G.PULLAIAH COLLEGE OF ENGINEERING & TECHNOLOGY (AT)

I B.Tech-SEM II-MID Exam (Descriptive) 2016-17

(Common to all Branches) **SET NO-I**

Sub: MATHEMATICS-I **Date: 22/11/2016**

Time: 90mins **Max.Marks: 30**

PART – A

ANSWER ALL QUESTIONS.EACH QUESTION CARRY EQUAL MARKS **6 X 1 = 6M**

1. A) If $x = r\cos\theta\,, y = r\sin\theta$ then find $\frac{\partial(x,y)}{\partial(r,\theta)}$

B) Find Radius of curvature of the curve $y = \cosh\frac{x}{c}$ at any point

C) Evaluate $\int_0^1\int_0^1\int_0^1 e^{x+y+z}dxdydz$

D) Evaluate $\int_0^\infty\int_0^\infty e^{-(x^2+y^2)}\,dxdy$

E) Prove that $\nabla(r^n) = nr^{n-2}\bar{r}$

F) Find the directional derivative of $f = xy + yz + zx$ in the direction of vector $\bar{\imath} + 2\bar{\jmath} + 2\bar{k}$ at the point (1,2,0)

PART – B

ANSWER ALL QUESTIONS **3 x 8= 24M**

2. A) Discuss Maxima & Minima values of $\sin x\sin y\sin(x+y)$ where 0<x<π, 0<y< π

B) Find the Radius of curvature of $x = a(\theta+\sin\theta), y = a(1-\cos\theta)$ at any point and at $\theta = \frac{\pi}{2}$

(OR)

3. A) Find maximum of $u = x^2y^3z^4$, if $2x + 3y + 4z = a$

B) If $x = r\sin\theta\cos\emptyset\,, y = r\sin\theta\sin\emptyset\,, z = r\cos\theta$, Show that $\frac{\partial(x,y,z)}{\partial(r,\theta,\emptyset)} = r^2\sin\theta$ and also find $\frac{\partial(r,\theta,\emptyset)}{\partial(x,y,z)}$

4. A) By changing into polar coordinates, evaluate $\iint\frac{x^2y^2}{x^2+y^2}dxdy$ over the annular region between the circles $x^2 + y^2 = a^2$ and $x^2 + y^2 = b^2$ (b>a).

B) Evaluate $\int_1^e\int_1^{\log y}\int_1^{e^x}\log z\,dx\,dz\,dy$

(OR)

5. By changing the order of integration, evaluate $\int_0^3\int_1^{\sqrt{4-y}}(x+y)dxdy$

6. A) Evaluate The line integral $\int(x^2+xy)\,dx + (x^2+y^2)dy$ where c is the square formed by the lines $x = \pm1, y = \pm1$

B) Evaluate $\nabla.\left(\frac{\bar{r}}{r^3}\right)$ where $\bar{r} = x\bar{\imath} + y\bar{\jmath} + z\bar{k}$

(OR)

7. Verify Stoke's theorem for $\bar{F} = (x^2+y^2)\bar{\imath} - 2xy\bar{\jmath}$ taken round the rectangle bounded by the lines $x = \pm a\,, y = 0, y = b$

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I B.Tech-SEM II-MID Exam (Descriptive) 2016-17

(Common to all Branches) **SET NO-2**

Sub: MATHEMATICS-I **Date: 22/11/2016**

Time: 90mins **Max.Marks: 30**

PART – A

ANSWER ALL QUESTIONS.EACH QUESTION CARRY EQUAL MARKS **6 X 1 = 6M**

1. A) Obtain Maclaurins Series expansion $\log_e(1+x)$ in powers of x

B) Find Radius of curvature of the curve $y = \cosh\frac{x}{c}$ at any point

C) Evaluate $\int_{-c}^{c}\int_{-b}^{b}\int_{-a}^{a}(x^2+y^2+z^2)dxdydz$

D) Evaluate $\int_0^5\int_0^{x^2} x(x^2+y^2)\,dx\,dy$

E) Evaluate the angle b/w the normals to the surface $xy = z^2$ at the points $(4,1,2) and (3,3,-3)$

F) Define Gauss theorem.

PART – B

ANSWER ALL QUESTIONS **3 x 8= 24M**

2. A) Find minimum of $x^2+y^2+z^2$, given that $x + y + z = 3a$

B) For the cardioid $r = a(1+\cos\theta)$, prove that $\frac{\rho^2}{r}$ is constant where ρ is the radius of curvature.

(OR)

3. A) Show that $\log(1+e^x) = \log 2 + \frac{x}{2} + \frac{x^2}{8} - \frac{x^4}{192} + \cdots$ and hence deduce that $\frac{e^x}{e^x+1} = \frac{1}{2} + \frac{x}{4} - \frac{x^3}{48} + \cdots$

B) If $u = \frac{x+y}{1-xy}$ and $\theta = \tan^{-1}x + \tan^{-1}y$, find $\frac{\partial(u,\theta)}{\partial(x,y)}$

4. Change the order of integration evaluate $\int_0^a\int_{x/a}^{\sqrt{x/a}}(x^2+y^2)\,dxdy$

(OR)

5. A) Evaluate the following integral by transforming into polar co-orindates $\int_0^a\int_0^{\sqrt{a^2-x^2}} y\sqrt{x^2+y^2}dxdy$

B) Evaluate $\int_0^1\int_0^{\sqrt{1-x^2}}\int_0^{\sqrt{1-x^2-y^2}}\frac{1}{\sqrt{1-x^2-y^2-z^2}}dz\,dy\,dx$

6. A) Show that curl$(r^n\bar{r}) = 0$

B) If $\bar{F} = (5xy-6x^2)\bar{\imath} + (2y-4x)\bar{\jmath}$, Evaluate $\int\bar{F}.\overline{dr}$, Where C is the curve in the XY-plane $y = x^3$ from (1,1) to (2,8)

