Slope Defl^n method

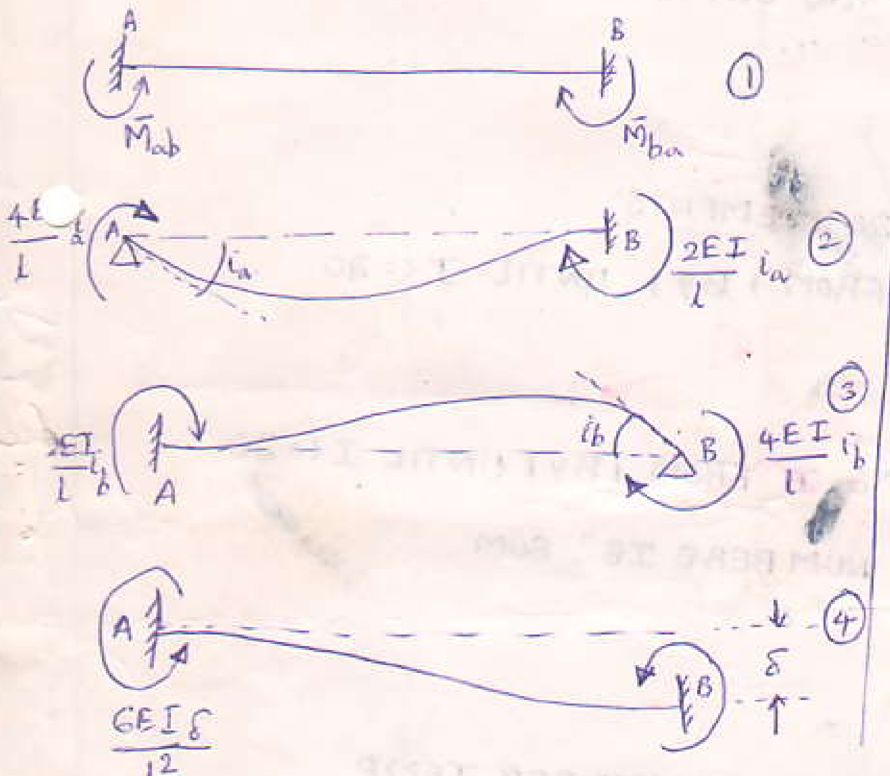
presented by Prof. George A. Maney.

Slope Defl^n method is used to analyse statically indeterminate beams & frames with rigid jts.

Sign conventions : i) At the end of any span clockwise end moments & clockwise slopes are +ve

ii) The downward defl^n of the right end of a span wrt its left end is +ve

iii) Defl^n of upper end towards the right relative to lower end is +ve.



AB = intermediate span of a continuous beam

i_a & i_b = slopes @ A & B

δ = downward defl^n of right end B w.r.t left end A

M_{ab} & M_{ba} be final moments @ A & B

(1) Consider AB as fixed beam. $\bar{M}_{ab}$ & $\bar{M}_{ba}$ are FEM's @ A & B

(2) Release fixity @ A; Maintain fixity @ B, Apply moment $\frac{4EI}{l} i_a$ at A to produce a slope i_a at A. This will induce a moment $\frac{2EI}{l} i_a$ at B.

(3) Maintain fixity @ A; Release fixity @ B; Apply a moment $\frac{4EI}{l} i_b$ @ B to produce a slope i_b at B. This will induce a moment $\frac{2EI}{l} i_b$ at A.

(4) Maintain fixity @ A & B; provide a vertical defl^n δ for the end B w.r.t the end A. For this condition, moment @ each end will be $\frac{6EI\delta}{l^2}$.

