G PULLAIAH COLLEGE OF ENGINEERING AND TECHNOLOGY

DEPARTMENT OF CIVIL ENGINEERING

SURVEYING-II

LECTURE NOTES

ON

UNIT-IV

CURVES

Introduction:

Curves are generally in bends used on highways, railways, canals due to natures of terrain, cultural features and some other unavoidable reasons to bring out the gradual change of direction such bends is called curves .basically curves are very useful in highway for the purposes of to keep the diver alert while driving. Curves are mainly divided into teo types they are, Horizontal curve and Vertical curve.

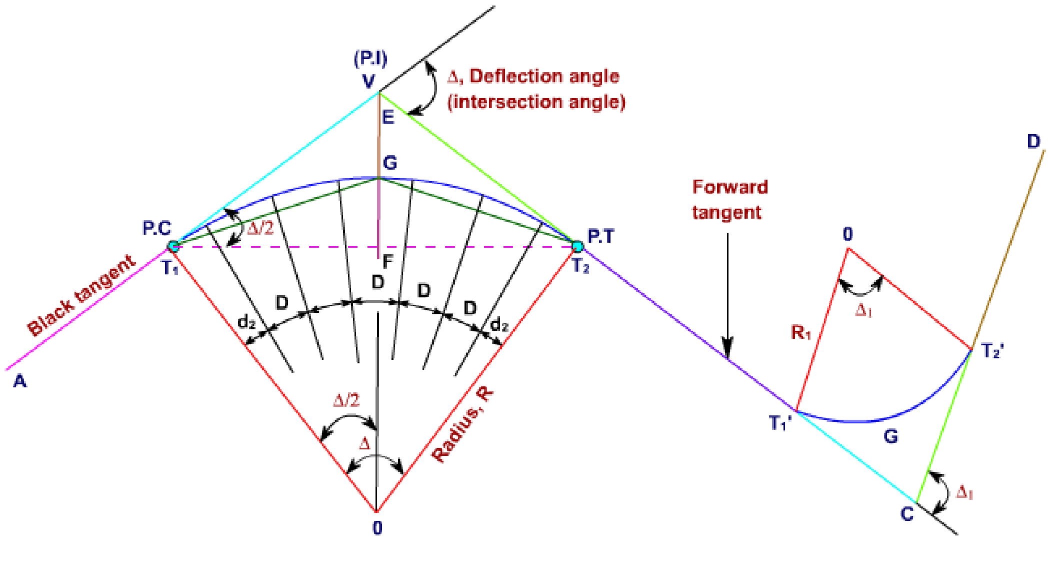
Horizontal Curve: These curves mainly used in plane surface and horizontal direction and it can be sub divided into 4 types they are:

- Simple Curves
- Compound Curves
- Reverse Curves
- Transition curves

Vertical Curve: These curves mainly sub divided into 2 types they are:

- Valley Curves
- Summit Curves

Simple Curves: A Simple Curve consists of a single arc of a circle and the curve is tangential of two straight line of a route.



Basic Definitions and Notations:

Radius: The radius of circle of which curve is an arc, and it is denoted by' R'

Back Tangent: The tangent line before the beginning of the curve is called back tangent (or) rear tangent. (AT_1)

Forward Tangent: The tangent line after end of the curve is called forward tangent (T_2B)

Vertex: The point at which extension of the back tangent and the forward tangent meet is known as the Vertex (V) or point of intersection (P.I.).

Intersection angle (I): The exterior angle at the vertex or point of intersection is known as the Intersection angle (I). It is also known as Deflection angle (D) as it represents the deflection angle between the back tangent and the forward tangent. Thus, angle between the line AV produced beyond the vertex V and the line VC represents I (or D).

Point of Curvature (P.C.) : The point on the back tangent where the curve begins is known as the Point of Curvature (P.C.). At this point, the alignment of the route changes from a straight line to a curve. This is represented by T1.