(OR)

7. Evaluate by Green's theorem $\oint(y-\sin x)dx + \cos x\,dy$ where C is the triangle enclosed by the lines $y = 0, x = \frac{\pi}{2}, \pi y = 2x$

G.PULLAIAH COLLEGE OF ENGINEERING & TECHNOLOGY (AT)
I B.Tech-SEM II-MID Exam (Descriptive) 2016-17
(Common to all Branches) **SET NO-3**

Sub: MATHEMATICS-I **Date: 22/11/2016**
Time: 90mins **Max.Marks: 30**

PART – A

ANSWER ALL QUESTIONS.EACH QUESTION CARRY EQUAL MARKS **6 X 1 = 6M**

1 A) If $x = u(1 - v), y = uv$ Prove that $JJ' = 1$
B) Find Radius of Curvature at any point of the curve $xy = c^2$ at any point.
C) Evaluate $\int_0^{\pi/2} \int_{a(1-\cos\theta)}^{a} r^2 dr\, d\theta$
D) Evaluate $\int_0^1 \int_0^1 \frac{dx\, dy}{\sqrt{(1-x^2)(1-y^2)}}$
E) Prove that Curl $\overline{(r)} = 0$
F) Define Stoke's Theorem

PART – B

ANSWER ALL QUESTIONS **3 x 8= 24M**

2. A) Discuss Maxima & Minima of $x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x$
B) If $x = a\cosh\theta\cos\emptyset , y = a\sinh\theta\sin\emptyset$, show that $\frac{\partial(x,y)}{\partial(\theta,\emptyset)} = \frac{1}{2}a^2(\cosh 2\theta - \cos 2\emptyset)$

(OR)

3. A) For the cardioid $r = a(1 + \cos\theta)$, prove that $\frac{\rho^2}{r}$ is constant where ρ is the radius of curvature
B) Find the point on the plane $x + 2y + 3z = 4$ that is closest to the origin
4. A) Evaluate $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)}\, dxdy$ by changing the polar co-ordinates.
B) Evaluate $\int_0^{\log 2} \int_0^x \int_0^{x+\log y} e^{x+y+z}\, dz\, dy\, dx$

(OR)

5. Change the order of integration solve $\int_0^a \int_{x^2/a}^{2a-x} xy^2 dydx$
6. Evaluate by Green's theorem $\oint(y - \sin x)dx + \cos x\, dy$ where C is the triangle enclosed by the lines $y = 0, x = \frac{\pi}{2}, \pi y = 2x$

(OR)

7. Use Divergence theorem to evaluate $\iint \bar{F}.\bar{N}ds$ where $\bar{F} = 4x\bar{\imath} - 2y^2\bar{\jmath} + z^2\bar{k}$ and 'S' is the surface bounded by the region $x^2 + y^2 = 4, z = 0$ & $z = 3$

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I B.Tech-SEM II-MID Exam (Descriptive) 2016-17
(Common to all Branches) **SET NO-4**

Sub: MATHEMATICS-I **Date: 22/11/2016**
Time: 90mins **Max.Marks: 30**

PART – A

ANSWER ALL QUESTIONS.EACH QUESTION CARRY EQUAL MARKS **6 X 1 = 6M**

1. A) If $x = r\cos\theta , y = r\sin\theta$ then find $\frac{\partial(x,y)}{\partial(r,\theta)}$
B) Find Radius of curvature of the curve $y = \cosh\frac{x}{c}$ at any point
C) Evaluate $\int_{-c}^{c} \int_{-b}^{b} \int_{-a}^{a} (x^2 + y^2 + z^2)dxdydz$
D) Evaluate $\int_0^5 \int_0^{x^2} x(x^2 + y^2)\, dx\, dy$
E) Prove that Curl $\overline{(r)} = 0$
F) Define Stoke's Theorem

PART – B

ANSWER ALL QUESTIONS **3 x 8= 24M**

2. A) Discuss Maxima & Minima values of $\sin x \sin y \sin(x + y)$ where 0<x<π, 0<y< π
B) Find the Radius of curvature of $x = a(\theta + \sin\theta), y = a(1 - \cos\theta)$ at any point and at $\theta = \frac{\pi}{2}$

(OR)

3. A) Show that $\log(1 + e^x) = \log 2 + \frac{x}{2} + \frac{x^2}{8} - \frac{x^4}{192} + \cdots$ and hence deduce that $\frac{e^x}{e^x+1} = \frac{1}{2} + \frac{x}{4} - \frac{x^3}{48} + \cdots$
B) If $u = \frac{x+y}{1-xy}$ and $\theta = \tan^{-1} x + \tan^{-1} y$, find $\frac{\partial(u,\theta)}{\partial(x,y)}$
4. A) Evaluate $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)}\, dxdy$ by changing the polar co-ordinates.
B) Evaluate $\int_0^{\log 2} \int_0^x \int_0^{x+\log y} e^{x+y+z}\, dz\, dy\, dx$

(OR)

5. Find the volume of largest rectangular parallelopiped that can be inscribed in the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$
6. Verify Stoke's theorem for $\bar{F} = (x^2 + y^2)\bar{\imath} - 2xy\bar{\jmath}$ taken round the rectangle bounded by the lines $x = \pm a , y = 0, y = b$

(OR)

7. A) Show that curl$(r^n\bar{r}) = 0$
B) If$\bar{F} = (5xy - 6x^2)\bar{\imath} + (2y - 4x)\bar{\jmath}$, Evaluate$\int \bar{F}.\overline{dr}$, Where C is the curve in the XY-plane$y = x^3$ from (1,1) to (2,8)