Final moment @ end A;

$$M_{ab} = \bar{M}_{ab} + \frac{4EI}{l} i_a + \frac{2EI}{l} i_b - \frac{6EI\delta}{l^2}$$

$$= \bar{M}_{ab} = \frac{2EI}{l}\left(2i_a + i_b - \frac{3\delta}{l}\right)$$

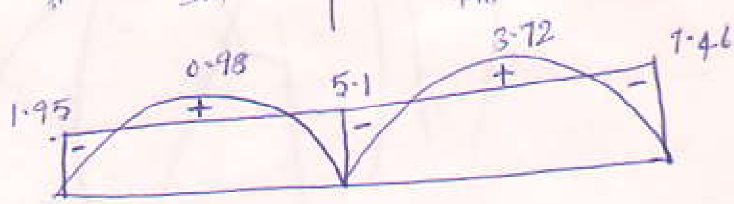
Final moment @ end B;

$$M_{ba} = \bar{M}_{ba} + \frac{4EI}{l} i_b + \frac{2EI}{l} i_a - \frac{6EI\delta}{l^2}$$

$$= \bar{M}_{ba} + \frac{2EI}{l}\left(2i_b + i_a - \frac{3\delta}{l^2}\right)$$

These are S.D. eq^{ns}. These eq^{ns} along with conditions of eq^{bm} will be sufficient to determine the unknowns.

Prob: Find support moments & draw BMD.



Sol^n: FEM:

$\bar{M}_{ab} = -\frac{4 \times 3^2}{12} = -3\ KN\cdot m$

$\bar{M}_{ba} = +\frac{wl^2}{12} = +3\ KN\cdot m$

$\bar{M}_{bc} = \frac{-wl^2}{12} = -6.67\ KN\cdot m$

$\bar{M}_{cb} = +\frac{wl^2}{12} = +6.67\ KN\cdot m$

Slope $i_a = i_c = 0$

$$M_{ba} = \bar{M}_{ba} + \frac{2EI}{l}(2i_b + i_a)$$

$$= 3 + 4/_3\ EI i_b$$

Span BC

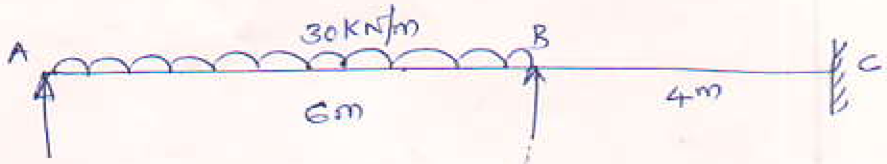
$$M_{bc} = \bar{M}_{bc} + \frac{2EI}{l}(2i_b + i_c)$$

$$= -6.67 + EI i_b$$

Eq^{bm} condition @ B ; $M_{ba} + M_{bc} = 0 \Rightarrow EIi_b = 1.57286$

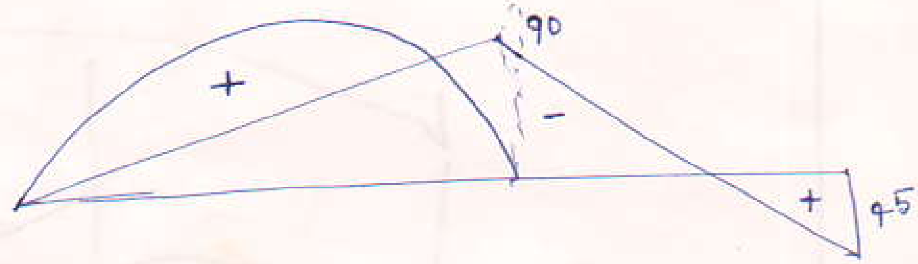
$M_{ab} = -1.95 \quad M_{ba} = +5.10 \quad M_{bc} = -5.10 \quad M_{cb} = +7.46$ kN·m

Prob :



$\bar{M}_{ab} = \frac{-wl^2}{12} = -90$ kN·m

$\bar{M}_{ba} = \frac{+wl^2}{12} = +90$ kN·m



$\bar{M}_{bc} = \bar{M}_{cb} = 0$

$i_c = 0$

span AB $\quad M_{ab} = 0$

$$M_{ba} = \bar{M}_{ba} - \frac{\bar{M}_{ab}}{2} + \frac{3EI}{l}\left(i_b - \frac{\delta}{l}\right)$$

$$= +90 - \frac{-90}{2} + \frac{3EI}{6}(i_b) = 135 + \frac{EIi_b}{2}$$

$$M_{bc} = \bar{M}_{bc} + \frac{2EI}{l}(2i_b + i_c) = 0 + \frac{2EI}{4}(2i_b + 0) = EIi_b$$

$$M_{cb} = \bar{M}_{cb} + \frac{2EI}{l}(2i_c + i_b) = 0 + \frac{2EI}{4}(0 + i_b) = \frac{EIi_b}{2}$$

