1

Structural Analysis of st's deals with the eq^{bm} of solid bodies basically subjected to stationary loads.

S.A involves the determination of forces developed in various components of str for the given loading.

Types of st^r

St's are classified in several ways depending upon the parameters involved;

i) structural action → Axial → Tensile & comp
→ M → Bending & Torsion

ii) Structural geometry.

iii) structural loading.

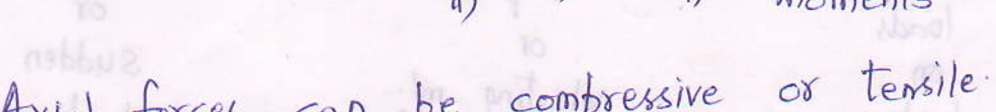
iv) structural systems.

Structural action : There are 2 types of structural elements, these
i) subjected to axial forces (AF)
ii) " " moments

Axial forces can be compressive or tensile.

Moments can be bending or Torsional.

Some c/s used to resist these forces are;



A member can be subjected AF+M; AF+TM or AF+BM+TM.

ii) Structural Geometry

One Dimensional form ; 2D form ; & 3D forms

Structures

- Linear (1-D)
  - Straight
  - Curved
- Plane (2-D)
  - Planar
  - Curved
- Space (3-D)
  - Coupled
  - Thick elements

Examples:

Straight	Curved	Planar	Curved	Coupled	Thick elements
Beam	Arch	Wall	Membrane	space frame	Foundation
column	cable	Plate	Shell	space truss	Dam
strut	Beam curved in plan	Grid	Cable network	Folded plate	Pressure vessel
Truss					

iii) Structural loading

St^r are usually subjected to gravity (or self) loading & superimposed (or external) loading.

Superimposed loads

Static	Moving	Dynamic	Impact
stationary loads fixed in position such as furniture and Partition walls	Vehicular loads on bridges	Earthquake or vibrating m/c	Explosion or sudden loading

iv) Structural systems

a) Statically determinate

b) Statically indeterminate

c) kinematically determinate

d) kinematically indeterminate

SA of st^r is based on – eq^{bm} of forces
– compatibility of displacements.

Statically determinate st^r : Can be analysed using conditions of static eq^{bm} alone.

Statically indeterminate st^r
(or)
Indeterminate st^r
(or)
Hyperstatic st^r
(or)
Redundant st^r

} Eq^{ns} of eq^{bm} alone are inadequate to analyse

Eq^{ns} of eq^{bm} + compatability conditions of displacements with supports

Degree of indeterminancy
(or)
Degree of Redundancy (D·O·R)

} No·of additional eq^{ns} necessary for solving hyperstatic st^{rs}.
DOR indicates no·of eq^{ns} req^d to analyse a st^r in addition to those provided by conditions of static eq^{bm}.

Kinematic indeterminacy
(or)
Degree(s) of freedom (D·O·F)

} A st^r may be capable of movements (displacements and rotations) at its joints and supports
Total no·of such possible movements in a st^r is known as D·O·F.

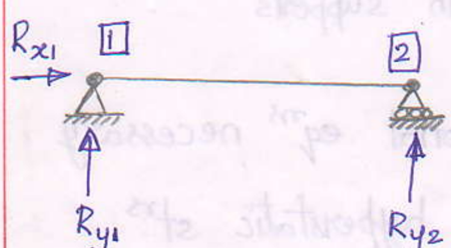
Static Indeterminacy (or) DOR : It can be visualised in terms of support reactions and internal member forces. In the former case it is termed external redundancy; and in the latter case internal redundancy.

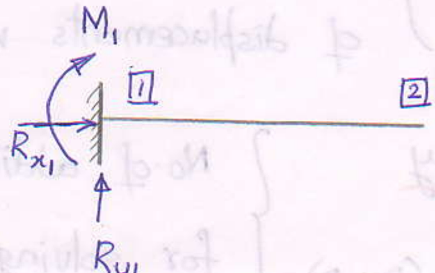
Degree of external redundancy : No. of additional eq^{ns} req^d to determine the external forces i.e. support reactions.

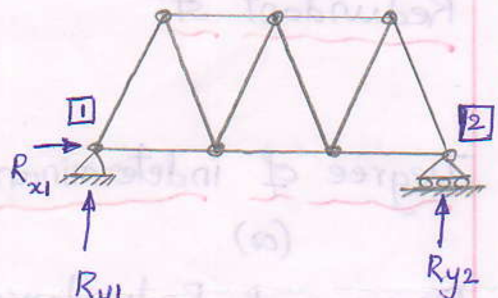
Degree of internal redundancy : No. of additional eq^{ns} req^d to determine the internal (member) forces.

Total DOR : External DOR + Internal DOR.

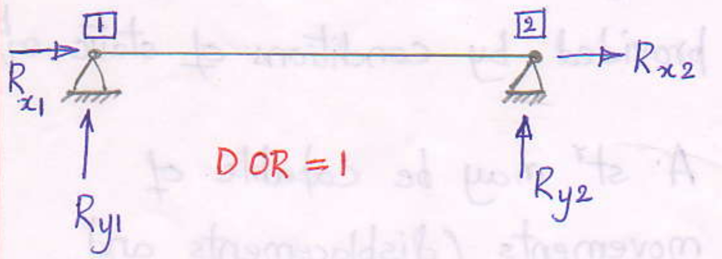
Statically determinate structures [1 & 2 are supports]

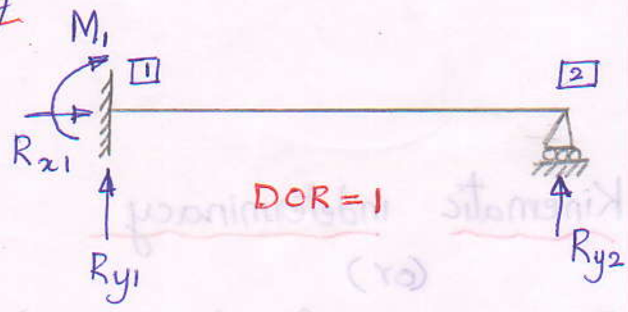


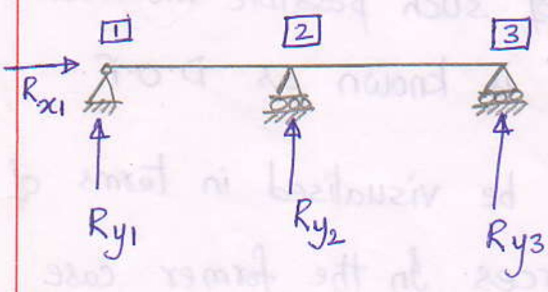




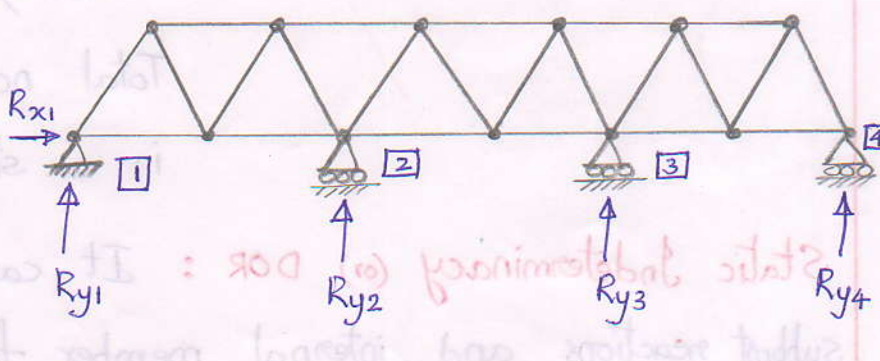
Structures with external indeterminacy





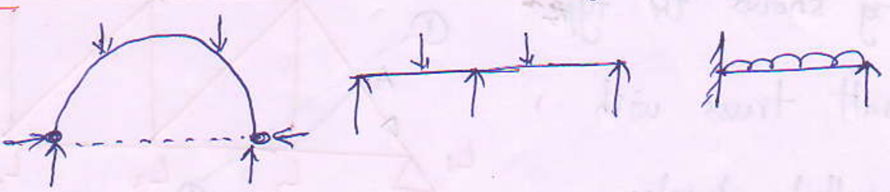


DOR = 1



DOR = 2

Statically Indet st^r : which can not be analysed by conditions of Eq^{bm}. Ex :



Stati det st^r	Stat. indet. st^r
1) Conditions of eq^{bm} are sufficient to analyse	Not sufficient
2) BM @ a section or force in any member is independent of material of components.	Depends
3) BM @ a section or force is independent of c/s area of components	Depends
4) No stresses are caused due to temp changes	are caused.