Point of tangency (P.T.): The point on the forward tangent where the curve ends is known as the Point of tangency (P.T.). At this point, the alignment of the route changes from a curve to a straight line. It is represented by T2

Tangent distance (T): The distance between the point of curvature (T1) to the point of intersection (V) along the extension of back tangent is known as Tangent distance (T). It is also equal to the distance between the points of tangency (T2) to the point of intersection along the extension of forward tangent.

External distance (E): The distance between the point of intersection (V) and the middle point of the curve is called as External distance (E).

Long chord (L) : The longest possible chord of the circular curve is known as Long chord (L). It is the line joining the point of curvature (T1) and the point of tangency (T2).

Mid-ordinate (M): The distance between the middle point of the curve and the middle point of the long chord is Mid-ordinate (M).

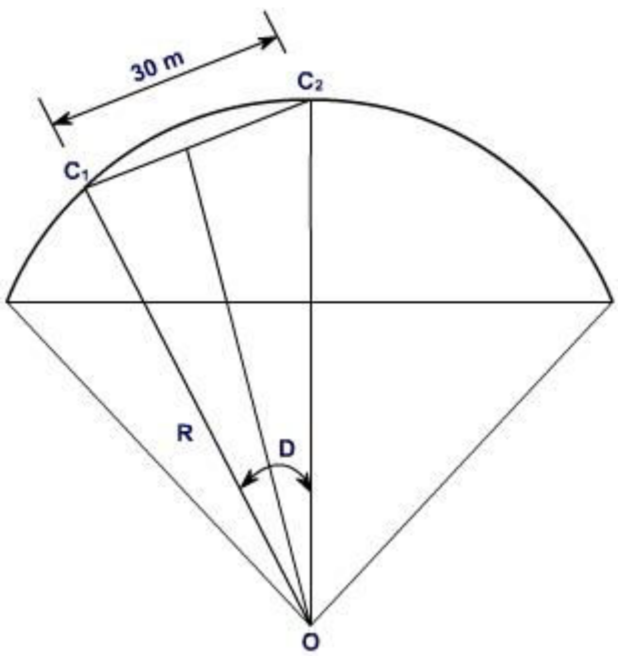
Length of curve (l): The length of the alignment along the curve between the point of curvature (T1) and the point of tangency (T2) is known as the Length of curve (l).

Left -hand curve and Right hand curve: During the progress of the route, if the direction of deflection is to the right then it is called Right-hand curve (T1GT2) and it is called left -hand curve, if the curve deflects to the left T'1G'T'2.

Designation of a Curve:

- A curve is designated either in terms of its degree (D) or by its radius (R).
- The degree of a curve (D) is defined as the angle subtended at the centre of the curve by a chord or an arc of specified length
- The degree of a curve is defined as the angle subtended at the centre of the curve by a chord of 30 m length.

  Chord Definition:



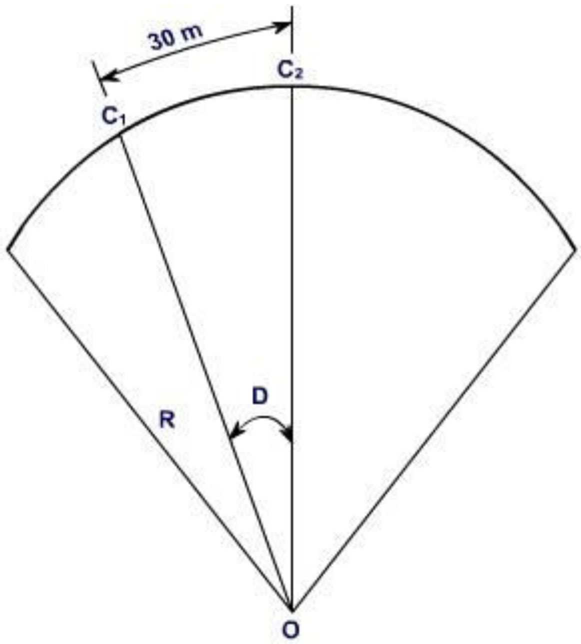
- Let D be the degree of a curve i.e., it is the angle subtended at its centre O by a chord C1C2 of 30 m length as shown in Thus

$$\sin\frac{D^\circ}{2} = \frac{(30/2)}{R}, \text{ where R is the radius of the circular curve}$$

$$\therefore R = \frac{15}{\sin\left(\frac{D^\circ}{2}\right)}$$

Arc Definition:

The degree of a curve is defined as the angle subtended at its centre of the curve by an arc of 30 m length.