Eq^{bm} condition @ B ;

$M_{ba} + M_{bc} = 0$

$135 + \frac{EIi_b}{2} + EIi_b = 0 \Rightarrow EIi_b = -90$

$M_{ab} = 0$

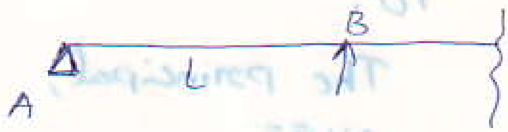
$M_{ba} = +90$

$M_{bc} = -90$

$M_{cb} = -45$

Modification for simply supported end of a continuous beam

$$M_{ab} = \bar{M}_{ab} + \frac{2EI}{l}\left(2i_a + i_b - \frac{3\delta}{l}\right) = 0$$

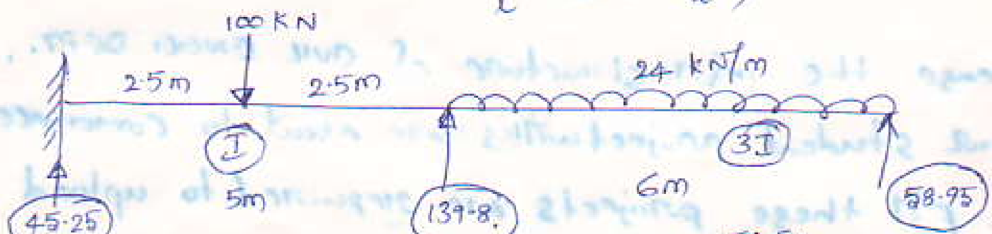


$$M_{ba} = \bar{M}_{ba} - \frac{\bar{M}_{ab}}{2} + \frac{3EI}{l}\left(i_b - \frac{\delta}{l}\right)$$

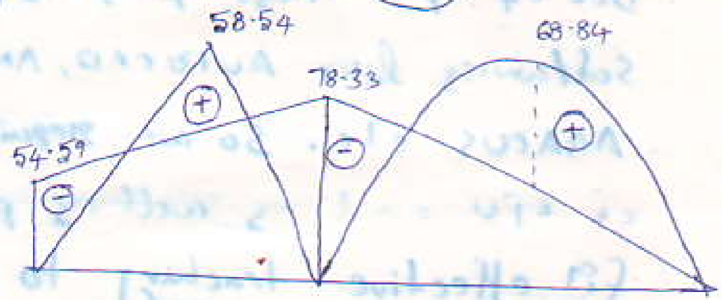
Modification when beam over hangs over a support

$$M_{ba} = \bar{M}_{ba} - \frac{M_a}{2} + \frac{3EI}{l}\left(i_b - \frac{\delta}{l}\right)$$

Prob:



Sol^n: FEM



$$\bar{M}_{ab} = \frac{-Wl}{8} = -62.5$$

$$\bar{M}_{ba} = +Wl/8 = +62.5$$

$$\bar{M}_{bc} = \frac{-wl^2}{12} =$$

$$\bar{M}_{cb} = \frac{+wl^2}{12} =$$

Span AB $\quad M_{ab} = \bar{M}_{ab} + \frac{2EI}{l}(2i_a + i_b) = -62.5 + 2/5\, EI i_b \quad (i_a = 0)$

$$M_{ba} = +62.5 + \frac{2EI}{5}(2i_b + 0) = 62.5 + 4/5\, EI i_b$$

Span BC $\quad M_{bc} = \bar{M}_{bc} - \frac{\bar{M}_{cb}}{2} + \frac{3E(3I)}{6}(i_b)$