Second theorem of Castigliano's (or) (Principle of least work) : In any case of statical indetermination, where no. of redundant forces satisfy the conditions of eq^{bm}; their actual values are those that ~~render~~ cause the total S.E stored to a min^m.

$$\frac{\partial W_i}{\partial X} = 0$$

IL for Pratt truss with parallel chords.

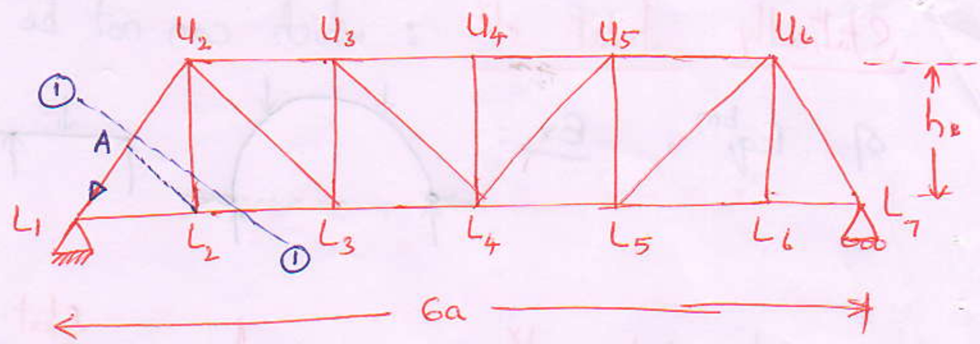


Fig shows the type pratt truss with parallel chords. of panels each of length 'a' and of height 'h'.

Let unit load roll from left to right

Top members → compr

Bottom chord members → Ten.

IL for L_1U_2

Pass section 1-1 cutting the member L_1U_2 as shown in fig.

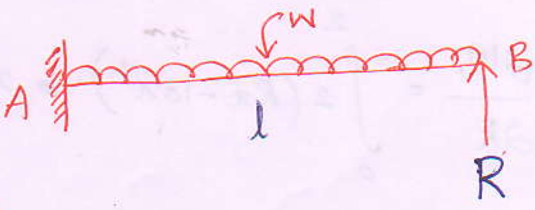
Let unit load move from L_1 to L_7

Assume dirn of force in L_1U_2 say compr.

T.M ab. opposite joint;

$$V_1 \times a = f_{L_1U_2} \times AL_2 \Rightarrow f_{L_1U_2} = \frac{V_1 \times a}{AL_2} = \frac{V_1 \times a}{\frac{a \times h}{\sqrt{a^2+h^2}}}$$

Prob: Find the reaction @ S.S. end (or) propped end using Castig 2nd theorem or principle of least work. (4)



Sol^n Let R = reaction @ prop.

BM @ any section x from B $\Big\}$ $M_x = Rx - \frac{wx^2}{2}$

S.E stored by beam $W_i = \int \frac{M^2 \cdot dx}{2EI}$

$$W_i = \int_0^l \left(Rx - \frac{wx^2}{2}\right)^2 \frac{dx}{2EI}$$

From 2nd theorem of Castigliano; reaction 'R' shall have a value that the S.E stored is a minimum.

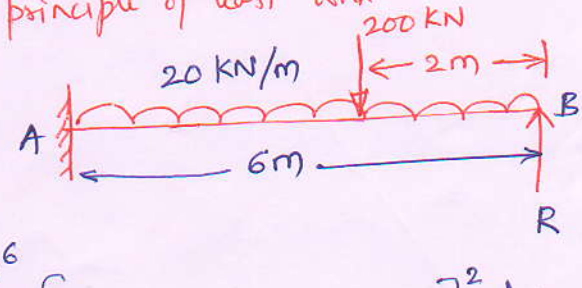
i.e. $\frac{\partial W_i}{\partial R} = 0$

$$\int_0^l \cancel{2}\left[Rx - \frac{wx^2}{2}\right] x \cdot \frac{dx}{\cancel{2}EI} = 0$$

$$\frac{R}{EI}\int_0^l x^2 \cdot dx - \frac{w}{2EI}\int_0^l x^3 \cdot dx = 0 \implies \frac{R}{EI}\left[\frac{x^3}{3}\right]_0^l - \frac{w}{2EI}\left[\frac{x^4}{4}\right]_0^l = 0$$

$$\frac{Rl^3}{3EI} - \frac{wl^4}{8EI} = 0 \implies R = \frac{3}{8} wl$$

Prob: Find the propped reaction using principle of least work.



S.E $W_i = \sum \int \frac{M^2 \cdot dx}{2EI}$

$$W_i = \int_0^2 \left(Rx - 20x \cdot \frac{x}{2}\right)^2 \frac{dx}{2EI} + \int_2^6 \left[Rx - 10x^2 - 200(x-2)\right]^2 \frac{dx}{2EI}$$

From principle of least work; $\frac{\partial W_i}{\partial R} = 0$

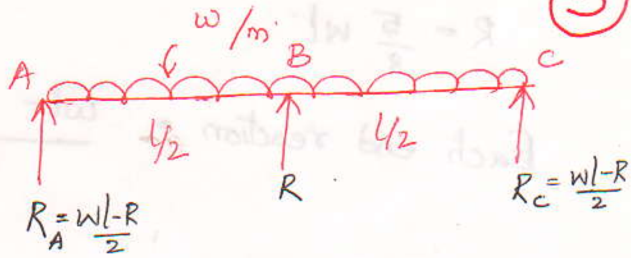
$$\frac{\partial W_i}{\partial R} = \int_0^2 2\left(Rx - 10x^2\right) \cdot \frac{x \cdot dx}{2EI} + \int_2^6 2\left[Rx - 10x^2 - 200(x-2)\right. \frac{x \cdot dx}{2EI} = 0$$

$$\int_0^2 \left(Rx - 10x^2\right) x \cdot dx + \int_2^6 \left[Rx - 10x^2 - 200(x-2)\right] x \cdot dx = 0$$

$$R \cdot \frac{2^3}{3} - 10 \cdot \frac{2^4}{4} + \frac{R}{3}\left(6^3 - 2^3\right) - \frac{10}{4}\left(6^4 - 2^4\right) - \frac{200}{3}\left(6^3 - 2^3\right) + \frac{400}{2}\left(6^2 - 2^2\right) = 0$$

$R = 148.70$ KN.

Prob: Determine the reactions @ supports by principle of least work.