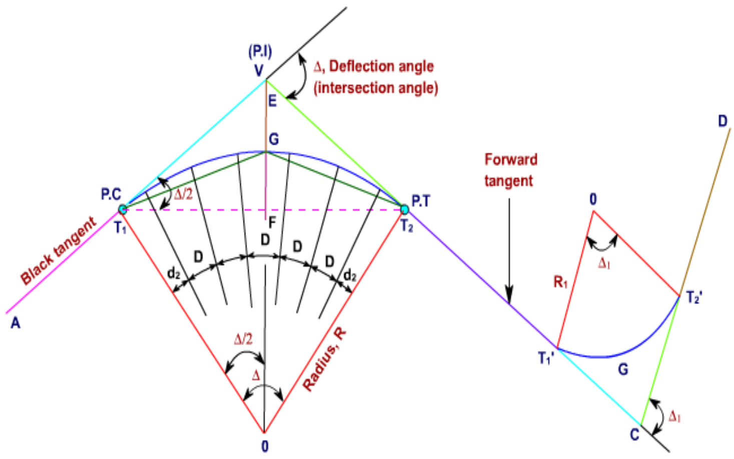


$$\frac{D^\circ}{30} = \frac{360^\circ}{2\pi R}$$

$$\text{Or, } D^\circ = \frac{1718.9}{R} \text{ degrees}$$

Radius of Curve: In this convention, a curve is designated by its radius. The sharpness of the curve depends upon its radius. A sharp curve has a small radius. On the other hand, a flat curve has a large radius. Moreover, from (Equation) it can be found that the degree of curve is inversely proportional to the radius of curve. Thus, a sharp curve has a large degree of curve, whereas a flat curve has a small degree of curve.

Elements of Curve:



Let T1GT2 be the circular curve that has been provided between the tangents AV and VC. The deflection angle, D between the tangents is measured in the field. The radius of curvature is the design value as per requirement of the route operation and field topography. The line joining O and V bisects the internal angles at V and at O, the chord T1T2 and arc T1GT2. It is perpendicular to the chord T1T2 at FRT1 O T2 = D and

$$\angle T_1OV = \angle VOT_2 = \angle VT_1T_2 = \angle VT_2T_1 = \frac{\Delta}{2}$$

To compute the elements of a circular curve, consider the radius of the curve OT1 = OT2 = R. Further, it is known that the RVT1 O = RVT2 O = 90° (since the tangent to a circle is perpendicular to the radius at the point of tangency). The elements of a circular curve required to lay it out in the field with reference to Figure 37.1 are as follows :

Length of Curve,
l = T1 GT2

$$= \left(\frac{2\pi R}{360^\circ}\right) \times \Delta$$

$$= \frac{\pi R \Delta}{180^\circ}$$

Tangent Length,

T = length T1 V = length T2 V

$$= O T_1 \tan\frac{\Delta}{2} = R \tan\frac{\Delta}{2}$$

Chainages of tangent point: The chainage of the point of intersection (V) is generally known.
Chainage of T1 = Chainage of V - tangent length (T)
Chainage of T2 = Chainage of T1 + length of curve (l)

Length of the long chord (L) : Length of the long chord,
L = length T1 FT2

$$= 2 \times R \sin \frac{\Delta}{2}$$

External distance (E) :
E = length VG
= VO - GO

$$= R \sec \frac{\Delta}{2} - R$$

$$= R [\sec \frac{\Delta}{2} - 1]$$

Mid-ordinate (M) :
M = length GF = OG-OF

$$= R - R\cos \frac{\Delta}{2}$$

$$= R(1 - \cos \frac{\Delta}{2})$$

Location of tangent points:

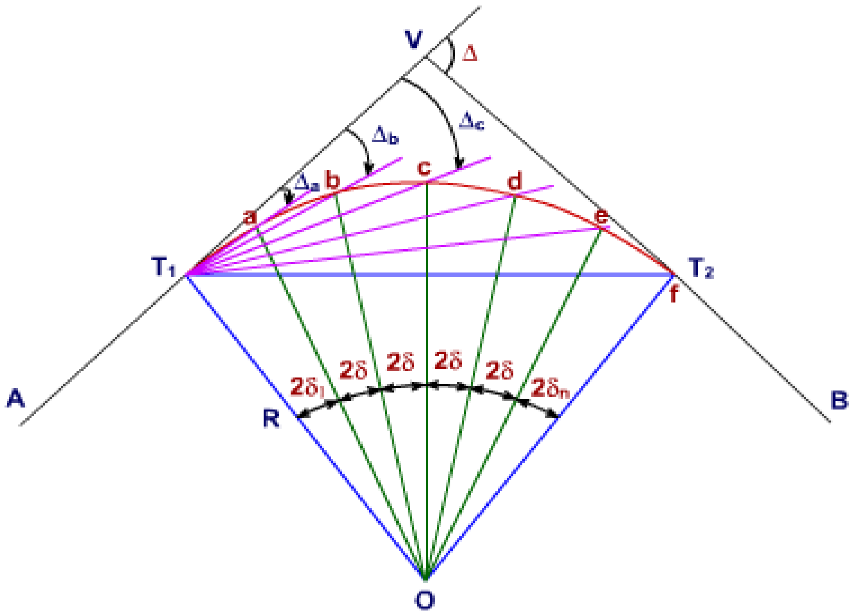
- Set up a t P.I. and layout tangents to establish P.C. and P. T.
- Set up a t P.C. and sight back to P.I. Set plates to 00 o 00’ 00”.
- Turn off first deflection angle on transit and measure out the first sub chord distance, pound in hub and write stationing on it.
- Continue laying out chords and placing hubs. When the last station (P.T. Is reached the last sub chord measured out should be quite close t o the P.T. hub established earlier. If it is not, an error has been made and the curve has to be run in all over again
- It is advisable to use intermediate s e t u p s when setting out the curve. They prevent communication difficulties and tend to reduce the number of errors.

Selection of peg intervals:
Methods of Simple Curve: There are totally five methods of simple circular curves they are mainly:

- Tape method
- Tape and Theodolite method
- Two Theodolite method
- Tachometric method
- Total Station method

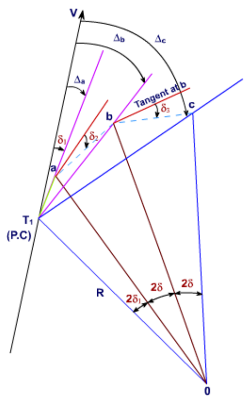
Tape and Theodolite method: In this method, both the linear and angular measurements are carried out simultaneously to stake points along which curve will be set out. A tape is used for the linear measurements, whereas a theodolite is used for the angular measurements. This method is quite accurate and is commonly used in practice.

In this method, curves are staked out by use of deflection angles turned at the point of curvature from the tangent to points along the curve. The curve is set out by driving pegs at regular interval equal to the length of the normal chord. Usually, the sub-chords are provided at the beginning and end of the curve to adjust the actual length of the curve. The method is based on the assumption that there is no difference between length of the arcs and their corresponding chords of normal length or less. The underlying principle of this method is that the deflection angle to any point on the circular curve is measured by the one-half the angle subtended at the centre of the circle by the arc from the P.C. to that point



Let points a, b, c, d, e are to be identified in the field to layout a curve between T1 and T2 to change direction from the straight alignment AV to VB as in Figure . To decide about the points, chords ab, bc, cd, de are being considered having nominal length of 30m. To adjust the actual length of the curve two sub-chords have been provided one at the beginning, T1 a and other, eT2 at the end of the curve. The amount of deflection angles that are to be set from the tangent line at the P.C. are computed before setting out the points. The steps for computations are as follows

