MSMF GATE CENTRE

Subject: Signals & Systems

Laplace Transform

Time: 30 min **Marks= 15**

01. The final value of $L^{-1}\ \dfrac{2S+1}{S^4+8S^3+16S^2+S}$ is ________

(a) infinity (b) 2 (c) 1 (d) zero

02. If x(t) and $\dfrac{dx(t)}{dt}$ are Laplace transformable and $\lim\limits_{t\to\infty} x(t)$ exists, then $\lim\limits_{t\to\infty} x(t)$ is equal to ___

(a) $\lim\limits_{S\to\infty} S\,X(S)$ (b) $\lim\limits_{S\to 0} S\,X(S)$ (c) $\lim\limits_{S\to\infty} \dfrac{X(S)}{S}$ (d) $\lim\limits_{S\to 0} \dfrac{X(S)}{S}$

03 The inverse Laplace transform of $\left(\dfrac{s+3}{s^2+9}\right)e^s$ is _______

(a) cos 3 (t+1) + 3 sin 3(t-1) (b) sin 3(t+1)+3 cos 3(t-1)
(c) sin 3 (t+1) + cos 3(t+1) (d) sin 3 (t-1)+ cos 3 (t-1)

04. The final value of $F(s)=\dfrac{1}{(s+j)(s-j)}$ is ________

(a) 1 (b) 0 (c) -1 (d) Not defined

05 The Laplace transform of $e^{-t}\,u(t-\tau)$ is _________

(a) $\dfrac{e^{-\tau}}{s+1}$ (b) $\dfrac{e^{-s}}{s(s+1)}$ (c) $\dfrac{e^{-(s+1)\tau}}{s+1}$ (d) $\dfrac{e^{-(s-1)\tau}}{s+1}$

06. The impulse response of a first order system is $K\,e^{-2t}$. If the input signal is sin 2t, then the steady state response will be given by

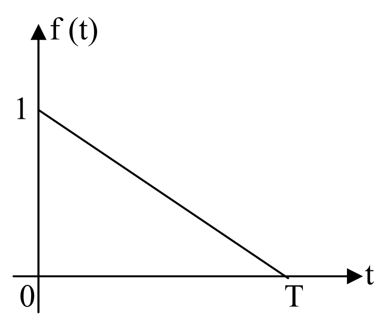
(a) $\dfrac{1}{2\sqrt{2}}\,\text{Sin}\left(2t+\dfrac{\pi}{4}\right)$ (b) $\dfrac{1}{4}\,\text{Sin}\,2t$

(c) $\dfrac{K}{2\sqrt{2}}\,\text{Sin}\left(2t-\dfrac{\pi}{4}\right)$ (d) $\dfrac{1}{2\sqrt{2}}\,\text{Sin}\left(2t-\dfrac{\pi}{4}\right)+Ke^{-2t}$

07. Which one of the following is the correct Laplace transform of the signal in the given figure ?

(a) $\dfrac{1}{TS^2}\left[1-e^{-TS}(1+TS)\right]$

(b) $\dfrac{1}{TS^2}\left[e^{-TS}-1+TS\right]$



(c) $\frac{1}{TS^2}\left[e^{-TS} + 1 - TS\right]$

(d) $\frac{1}{TS^2}\left[1 - e^{-TS} + TS\right]$

08. Which one of the following is the response y (t) of a causal LTI system described by $H(S) = \frac{(S+1)}{S^2 + 2S + 2}$ for a given input x (t) = e^{-t} u (t) ?

(a) y (t) = e^{-t} sin t u(t)
(b) y (t) = $e^{-(t-1)}$ sin (t – 1) u (t – 1)
(c) y (t) = sin (t – 1) u (t – 1)
(d) y (t) = e^{-t} cos t u(t)

09. Which of following represent a stable system ?

1. Impulse response of the systems decreases exponentially
2. Area within the impulse response is finite
3. Eigen values of the system are positive and real
4. Roots of the characteristic equation of the system are real and negative

Select the correct answer using the codes given below :

Codes :

(a) 1 and 4 (b) 1 and 3 (c) 2, 3 and 4 (d) 1, 2 and 4

10. If input – output relation is given by $y(t) = x(t) - 2\int_{-\infty}^{t} y(\lambda)e^{-(t-\lambda)}u(t-\lambda)d\lambda$ then impulse response is given by ________

(a) -3e^{-3t}u(t) (b) δ(t)-2 e^{-3t}u(t) (c) 2δ(t)+ e^{-3t}u(t) (d) e^{-3t}u(t)

11. Given $L\{x(t)\} = \frac{se^{-2s} + 1}{(s+1)(s+2)}$ then x(t) is ________

(a) $[e^{-(t-2)} - e^{-(t-1)}]u(t) + \left(e^{-t} - e^{-2t}\right)u(t)$
(b) $[2e^{-2(t-2)} - e^{-(t-2)}]u(t-2) + (e^{-t} - e^{-2t})u(t)$
(c) $[e^{-2(t-2)} + e^{-(t-2)})0]u(t-2) + (e^{-t} - e^{-2t})u(t)$
(d) $[2e^{-2(t-2)} - e^{-(t-2)}]u(t) + (e^{-t} - e^{-2t})u(t)$

12. The minimum-phase transfer function corresponding to $|H(\omega)|^2 = \frac{4(9+\omega^2)}{4 + 5\omega^2 + \omega^4}$ is ______

(a) $\frac{2(s+3)}{(s+1)(s+2)}$ (b) $\frac{4(s+3)}{(s+1)(s+2)}$ (c) $\frac{4(s+3)}{(s+1)(s-2)}$ (d) $\frac{4(s+3)}{(s-1)(s+2)}$