Sol^n Reaction @ B = R

$$R_A + R + R_C = wl$$

$$R_A + R_C = wl - R$$

$$2R_A = wl - R \Rightarrow R_A = R_C = \frac{wl-R}{2}$$

Total SE stored W_i = twice energy stored by AB

$$\left.\begin{array}{l}\text{BM @ any section in AB, distant} \\ x \text{ from A}\end{array}\right\} = \frac{wl-R}{2}x - \frac{wx^2}{2}$$

$$\text{Total St. eng stored } W_i = \sum \int \frac{M^2\,dx}{2EI}$$

$$= 2\int_0^{l/2} \left[\frac{wl-R}{2}x - \frac{wx^2}{2}\right]^2 \frac{dx}{2EI} = \frac{1}{EI}\int_0^{l/2} \left[\frac{wl-R}{2}x - \frac{wx^2}{2}\right]^2 dx$$

By principle of least work; 'R' is given by condition of min^m strain energy i.e. $\frac{\partial W_i}{\partial R} = 0$

$$\frac{\partial W_i}{\partial R} = \frac{1}{EI}\int_0^{l/2} 2\left[\frac{wl-R}{2}x - \frac{wx^2}{2}\right]\left(-\frac{x}{2}\right) dx = 0$$

$$= \frac{1}{EI}\int_0^{l/2} \left[\frac{wx^3}{2} - \frac{wl-R}{2}x^2\right] dx = 0$$

$$= \frac{1}{EI}\left[\frac{wx^4}{8} - \left(\frac{wl-R}{2}\right)\frac{x^3}{3}\right]_0^{l/2}$$

$$= \frac{1}{EI}\left[\frac{w}{8}\left(\frac{l}{2}\right)^4 - \left(\frac{wl-R}{2}\right)\cdot\frac{1}{3}\left(\frac{l}{2}\right)^3\right] = 0$$

$$\Rightarrow \frac{wl^4}{128} - \frac{wl^4}{48} + \frac{Rl^3}{48} = 0 \Rightarrow \frac{Rl^3}{48} = \frac{5}{384}wl^4$$

$R = \frac{5}{8} wl$

Each end reaction $= \dfrac{\omega l - \frac{5}{8}\omega l}{2} = \frac{3}{16} wl$

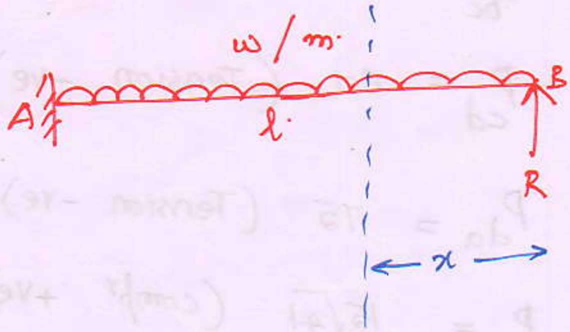
ANALYSIS OF INDETERMINATE STRUCTURES:

Statically indet. st^r may be analysed by the principle of least work or second theorem of Castigliano.

"In any case of statical indetermination where in, the redundant forces satisfy the conditions of statical eq^bm, their actual values are those that make the total strain energy stored to a min^m."

$$\frac{\partial W_i}{\partial X} = 0$$

Prob: A beam of span 'l' is fixed at one end and simply supported at the other end. It carries udl thr the span. Find reaction @ prop by using principle of least work.



Sol^n BM @ X $M_x = Rx - \frac{wx^2}{2}$

St. eng stored by the beam;

$$W_i = \int \frac{M^2 \cdot dx}{2EI}$$

$$W_i = \int_0^l \left(Rx - \frac{wx^2}{2}\right)^2 \cdot \frac{dx}{2EI}$$

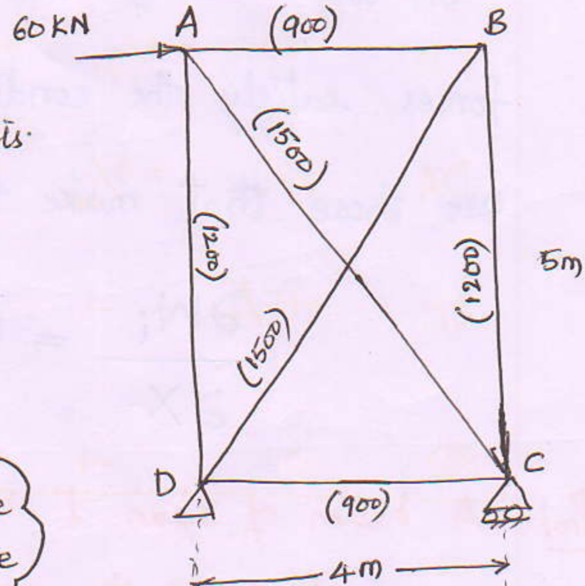
From 2^nd th. of Castigliano; $\frac{\partial W_i}{\partial R} = 0$

$$\int_0^l 2\left[Rx - \frac{wx^2}{2}\right] x \cdot \frac{dx}{2EI} = 0$$

$$\frac{R}{EI}\int_0^l x^2\,dx - \frac{w}{2EI}\int_0^l x^3\,dx = 0$$

$$\frac{Rl^3}{3EI} - \frac{wl^4}{8EI} = 0 \Rightarrow R = \tfrac{3}{8}wl$$

Prob Determine forces in the members of frame. Area is given in paranthesis.



Solⁿ Consider 'BD' as redundant.

Remove this member & find the forces in all member.

Compʳ : +ve

Tens : -ve

$P_{ab} = 0$

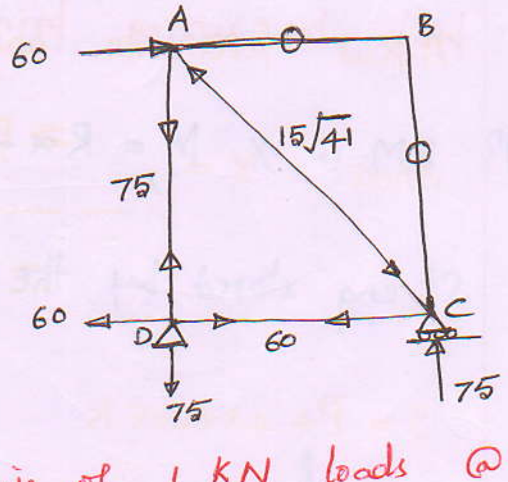
$P_{bc} = 0$

$P_{cd} = 60$ (Tension -ve)

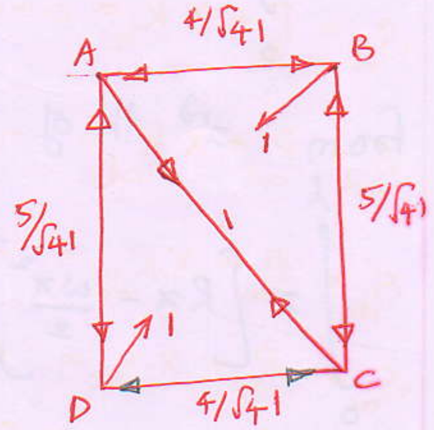
$P_{da} = 75$ (Tension -ve)

$P_{ac} = 15\sqrt{41}$ (compʳ +ve)

$P_{bd} = 0$



Remove given load system & apply a pair of 1 KN loads @ B & D in place of member BD. Obviously there will be no reactions @ the supports.



~~Member~~ $K_{ab} = +4/\sqrt{41}$ $\quad K_{da} = +5/\sqrt{41}$

$K_{bc} = +5/\sqrt{41}$ $\quad K_{ac} = -1$

$K_{cd} = +4/\sqrt{41}$ $\quad K_{bd} = -1$

Member	P	K	L (mm)	A (mm²)	$\frac{PKL}{A}$	$\frac{K^2L}{A}$
AB	0	$+\frac{4}{\sqrt{41}}$	4000	900	0	1.7344
BC	0	$+5/\sqrt{41}$	5000	1200	0	2.5407
CD	−60	$+4/\sqrt{41}$	4000	900	−166.585	1.7344
DA	−75	$+5/\sqrt{41}$	5000	1200	−244.022	2.5407
AC	$+15\sqrt{41}$	−1	$1000\sqrt{41}$	1500	−410	4.2687
BD	0	−1	$1000\sqrt{41}$	1500	0	4.2687
			Total		−820.607	17.0876

Actual force in any member is given by $S = P + XK$

where $X = -\dfrac{\sum \frac{PKL}{A}}{\sum \frac{K^2L}{A}} = -\dfrac{-820.607}{17.0876} = 48.0235$

$S = P + 48.0235\,K$

$S_{ab} = P_{ab} + X K_{ab} = 0 + 48.0235 \times \frac{4}{\sqrt{41}} = +30 \text{ KN}$ (compr)

$S_{bc} = P_{bc} + X K_{bc} = 0 + 48.0235 \times \frac{5}{\sqrt{41}} = +37.5 \text{ KN}$ (compr)

$S_{cd} = P_{cd} + X K_{cd} = -60 + 48.0235 \times \frac{4}{\sqrt{41}} = -30 \text{ KN}$ (tens)

$S_{da} = P_{da} + X K_{da} = -75 + 48.0235 \times \frac{5}{\sqrt{41}} = -37.5 \text{ KN}$ (tens)

$S_{ac} = P_{ac} + X K_{ac} = 15\sqrt{41} + 48.0235 \times (-1) = +48.023 \text{ KN}$ (comp)

$S_{bd} = P_{bd} + X K_{bd} = 0 + 48.0235(-1) = -48.023 \text{ KN}$ (ten)

Degree of Redundancy of pin jointed plane trusses.