Let the tangential angles for points a, b, c,… be δ_1, δ,..., δ, δ_n and their deflection angles (from



the tangent at P.C.) be Δ_a, Δ_b, ….. , Δ_n

Now, for the first tangential angle d1, from the property of a circle

Arc T1 a = R x 2d1 radians

Assuming the length of the arc is same as that of its chord, if C1 is the length of the first chord i.e., chord T1 a, then

$$\delta_1 = \frac{C_1}{2R} \text{ radians}$$

$$= \frac{180^\circ C_1}{2\pi R} \text{ degrees}$$

$$= \frac{180 \times 60 C_1}{2\pi R} \text{ minutes}$$

$$= 1718.9 \frac{C_1}{R} \text{ minutes}$$

Similarly, tangential angles for chords of nominal length, say C,

$$\delta = 1718.9 \frac{C}{R} \text{ minutes}$$

And for last chord of length, say Cn

$$\delta_n = 1718.9 \frac{C_n}{R} \text{ minutes}$$

The deflection angles for the different points a, b, c, etc. can be obtained from the tangential angles. For the first point a, the deflection angle Da is equal to the tangential angle of the chord to this point i.e., d1. Thus,

$$\Delta_a = \delta_1.$$

The deflection angle to the next point i.e., b is Db for which the chord length is T1 b. Thus, the deflection angle

$$\Delta_b = \frac{1}{2}\angle T_1Ob$$

$$= \frac{1}{2}(2\delta_1 + 2\delta)$$

$$= \Delta_a + \delta$$

$$\text{Similarly,}\quad \Delta_c = \frac{1}{2}\angle T_1Oc$$

$$= \frac{1}{2}(2\delta_1 + 2\delta + 2\delta).$$

$$= \Delta_b + \delta$$

$$\text{Like wise,}\quad \Delta n = \Delta_{n-1} + \delta_n$$

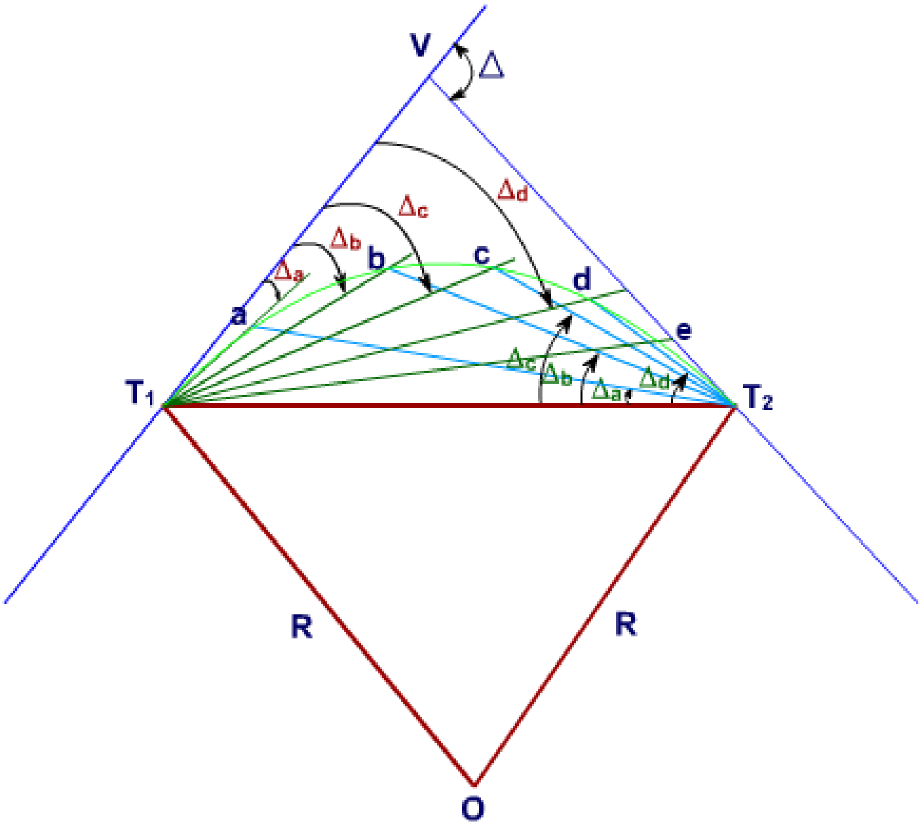
Thus, the deflection angle for any point on the curve is the deflection angle up to previous point plus the tangential angle at the previous point.