13. The inverse Laplace transform of $x(s) = \frac{4s^2 + 15s + 8}{(s+2)^2(s-1)}$ if the Fourier transform of x(t) exists is __

(a) $e^{-2t}u(t)+2te^{-2t}u(t)-3e^{t}u(-t)$
(b) $e^{-2t}u(-t)-2te^{-2t}u(t)+3e^{t}u(t)$
(c) $-e^{-2t}u(t)-2te^{-2t}u(-t)+3e^{t}u(-t)$
(d) None of these

14. Given cos (2t) u(t) ↔ x(s), then time-domain signal corresponding to x(2s) is ______

(a) cos 2t u(t-1) (b) $\frac{1}{2}\cos t u(t)$ (c) cos t u(t) (d) $\cos\left(\frac{t}{2}\right)u(t)$

15. A system is represented by $\frac{dy}{dx} + 2y(t) = 4t\,u(t)$ the ramp component in the forced response will be (a) t u(t) (b) 2t u(t) (c) $-u(t) + 2tu(t) + e^{-2t}u(t)$ (d) 4t u(t)

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SOLUTIONS

01. Ans : (c)

Hint : Final value theorem $x(\infty) = \lim_{s\to 0} SX(S)$

$$= \lim_{S\to 0} \frac{2S+1}{S^3 + 8S^2 + 16S + 1}$$

= 1

02. Ans : (b)

Hint : Final value theorem : $\lim_{t\to\infty} x(t) = \lim_{S\to 0} S\,X(S)$

03. Ans : (c)

Hint : $\cos \omega_0 t \xleftrightarrow{L.T} \frac{S}{S^2 + \omega_0^2}$

$\sin \omega_0 t \xleftrightarrow{L.T} \frac{\omega_0}{S^2 + \omega_0^2}$

Shift in time – domain : $x(t - t_0) \xleftrightarrow{L.T} X(S)\, e^{-st_0}$

04. Ans : (d)

Hint : Final value theorem can't be applied if poles lie on imaginary axis.
Final value theorem is valid of all poles have negative real parts except a simple pole at origin.

05. Ans : (c)

Hint : $e^{-(t-\tau+\tau)}\, u(t-\tau)$

Use time shifting property of Laplace transform to above function.

06. Ans : (c)

Hint : If input = A Sin ($\omega t + \theta$)

calculate $H(S)\big|_{S=j\omega_0} = K\angle\phi$

Steady state output $y_{s.s}(t) = KA \sin(\omega t + \theta + \phi)$

07. Ans : (b)

Hint : $f(t) = \left(-\frac{t}{T} + 1\right)\left[u(t) - u(t-T)\right]$

To get F (S) use shift in time domain property of Laplace transform

08. Ans : (a)

Hint : Y (S) = X (S) H (S)

$$= \frac{1}{(S+1)^2 + 1}$$

$\therefore y(t) = e^{-t} \sin t\, u(t)$

Use $\sin \omega_0 t \xleftrightarrow{L.T} \dfrac{\omega_0}{S^2+\omega_0^2}$ and $x(t)e^{S_0 t} \xleftrightarrow{L.T} X(S-S_0)$

09. Ans : (d)

10. Ans : (b)

Hint : In above given question y (t) convolved with e^{-t} u (t)

Convolution of 2 signals in time domain equal to multiplication of their Laplace transforms in S – domain

→ In transfer function make improper Laplace function to proper Laplace function and apply inverse Laplace transform

11. Ans : (b)

Hint : $X(S) = e^{-2S}\left[\dfrac{-1}{S+1}+\dfrac{2}{S+2}\right]+\left[\dfrac{1}{S+1}-\dfrac{1}{S+2}\right]$

Apply shift in S domain property of Laplace transform to above X (S) to get x (t)

12. Ans : (a)

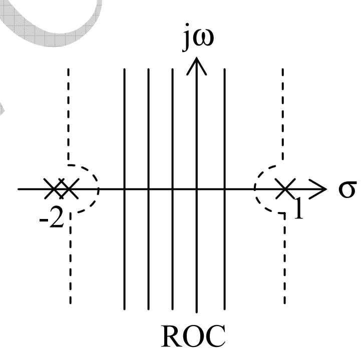
Hint : $|H(\omega)|^2 = H(j\omega)\, H^*(j\omega)$

For minimum phase function all poles lie in left half of S – Plane

13. Ans : (a)

Hint : 2 poles lie at -2 & 1 pole lie at 1

$$X(S) = \frac{2}{(S+2)^2}+\frac{1}{(S+2)}+\frac{3}{S-1}$$



If Fourier transform exists ROC must include jω axis

For pole at -2 ROC is right sided

For pole at 1 ROC is left sided

$\therefore$ x (t) = $e^{-2t}u(t)+2te^{-2t}u(t)-3e^{t}u(-t)$

14. Ans : (b)

Hint : $x(at) \xleftrightarrow{L.T} \dfrac{1}{|a|}\, x\left(\dfrac{S}{a}\right)$

For X (2S) Here $a = \dfrac{1}{2}$

$\cos t\, u(t) \xleftrightarrow{L.T} 2X(2S)$

$X(2S) = \dfrac{1}{2}\cos t\, u(t)$

15. Ans : (b)

Hint : $Y(S)(S+2) = \dfrac{4}{S^2}$

$$Y(S) = \frac{4}{S^2(S+2)}$$

$$= \frac{-1}{S}+\frac{2}{S^2}+\frac{1}{(S+2)}$$

$$= -u(t)+2tu(t)+e^{-2t}\,u(t)$$

$\therefore$Ramp component = 2t u (t)