The stability of a pin jointed truss depends on number & arrangement of its members.

For the truss to be stable;

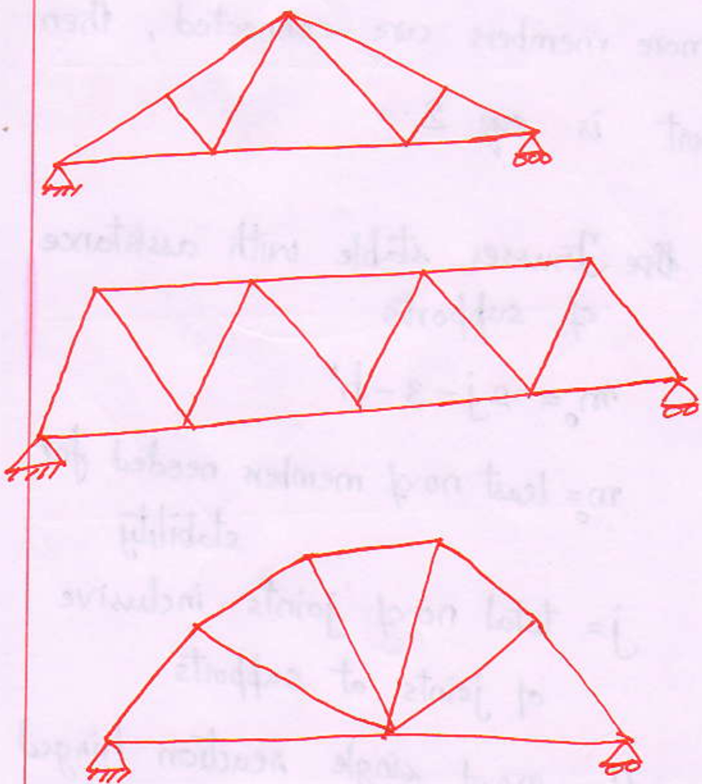
i) it should have certain min^m no. of members & the

ii) ~~geometry~~ Arrangement of members should be such that the geometry of the truss should be stable.

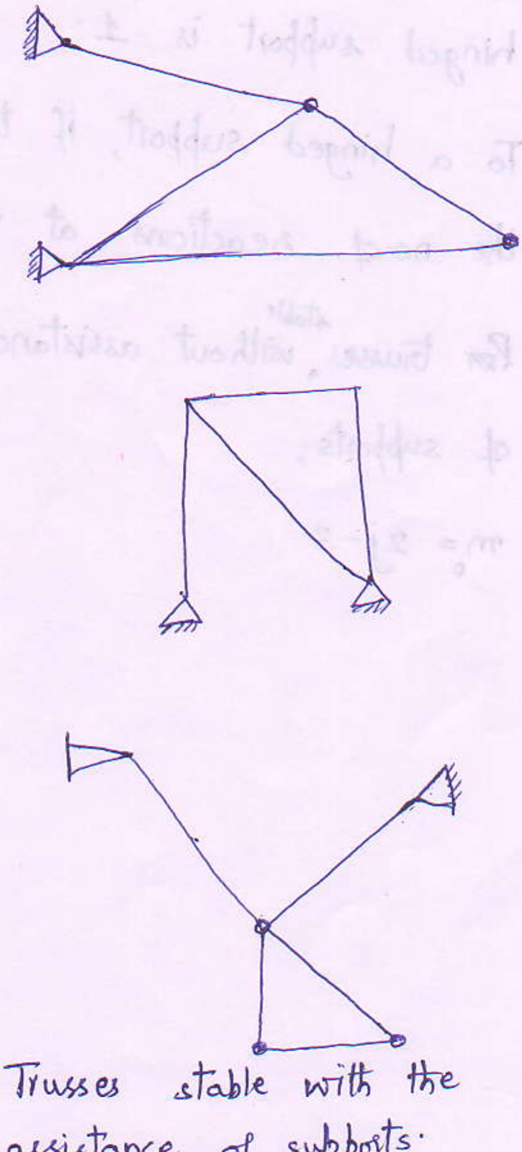
Pin jointed trusses may be classified –

i) trusses having stability of geometry without the need of supports.

ii) trusses whose geometry is stable only with the assistance of supports.



Trusses stable without the assistance of supports.



Trusses stable with the assistance of supports.

Trusses stable without the assistance of supports.

Let m = least no.of members needed for stability

j = total no.of joints (inclusive of joints at supports)

$$m = 2j - 3$$

Supports provided for a truss: i) Hinged support

ii) Roller support

At roller support, the dir^n of reaction is always normal to the roller base, irrespective of the loading on the truss. So no.of reactions at roller support is 1.

To a hinged support, if only one member is connected, the reaction at that support is always in line with the member. So no.of reactions at such hinged support is 1.

To a hinged support, if two or more members are connected, then the no.of reactions at the support is eq 2.

For trusses stable without assistance of supports;

$$m_0 = 2j - 3.$$

For the Trusses stable with assistance of supports

$$m_0 = 2j - 3 - h'$$

m_0 = least no.of members needed for stability.

j = total no.of joints inclusive of joints at supports.

h' = no.of single reaction hinged supports

Degree of external redundancy:

$= r + 2h + h' - 3$

r = no. of roller supports.

h = no. of double reaction hinge supports

h' = no. of single reaction hinge supports

Degree of internal redundancy:

$= m - m_0$

$= m - (2j - 3 - h')$

m = no. of members provided.

m_0 = least no. of members needed for stability.

Neglect h' if not applicable.

Total degree of redundancy is sum of degree of external & internal redundancy.

$= (r + 2h + h' - 3) + (m - (2j - 3 - h'))$

$= r + 2h + h' - 3 + m - 2j + 3 + h'$

$= r + 2h + 2h' + m - 2j$

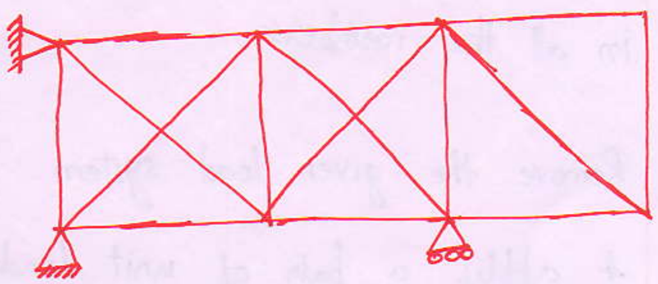
$= r + 2H + m - 2j$

where $H = h + h'$

= total no. of hinged supports.

This method of determining the degree of redundancy is called stability method.

Prob: Find the degree of redundancy for the truss shown.