Procedure:

- A theodolite is set up at the point of curvature T1, and gets it temporary adjusted.
- The vernier A is set to zero, and get the upper plate clamped. After opening the lower plate main screw, sight the point of intersection, V. Then the lower plate main screw gets tightened and get the point V bisected exactly using the lower plate tangent screw. Now the line of sight is in the direction of the rear tangent T1 V and the vernier a reads zero.
- Open the upper plate main screw, and set the vernier A to the deflection angle Da. The line of sight is now directed along the chord T1 a. Clamp the upper plate.
- Hold the zero end of the tape of a steel tape at T1. Note a mark equal to the first chord length C1 on the tape and swing an arrow pointed at the mark around ‘a' till it is bisected along the line of sight. The arrow point then indicates the position of the first peg ‘a'. Fix the first peg at ‘a'.
- Unclamp the upper plate, and set the vernier A to the deflection angle Db. The line of sight is now directed along T1 b.
- .With the zero end of the tape at a, and an arrow at a mark on the tape equal to the normal chord length C, swing the tape around b until the arrow is bisected along the line of sight. Fix the second peg at the point b at the arrow point.
- It may be noted that the deflection angles are measured from the tangent point T1 but the chord lengths are measured from the preceding point. thus, deflection angles observed are cumulative in nature but chord lengths swung are individual in nature.
- Repeat steps (5) and (6) till the last point is reached. The last point so located must coincide with the tangent point T2 already fixed from the point of intersection.

Two Theodolite Method:

In two theodolite method, curves are staked out by angular measurements only. Accuracy attained in this method is quite high. Thus, the method is used when higher accuracy is required and when the topography is rough or field condition is difficult. The underlying principle of this method is that the deflection angle between a tangent (at any point on a circle) and a chord is equal to the angle which the chord subtends in the alternate segment

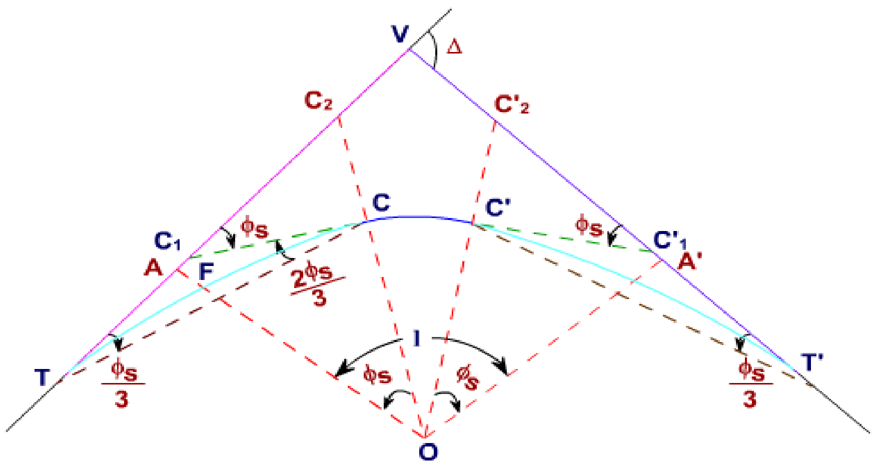


In two theodolite are used simultaneously placing one at the point of curvature (T1) and the other at the point of tangent (T2). Deflection angles for specified chord lengths are computed as defined in the Rankine's method. The deflection angles are set at the theodolite. Ranging from both the theodolite at the defined angles provides the location of the point along curve. Thus, the curve is set out by driving pegs at suitable location identified through the theodolite.

