^{-jn\omega_0 t}\, dt$

$= \frac{1}{\pi}\int_0^{\pi} V \sin t\, e^{-jn2t}\, dt$

14. **Ans:** (b)

Hint: $x(t) = \sum_{n=-\infty}^{+\infty} \frac{1}{1 + j\pi n} e^{\frac{j3\pi t}{2}} = \sum_{n=-\infty}^{+\infty} c_n e^{jn\omega_0 t}$

$\omega_0 = \frac{3\pi}{2} \Rightarrow T = \frac{4}{3}$

15. **Ans:** (c) from k = 0 to 17

Hint: Given C.T.L.T.I system allows i/p components from 0 to 250

$T = \frac{\pi}{7} \rightarrow \omega_0 = 14$

$\omega_0, 2\omega_0$ ----------$17\omega_0$ components are allowed

$\therefore a_k = 0$ for $|k| > 17$.