No. of roller supports $r = 1$

No. of single reaction hinged supports $h' = 0$

No. of double reaction hinged supports $h = 2$

No. of joints $= j = 8$

No. of members $m = 15$

Degree of External redundancy $= r + 2h + h' - 3$

$= 1 + 2(2) + 0 - 3$

$= 2$

Degree of internal redundancy $= m - m_0$

Min^m no. of members needed for stability $m_0 = 2j - 3 - h'$

$= 2(8) - 3 - 0$

$= 13$

No. of members provided $m = 15$

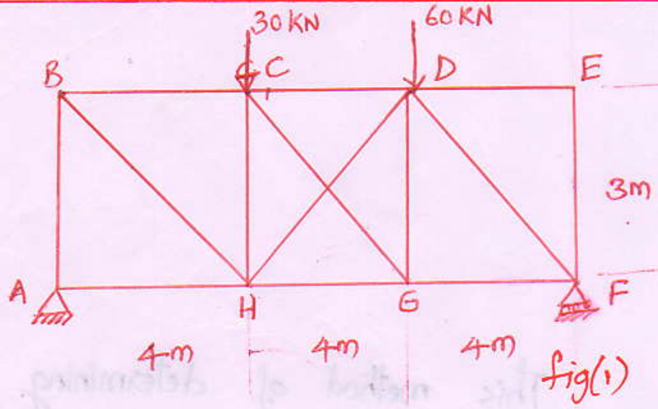
$m - m_0 = 15 - 13 = 2$

Total degree of redundancy $= 2 + 2 = 4$

Prob Find forces in members of truss shown.

$A_{HORI} = 4000\ mm^2$ $A_{ver} = 3000\ mm^2$

$A_{diag} = 5000\ mm^2$



fig(1)

Consider member DH as redundant.

Let this member be removed.

Now determine forces in all members.

Fig (2) shows the truss with forces in all the members.

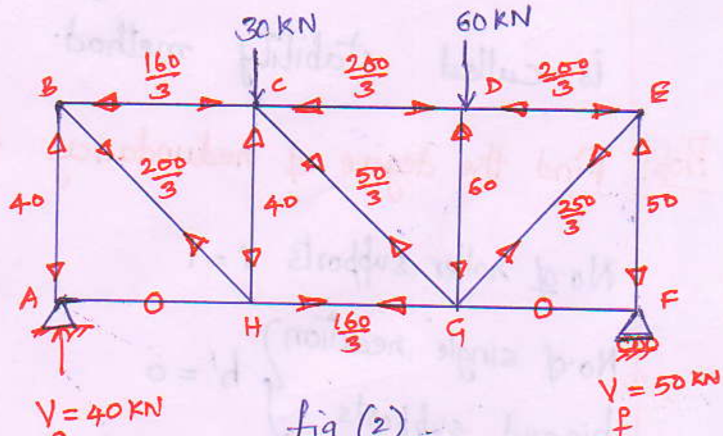


fig (2)

Remove the given load system & apply a pair of unit loads at D and H in place of the member DH (may be Compr or tensile)

There will be no reactions @ the supports.

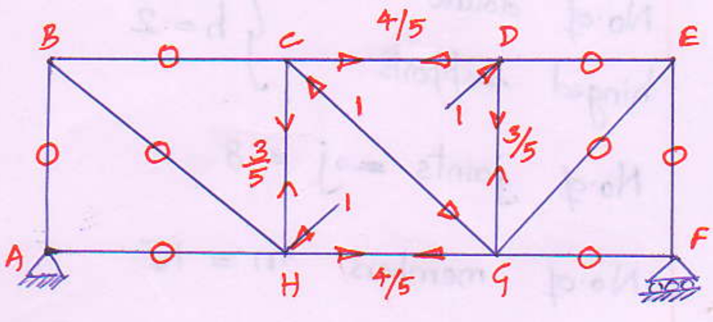


fig (3)

Ten: -ve
Comp: +ve

Member	P	K	L mm	A mm^2	$\frac{PKL}{A}$	$\frac{K^2L}{A}$
AB	40	0	3000	3000	0	0
BC	$\frac{160}{3}$	0	4000	4000	0	0
CD	$\frac{200}{3}$	$\frac{-4}{5}$	4000	4000	$\frac{-160}{3}$	$\frac{16}{25}$
DE	$\frac{200}{3}$	0	4000	4000	0	0
EF	50	0	3000	3000	0	0
FG	0	0	4000	4000	0	0
GH	$\frac{-160}{3}$	$-\frac{4}{5}$	4000	4000	$+\frac{128}{3}$	$\frac{16}{25}$
HA	0	0	4000	4000	0	0
BH	$\frac{-200}{3}$	0	5000	5000	-24	25
HC	40	$\frac{-3}{5}$	3000	3000	$\frac{-50}{3}$	1
CG	$\frac{-50}{3}$	1	5000	5000	-36	$9/25$
GD	60	$\frac{-3}{5}$	3000	3000	0	0
GE	$\frac{-250}{3}$	0	5000	5000	0	1
DH	0	1	5000	5000		
Total					$\frac{-518}{8}$	4

~~X~~ Correcting factor $X = -\dfrac{\Sigma P.KL/A}{\Sigma K^2L/A}$

$$X = -\left[\dfrac{\frac{-518}{8}}{4}\right] = 259/16$$

Actual force in any member $S = P + XK$

$$= P + \frac{259}{16}K$$

$S_{ab} = 40 + 0 = 40$ KN (compr)

$S_{bc} = \frac{160}{3} + 0 = \frac{160}{3}$ KN (compr)

$S_{cd} = \frac{200}{3} + \frac{259}{16}\left(-\frac{4}{5}\right) = 53.717$ KN (compr)

$S_{de} = \frac{200}{3} + 0 = \frac{200}{3}$ KN (compr)

$S_{ef} = 50 + 0 = 50$ KN (compr)

$S_{fg} = 0$

$S_{gh} = \frac{-160}{3} + \frac{259}{16}(4/5) = -40.383$ KN (ten)

$S_{ha} = 0$

$S_{bh} = \frac{-200}{3} + 0 = -\frac{200}{3}$ KN (ten)

$S_{hc} = 40 + \frac{259}{16}\left(-\frac{3}{5}\right) = 30.288$ KN (compr)

$S_{cg} = \frac{-50}{3} + \frac{259}{16}(1) = -0.479$ KN (tens)

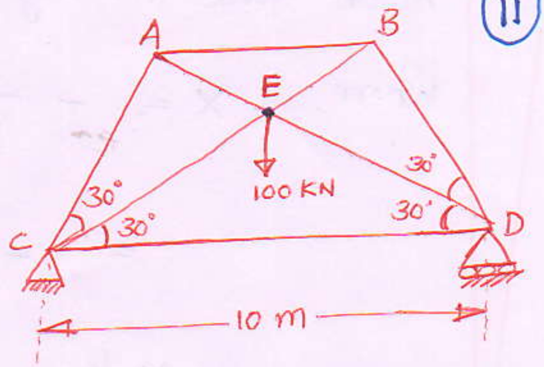
$S_{gd} = 60 + \frac{259}{16}\left(-\frac{3}{5}\right) = 50.288$ KN (compr)

$S_{ge} = -\frac{250}{3}$ KN (ten)

$S_{dh} = 0 + 1\left(\frac{259}{16}\right) = 16.188$ KN (compr)

— x — x —

Prob: Find the force in member AB of the truss if the value of $l/A = 0.16\ mm^{-1}$ ∀ members.



Solⁿ: Least no. of members for stability $m_0 = 2j - 3$

$= 2(5) - 3 = 7$

No. of members in truss = 8

No. of redundant members = 8 − 7 = 1

Consider AB as redundant.

Remove AB & analyse the truss as shown in fig (2).

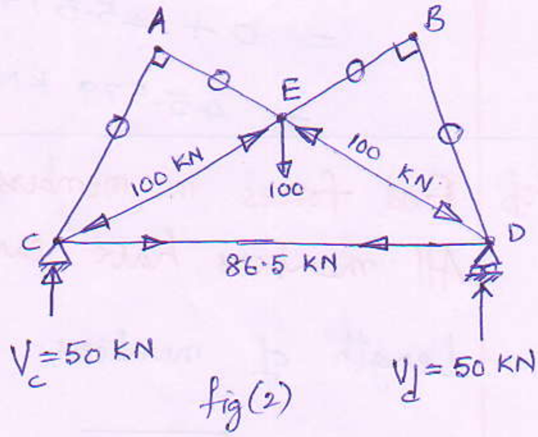


fig (2)

Now remove the given load & apply a pair of unit loads at A & B in place of member AB.