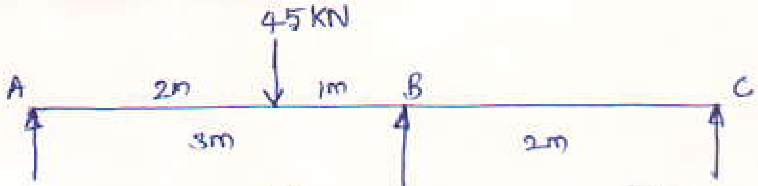
$$M_{bc} = \bar{M}_{bc} - \frac{\bar{M}_{cb}}{2} + \frac{3EI}{l}\left(i_b - \delta/l\right) = -72 - \frac{72}{2} + \frac{3}{2}EI i_b$$

$$= -108 + 3/2\, EI i_b$$

Eq^bm condition @ B; $\quad M_{ba} + M_{bc} = 0 \Rightarrow EI i_b = 19.78$

$M_{ab} = -54.59$ kN-m & $M_{bc} = -78.33$

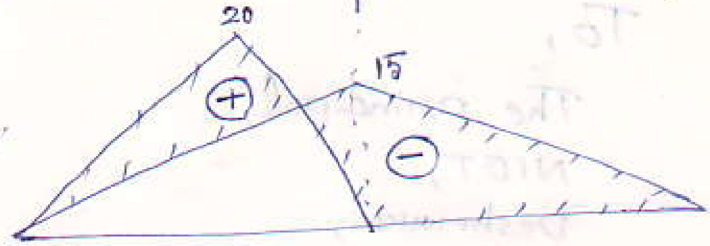
Prob:



$$\bar{M}_{ab} = \frac{-Wab^2}{l^2} = -10\ kN\cdot m$$

$$\bar{M}_{ba} = \frac{+45 \times 1 \times 2^2}{3^2} = +20\ kNm$$

$$\bar{M}_{bc} = \bar{M}_{cb} = 0$$



Span AB $\quad M_{ab} = 0$

$$M_{ba} = \bar{M}_{ba} - \frac{\bar{M}_{ab}}{2} + \frac{3EI}{l}\left(i_b - \frac{\delta}{l}\right) = 20 - \frac{-10}{2} + \frac{3EI}{3} i_b$$

$$= 25 + EI i_b$$

Span BC $\quad M_{bc} = \bar{M}_{bc} - \frac{\bar{M}_{cb}}{2} + \frac{3EI}{l}\left(i_b - \frac{\delta}{l}\right) = 0 - 0 + \frac{3EI}{2} i_b$

$$= 3/2\ EI i_b$$

Eq^{bm} condition @ B; $\quad M_{ba} + M_{bc} = 0 \Rightarrow EI i_b = -10$

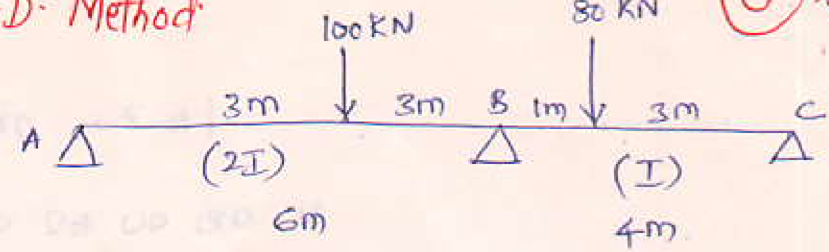
$M_{ab} = 0$

$M_{ba} = 25 - 10 = +15\ kN\cdot m$

$M_{bc} = \frac{3}{2}(-10) = -15\ kN\cdot m$

$M_{cb} = 0$

Prob Analyse this by S.D. Method



Sol^n:

$\bar{M}_{ab} = -\frac{Wl}{8} = -75$ KN·m

$\bar{M}_{bc} = -\frac{Wab^2}{l^2} = -45$

$M_{ba} = +75$ KN·m

$\bar{M}_{cb} = +\frac{Wba^2}{l^2} = +15$

A & C are hinged

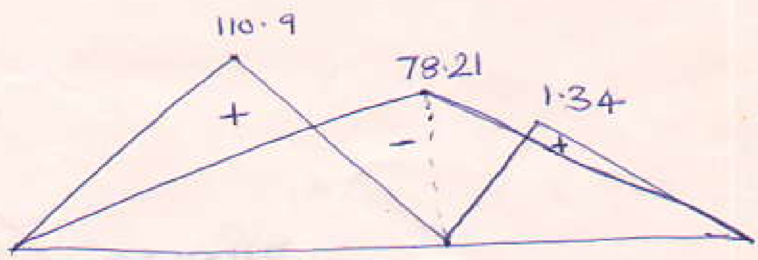
$\therefore M_{ab} = M_{cb} = 0$

$$M_{ba} = \bar{M}_{ba} - \frac{\bar{M}_{ab}}{2} + \frac{3EI_{ba}}{6} i_b = +75 - \frac{(-75)}{2} + \frac{3E(2I)i_b}{6} = +112.5 + EIi_b$$

$$M_{bc} = \bar{M}_{bc} - \frac{\bar{M}_{cb}}{2} + \frac{3EI_{bc}}{4} i_b = -45 - \frac{15}{2} + \frac{3EI}{4} i_b = -52.5 + \frac{3}{4} EIi_b$$

Eq^{bm} condition @ B; $M_{ba} + M_{bc} = 0 \Rightarrow EIi_b = \frac{-240}{7}$

$M_{ba} = +78.21$ KN·m $\quad M_{bc} = -78.21$ KN·m



BM @ B (LHS)

$V_a \times 6 - 100 \times 3 = -78.21$

$V_a = 36.97$ KN

BM @ B (RHS)

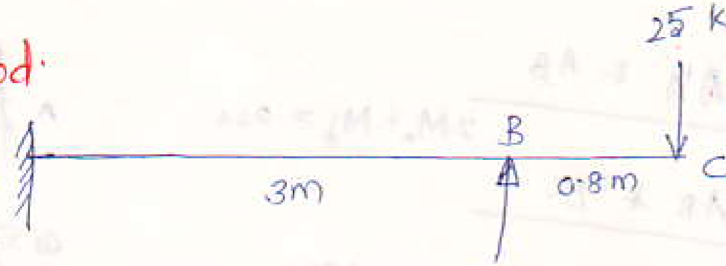
$V_c \times 4 - 80 \times 1 = -78.21$

$V_c = 0.45$ KN

$V_b = 142.58$ KN

Prob: Analyse by S.D. Method.

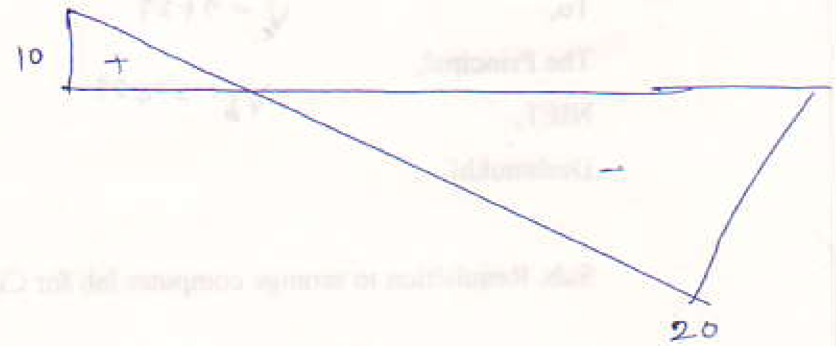
$M_b = 20$ KN·m (hogg)



A'A & AB

$0 + 2M_a(0+3) + (20 \times 3) = 0 \Rightarrow 6M_a = -60 \Rightarrow M_a = -10$ KN·m (sagg)