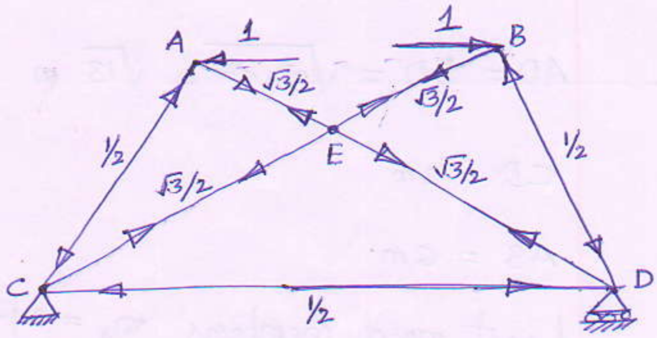


fig (3)

Comp +ve
Ten −ve

Member	P	K	l/A	$\frac{PKl}{A}$	$\frac{K^2 l}{A}$
AB	0	+1	0.16	0	0.16
BD	0	+1/2	0.16	0	0.04
DC	−86.6	+1/2	0.16	−6.928	0.04
CA	0	+1/2	0.16	0	0.04
AE	0	$-\sqrt{3}/2$	0.16	0	0.12
BE	0	$-\sqrt{3}/2$	0.16	0	0.12
CE	100	$-\sqrt{3}/2$	0.16	−13.856	0.12
DE	100	$-\sqrt{3}/2$	0.16	−13.856	0.12
		Total		−34.64	0.76

11

Actual force in any member $S = P + XK$

where $X = -\dfrac{\Sigma PKl/A}{\Sigma K^2 l/A}$

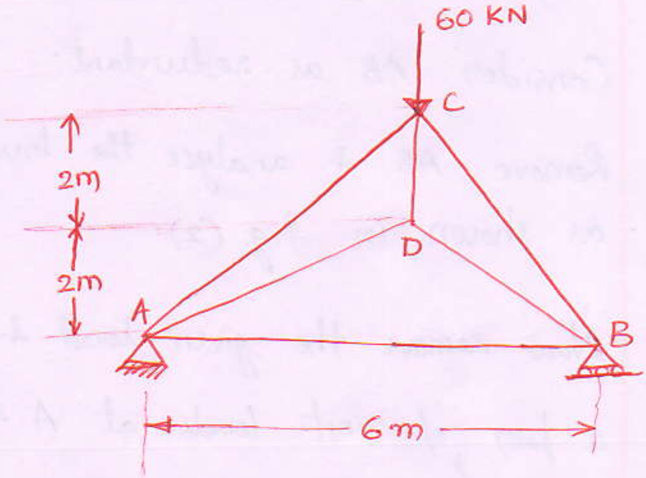
$= -\dfrac{(-34.64)}{0.76} = +45.579$

$S_{ab} = P_{ab} + X K_{ab}$

$= 0 + 45.579(1)$

$= 45.579$ KN [Comp].

Prob: Find forces in members of frame. All members have same c/s area



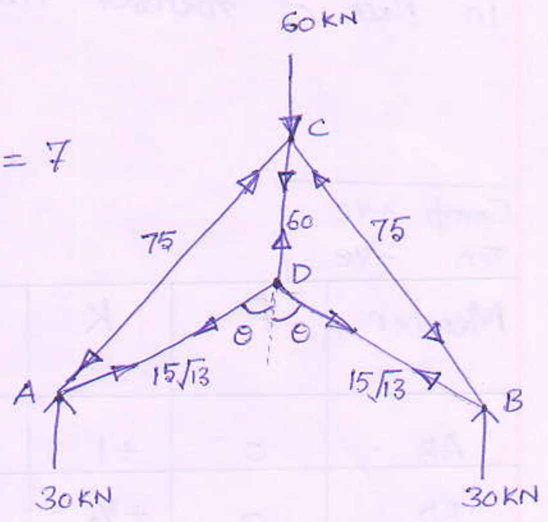
Length of members;

$AC = BC = \sqrt{3^2+4^2} = 5$ m.

$AD = BD = \sqrt{3^2+2^2} = \sqrt{13}$ m

$CD = 2$ m

$AB = 6$ m



Least no. of members $m_0 = 2j - 3 = 2(5) - 3 = 7$

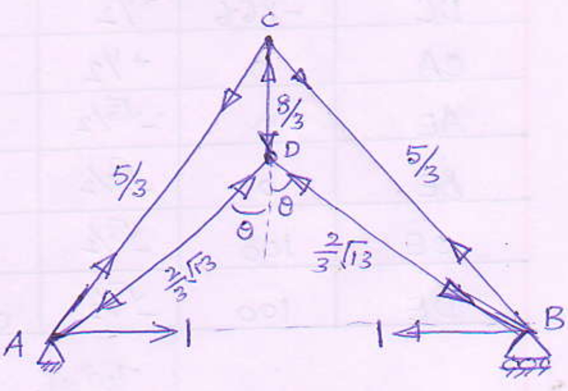
No. of members in truss = 6

No. of redundant members = 7 − 6 = 1.

Consider AB as redundant.

Remove the member AB & analyse the frame.

Now remove the external loading & apply a pair of unit loads at A & B in place of member AB.

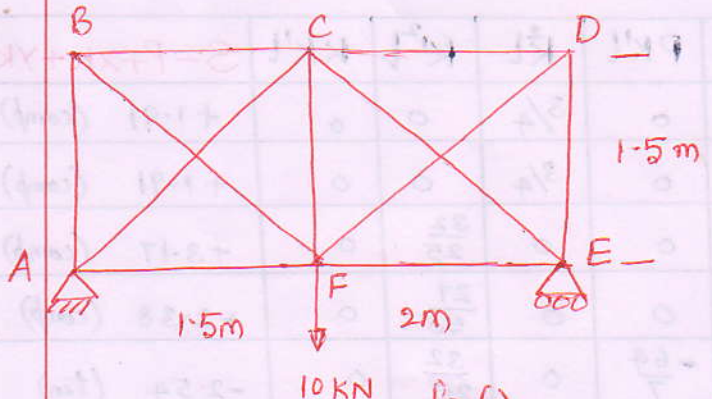


Member	P	K	l	PKl	K^2l	$S = P + XK$ $= P + 3.107K$
AC	75	$-5/3$	5	-625	$125/9$	69.82 (comp)
CB	75	$-5/3$	5	-625	$125/9$	69.82 (comp)
AD	$-15\sqrt{13}$	$2\sqrt{13}/3$	$\sqrt{13}$	$-130\sqrt{13}$	$\frac{52}{9}\sqrt{13}$	-46.61 (Ten)
BD	$-15\sqrt{13}$	$2\sqrt{13}/3$	$\sqrt{13}$	$-130\sqrt{13}$	$\frac{52}{9}\sqrt{13}$	-46.61 (Ten)
CD	-60	$8/3$	2	-320	$\frac{128}{9}$	-51.71 (Ten)
AB	0	-1	6	0	6	-3.11 (Ten)
Total				$-1570-260\sqrt{13}$	~~$-1570-260\sqrt{13}$~~ $432/9 + 104\sqrt{13}/9$	

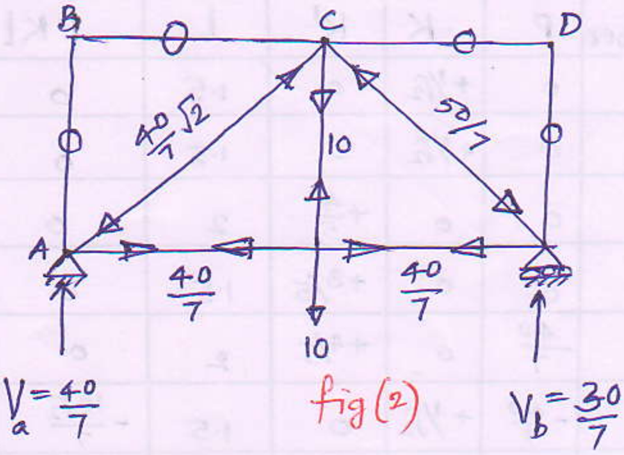
$$X = -\frac{\Sigma PKl}{\Sigma K^2 l} = \frac{-1570-260\sqrt{13}}{\frac{432}{9} + \frac{104\sqrt{13}}{9}} = 3.107$$

Actual forces in various members $S = P + XK$ [shown in last column of tabular column].

Prob Analyse the frame shown in the figure. Area of each member is $1000\ mm^2$



fig(1)



fig(2)

Consider the members FB & FD as redundants.

Min^m no. of members for stability $m_0 = 2j - 3 = 2(6) - 3 = 9$

No. of members in frame = 11

No. of redundants = $11 - 9 = 2$

Let X be tension in ~~B~~FB & Y be tension in FD.

Remove FB & FD & then analyse the frame as shown in fig (2).

Consider the effect of pair of unit loads at B & F in place of member FB. Let K_1 K_2 K_3 ... be the forces due to condition as shown in fig (3).

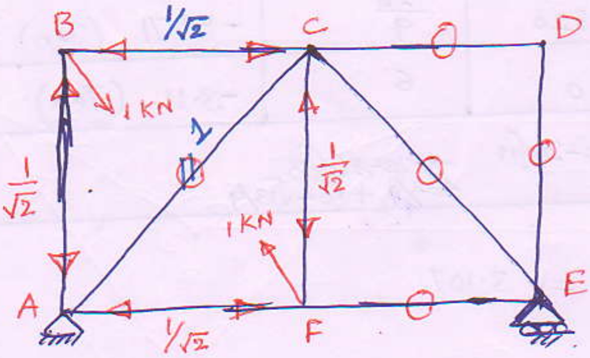


fig (3)

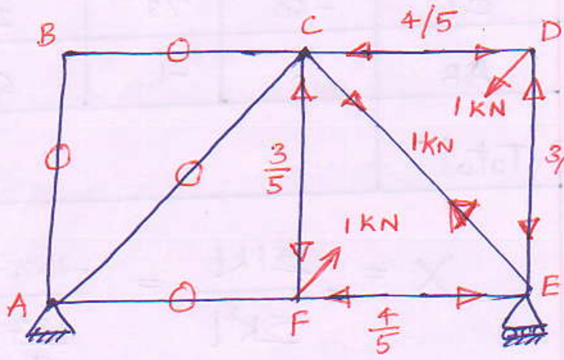


fig (4)

Consider the effect of pair of unit loads @ F & D in place of member FD. Let K_1' K_2' K_3' .... be the forces due to condition as shown in fig (4)

Member	P	K	K'	l	PKl	PK'l	K^2l	K'^2l	KK'l	S=P+XK+YK'
AB	0	$+1/\sqrt{2}$	0	1.5	0	0	3/4	0	0	+1.91 (comp)
BC	0	$+1/\sqrt{2}$	0	1.5	0	0	3/4	0	0	+1.91 (comp)
CD	0	0	$+\frac{4}{5}$	2	0	0	0	$\frac{32}{25}$	0	+3.17 (comp)
DE	0	0	$+3/5$	1.5	0	0	0	$\frac{27}{50}$	0	+2.38 (comp)
EF	$-\frac{40}{7}$	0	$+4/5$	2	0	$-\frac{64}{7}$	0	$\frac{32}{25}$	0	-2.54 (ten)
FA	$-\frac{40}{7}$	$+1/\sqrt{2}$	0	1.5	$-\frac{10\sqrt{2}}{7}$	0	3/4	0	0	-3.81 (ten)
FC	-10	$+1/\sqrt{2}$	$+3/5$	1.5	$-\frac{15\sqrt{2}}{2}$	-9	3/4	27/50	$\frac{9\sqrt{2}}{20}$	-5.72 (tens)
AC	$+\frac{40}{7}\sqrt{2}$	-1	0	$1.5\sqrt{2}$	-120/7	0	$3/2(\sqrt{2})$	0	0	+5.39 (comp)
EC	$+50/7$	0	-1	2.5	0	$-\frac{125}{7}$	0	5/2	0	+3.18 (comp)
BF	0	-1	0	$1.5\sqrt{2}$	0	0	$\frac{3}{2}(\sqrt{2})$	0	0	-2.693 (tens)
DF	0	0	-1	2.5	0	0	0	5/2	0	-3.968 (tens)
TOTAL					~~-42.39~~ -29.76	-36	7.242	8.64	0.636	

$$\sum PKl + X \sum K^2 l + Y \sum KK'l = 0 \quad \text{——} \quad (1)$$

$$\sum PK'l + Y \sum K'^2 l + X \sum KK'l = 0 \quad \text{——} \quad (2)$$

From eq^n (1); $-29.76 + 7.242X + 0.636Y = 0$ —— (3)

From eq^n (2); $-36 + 8.64Y + 0.636X = 0$ —— (4)

Solving eq^n (3) & (4);

$X = 2.693$

$Y = 3.968$

Actual force $S = P + XK + YK'$

$= P + 2.693K + 3.968K'$

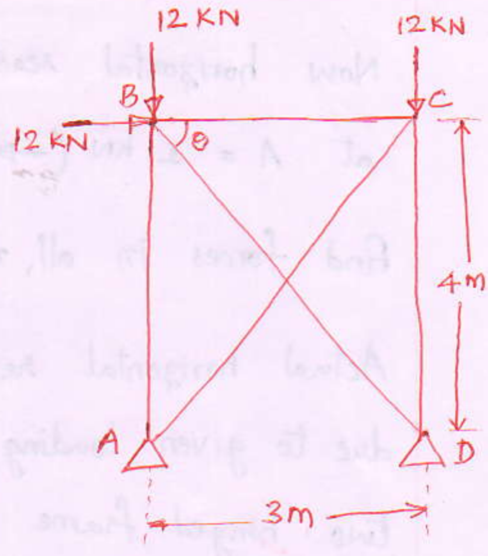
Prob: Fig shows frame with hinged supports. Sectional areas of members are as follows:

Hori members : 3000 mm^2

Vert members : 4000 mm^2

Diagonal members : 5000 mm^2

Find forces in all the members.



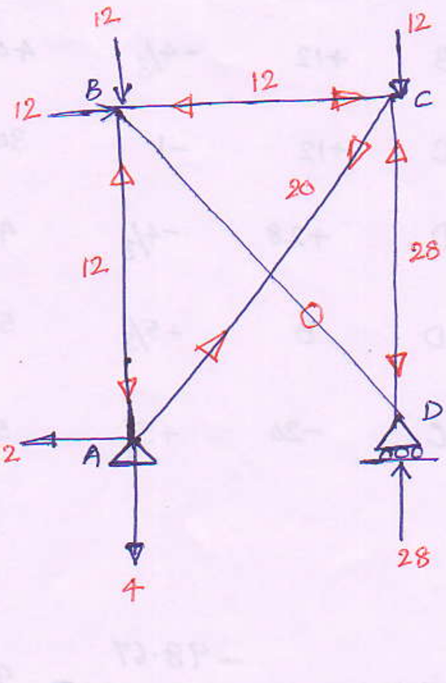
Soln: The st^r is externally redundant by one degree.

To make the st^r to a structure determinate state let one of the supports, say the support at D be changed to a roller support.

$\tan\theta = 4/3$

$\sin\theta = 4/5$

$\cos\theta = 3/5$



Taking moments about A;

$V_d \times 3 = (12\times3) + (12\times4)$

$V_d = 28$ KN (↑)

$V_a = 28-12-12 = 28-24 = 4$ KN (↓)

$H_a = 12$ KN (←)

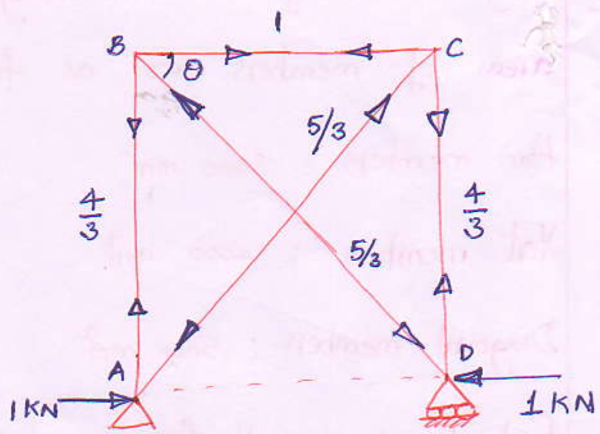
Now find forces in all members.