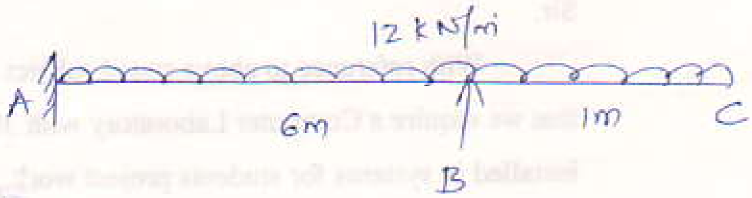
Reactions



$(V_b \times 3) - (25 \times 3.8) = 10$

$V_b = 35$ KN; $V_a = 10$ KN

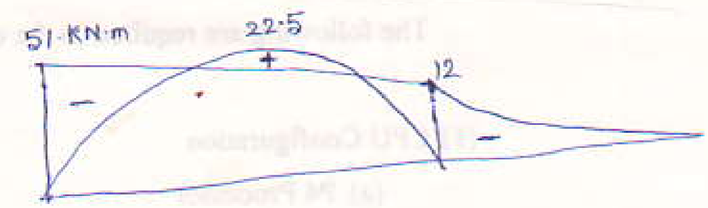
Prob: $M_b = \dfrac{-12 \times 1^2}{2} = 6$ KN·m (hogg)



Applying 3-Mom eq^n; A'A & AB

$0 + 2M_a(0+6) + 6 \times 6 = \dfrac{6a_1\bar{x}_1}{l}$

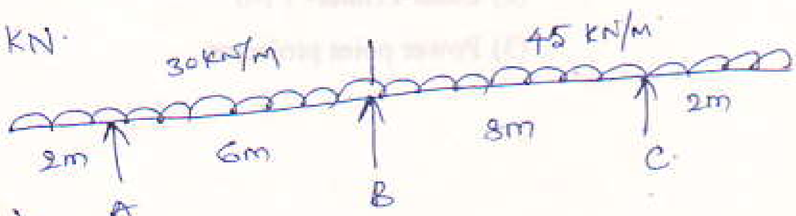
$M_a = 51$ KN·m



Reactions:

BM @ A; $V_b \times 6 - 12 \times 7 \times 7/_2 = -51 \Rightarrow V_b = 40.5$ KN

$V_a = (12 \times 7) - 40.5 = 43.5$ KN



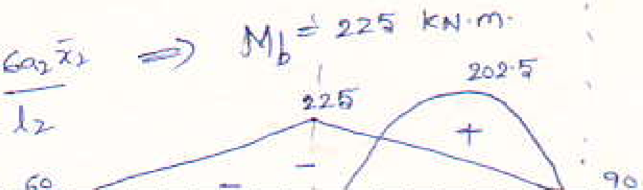
Prob:

$M_a = \dfrac{30 \times 2^2}{2} = 60$ KN·m (hogg)

$M_c = \dfrac{45 \times 2^2}{2} = 90$ KN·m (hogg)

AB & BC

$(60 \times 6) + 2M_b(6+8) + (90 \times 8) = \dfrac{6a_1\bar{x}_1}{l_1} + \dfrac{6a_2\bar{x}_2}{l_2} \Rightarrow M_b = 225$ KN·m



$(\ldots \times 8 \times 8/_2) = -225 \Rightarrow V_a = 122.5$ KN

Prob: Analyse the beam by S.D method if supports B & C sink by 3 mm & 5 mm ; $I = 4\times10^{7}\ mm^4$; $E = 200\ KN/mm^2$

$$EI = \frac{200\times4\times10^{7}}{10^{6}} = 8000\ KN\text{-}m^2$$

$$\frac{KN}{mm^2}\times mm^4 = KN\text{-}mm^2 = \frac{KN.m^2}{10^6}$$

* Defl^n of right end in downward dir^n

$i_a = i_d = 0$

Span AB

$$M_{ab} = \frac{2EI}{l}\left(2i_a + i_b - \frac{3\delta}{l}\right) = \frac{2\times8000}{4}\left(0 + i_b - \frac{3\times3}{4000}\right) = 4000i_b - 9$$

$$M_{ba} = \frac{2EI}{l}\left(2i_b + i_a - \frac{3\delta}{l}\right) = \frac{2\times8000}{4}\left(2i_b + 0 - \frac{3\times3}{4000}\right) = 8000i_b - 9$$

Span BC

$$M_{bc} = \frac{2EI}{l}\left(2i_b + i_c - \frac{3\delta}{l}\right) = \frac{2\times8000}{4}\left(2i_b + i_c - \frac{3\times2}{4000}\right) = 8000i_b + 4000i_c - 6$$

$$M_{cb} = \frac{2EI}{l}\left(2i_c + i_b - \frac{3\delta}{l}\right) = \frac{2\times8000}{4}\left(2i_c + i_b - \frac{3\times2}{4000}\right) = 4000i_b + 8000i_c - 6$$

Span CD

$$M_{cd} = \frac{2EI}{l}\left(2i_c + i_d - \frac{3\delta}{l}\right) = \frac{2\times8000}{6}\left(2i_c + 0 - \frac{3(-5)}{6000}\right) = \frac{16000}{3}i_c + \frac{20}{3}$$

$$M_{dc} = \frac{2EI}{l}\left(2i_d + i_c - \frac{3\delta}{l}\right) = \frac{2\times8000}{6}\left(0 + i_c - \frac{3(-5)}{6000}\right) = \frac{8000}{3}i_c + \frac{20}{3}$$

Eq^{bm} condition @ B; $M_{ba} + M_{bc} = 0 \Rightarrow 4i_b + i_c = \frac{3}{800}$

" " " C; $M_{cb} + M_{cd} = 0 \Rightarrow i_b + \frac{10}{3}i_c = \frac{-1}{6000}$

$i_b = 0.001027$
$i_c = -0.0003581$

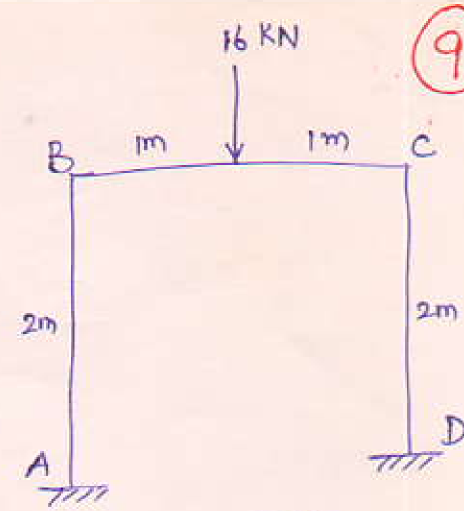
$M_{ab} = -4.89$ $M_{bc} = +0.784$ $M_{cd} = +4.75$

$M_{ba} = -0.784$ $M_{cb} = -4.75$ $M_{dc} = +5.712$

A		B		C	D
-4.892	-0.784	+0.784	-4.757	+4.757	+5.712

Slope Defl^n method

Prob: Analyse by " "