$P_{ab} = +12$ (comp) $P_{bc} = +12$ (comp) $P_{cd} = +28$ (C) $P_{bd} = 0$ $P_{ac} = -20$ (T)

Remove the given loading & apply 1 KN load at 'D' as shown in figure below.

Now horizontal reaction at $A = 1$ KN ($\rightarrow$)

Find forces in all members say K.



Actual horizontal reaction at 'D' due to given loading, for the two hinged frame

$$X = -\frac{\sum \frac{PKL}{A}}{\sum \frac{K^2L}{A}}$$

Member	P	K	L (mm)	A (mm²)	$\frac{PKL}{A}$	$\frac{K^2L}{A}$
AB	+12	−4/3	4000	4000	−16	16/9
BC	+12	−1	3000	3000	−12	1
CD	+28	−4/3	4000	4000	$-\frac{112}{3}$	16/9
BD	0	+5/3	5000	5000	0	25/9
AC	−20	+5/3	5000	5000	$-\frac{100}{3}$	25/9
					−98.67	10.11

$$\therefore X = -\frac{-98.67}{10.11} = 9.758 \text{ kN.}$$

Positive value of X indicates that horizontal reaction at 'D' acts in the dir^n in which 1 KN was applied at D. i.e. if we get −ve value of X, change the assumed dir^n.

Actual hori reaction at A = $12 - 9.758 = 2.242$ KN ←

The actual force in any member is;

$$S = P + X K.$$

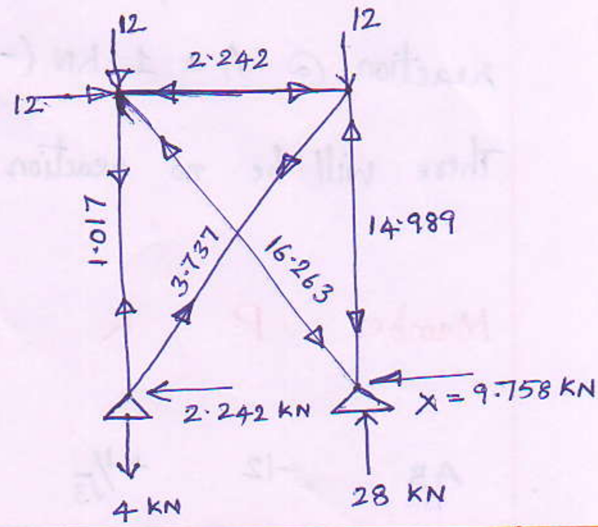
$$S_{ab} = +12 + 9.758\left(-\frac{4}{3}\right) = -1.017 \text{ KN (tensile)}$$

$$S_{bc} = 2.242 \text{ KN (C)}$$

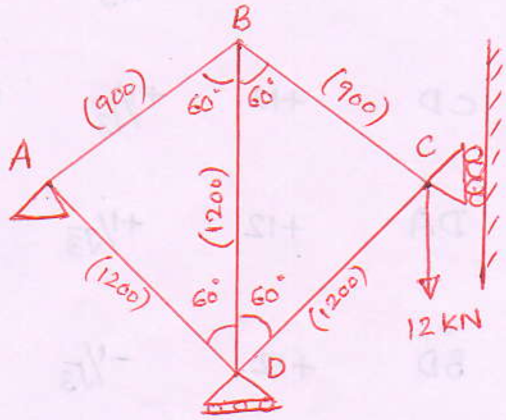
$$S_{cd} = +14.989 \text{ KN (C)}$$

$$S_{bd} = 16.263 \text{ KN (C)}$$

$$S_{ac} = -3.737 \text{ KN (T)}$$



Prob Determine the reactions at the supports & the forces in the members of truss shown in fig. The sectional areas of the members in mm^2 are shown in brackets.



Sol^n The truss is externally indeterminate by one degree.

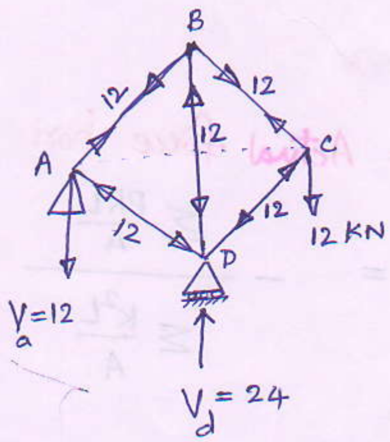
To make the truss determinate; let the roller support at C be removed.

For this condition;

$V_a = 12$ KN (↓)

$V_d = 24$ KN (↑)

Find forces in all members as shown in fig (P)

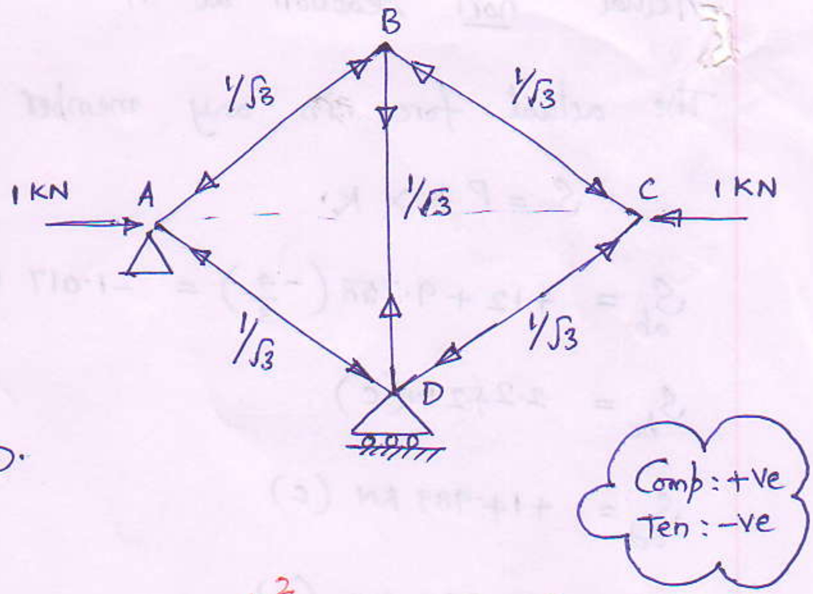


Remove given loading.

Let a unit horizontal load be applied at C as shown.

For this condition, hori reaction @ A = 1 KN (→)

There will be no reaction at D.



Member	P	K	A	$\frac{PK}{A}$	$\frac{K^2}{A}$	$S = P + XK$
AB	−12	$+1/\sqrt{3}$	900	−0.08	1/2700	−4.235 (T)
BC	−12	$+1/\sqrt{3}$	900	−0.08	1/2700	−4.235 (T)
CD	+12	$+1/\sqrt{3}$	1200	+0.01	1/3600	+19.765 (C)
DA	+12	$+1/\sqrt{3}$	1200	+0.01	1/3600	+19.765 (C)
BD	+12	$-1/\sqrt{3}$	1200	−0.01	1/3600	+4.235 (C).
		Total		−0.02 x	0.01 x	

X = Actual ~~force~~ hori reaction at C.

$$= -\frac{\sum \frac{PKL}{A}}{\sum \frac{K^2L}{A}} = -\frac{\sum \frac{PK}{A}}{\sum \frac{K^2}{A}} = -\frac{\cancel{-0.02}}{0.01} \neq \cancel{2\,KN}$$

$$= 13.44 \text{ KN}.$$