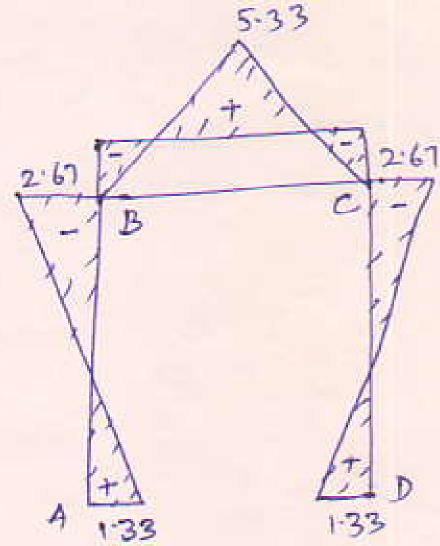
F.E.M

$\bar{M}_{ab} = \bar{M}_{ba} = \bar{M}_{cd} = \bar{M}_{dc} = 0$

$\bar{M}_{bc} = \frac{-WL}{8} = -\frac{16 \times 2}{8} = -4 \text{ KN·m}$

$\bar{M}_{cb} = +\frac{Wl}{8} = +\frac{16 \times 2}{8} = +4 \text{ KN·m}$

Due to symmetry $i_c = -i_b$ & $i_a = i_d = 0$



Span AB

$M_{ab} = \bar{M}_{ab} + \frac{2EI}{l}(2i_a + i_b) = EIi_b$

$M_{ba} = \bar{M}_{ba} + \frac{2EI}{l}(2i_b + i_a) = 2EIi_b$

$M_{bc} = \bar{M}_{bc} + \frac{2EI}{l}(2i_b + i_c) = -4 + EIi_b$

Eq^{bm} condi @ B $\quad M_{ba} + M_{bc} = 0 \Rightarrow EIi_b = 4/3$

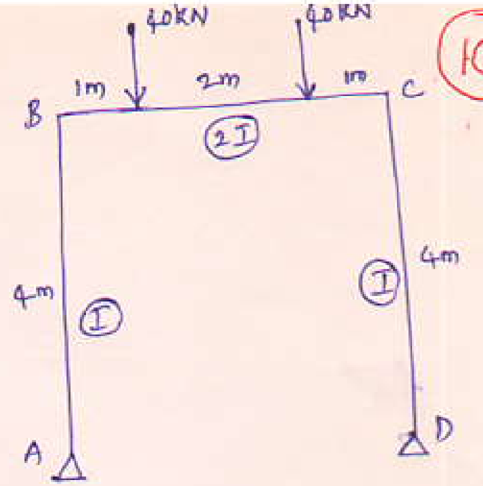
Substituting i

$M_{ab} = +1.33$

$M_{ba} = +2.67$

$M_{bc} = -2.67$

Analyse by S.D



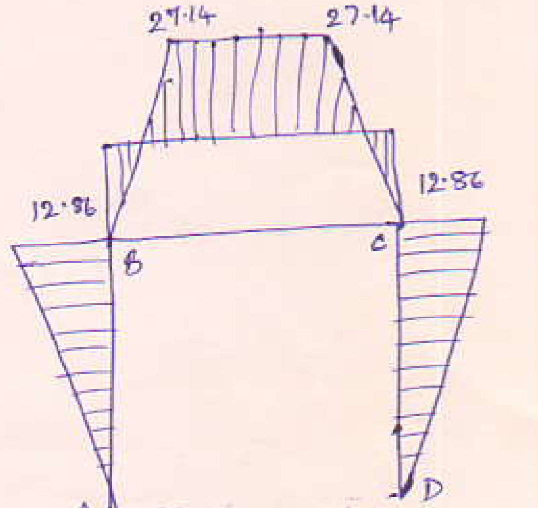
F.E.M

$\bar{M}_{ab} = \bar{M}_{ba} = \bar{M}_{cd} = \bar{M}_{dc} = 0$

$\bar{M}_{bc} = -\left[\dfrac{40\times1\times3^2}{4^2} + \dfrac{40\times3\times1^2}{4^2}\right] = -30\ KN\cdot m$

$\bar{M}_{cb} = +30\ KN\cdot m$

Symmetry $i_c = -i_b$



Span AB

$M_{ab} = 0$

$M_{ba} = \bar{M}_{ba} - \dfrac{\bar{M}_{ab}}{2} + \dfrac{3EI}{l}\left(i_b - \dfrac{\delta}{l}\right)$

$= 0 - \dfrac{0}{2} + \dfrac{3}{4}EI\,i_b = \dfrac{3}{4}EI\,i_b$

Span BC

$M_{bc} = \bar{M}_{bc} + \dfrac{2E(2I)}{4}\left(2i_b + i_c - \dfrac{3\delta}{l}\right) = -30 + EI\,(2i_b - i_b) = -30 + EI\,i_b$

$\begin{array}{r} 40.00 \\ 12.86 \\ \hline 27.14 \end{array}$

Eq^{bm} condition @ B; $M_{ba} + M_{bc} = 0 \Rightarrow EI\,i_b = 120/7$

Final moments

$M_{ab} = 0;\quad M_{ba} = \dfrac{3}{4}\cdot\dfrac{120}{7} = +12.86\ KN\cdot m;\quad M_{bc} = -12.86\ KNm$

A	B		C		D
0	+12.86	-12.86	+12.86	-12.86	

Hori Reaction @ A $= \dfrac{0 + 12.86}{4} = +3.215\ KN\ (\rightarrow)$

Hori Rea. @ D $= \dfrac{-12.86 + 0}{4} = -3.215\ KN\ (\leftarrow)$

Each vertical reaction $= 40\ KN\ (\uparrow)$ (symmetry)