Procedure:

- Two theodolite are placed, one at the point of curvature T1 and the other at the point of curvature T2. Get temporary adjustment in both. The vernier A of each theodolites set to zero and clamp the upper plates.
- Bisect the point of intersection, V from theodolite at T1 and T1 from the theodolite at T2 using the lower plate main screw and then its tangent screw. Now both the theodolite are properly oriented.
- Open the upper plate main screw of the theodolite at T1, and set the vernier A to the deflection angle D1. The line of sight is now directed along the chord T1 a. Clamp the upper plate.
- Release the upper clamp of the theodolite at T2 and set the vernier A to the angle D1. The line of sight is now directed along the chord T2 a.
- Thus the lines of sight of both the theodolite are directed towards the point 'a'.
- Now, move a ranging rod or an arrow near the expected point 'a' until it is bisected simultaneously by the cross-hairs of both the theodolite. Locate the point 'a' on the ground at the arrow point and fix a peg at that point.
- To locate the second point 'b', set the vernier of both the theodolite at angle Db and repeat steps (3) to (5) .
- Locate all other points' c, d, e..... in the same manner.

Compound Curve: Compound curve is two or more simple curves or simple arcs with different radii is known as Compound Curve



For setting out a combined transition curve and circular curve, data required to be known are Figure:

- The deflection angle D between the tangents;
- The radius R of the circular curve
- The length L of the transition curve;
- The chain-age of the point of intersection, V of the tangents

Elements of compound curve:

1. Spiral angle, $\varphi_s = \frac{L}{2R}$

2. Shift, $s = \frac{L^2}{24R}$

3. Total tangent length, $T_t = (R + s)\tan\frac{\Delta}{2} + (\frac{L}{2})$;

4. Length of the circular curve, $l = \frac{\pi R}{180}(\Delta - 2\Phi_s)$

5. Chainage of the salient points

   Chainage of T = Chainage of V - T_t

   Chainage of C = Chainage of T + L

   Chainage of C' = Chainage of C + l

   Chainage of T' = Chainage of C' + L

6. Lengths of the normal chords are generally considered as 10m for transition curve and 20m for circular curve. Lengths of sub - chords are as required to adjust field condition.

7. Deflection angles:

   (i) For transition curve

$$\alpha = \frac{1800 l^2}{\pi R L} \text{ minutes}$$

   Length l is measured from tangent point T. The deflection angle for each chord is the total angle referred to the initial tangents and not to the previous chord.

Check : $\alpha_n = \frac{1800 L^2}{\pi R L} = \frac{1800 L}{\pi R} = \frac{\Delta}{3}\left[\begin{array}{c}\text{Deflection angle at junction of} \\ \text{circular and transition curve}\end{array}\right]$

   (ii) For circular curve

   For a chord of length c (m), the deflection angle from the tangent at C is

$$\delta = \frac{1718.9c}{R} \text{ minutes},$$

$$\Delta_1 = \delta_1$$

$$\Delta_2 = \delta_1 + \Delta_1$$

$$\Delta_n = \Delta_{n-1} + \delta_n = \frac{1}{2}(\Delta - 2\Delta_s)$$

For a chord of length c (m), the deflection angle from the tangent at C is

$$\delta = \frac{1718.9c}{R} \text{ minutes,}$$

$$\Delta_1 = \delta_1$$

$$\Delta_2 = \delta_1 + \Delta_1$$

$$\Delta_n = \Delta_{n-1} + \delta_n = \frac{1}{2}(\Delta - 2\Delta_s)$$

Procedure:

- The procedure for setting out a combined curve (consisting of a simple circular curve with transition curve at each end) by method of deflection angles with reference to :
- Locate the tangent point T, by measuring back the total tangent length (Tt) along the back tangent, from the point of intersection V.
- Likewise, locate the tangent point T' by measuring along the forward tangent the distance Tt from V.
- **:**
- Set a theodolite over the point T. Set the vernier A to zero, and clamp the upper plate.
- Direct the line of sight of theodolite to the intersection point V, and clamp the lower plate.
- Release the upper plate. Set the vernier A to the first deflection angle (a1).
- The line of sight now points towards the first peg on the transition curve.
- With the zero of the tape pinned at T and an arrow kept at the mark corresponding to the first length of the chord, the assistant will swing the tape till the arrow is bisected by the line of sight.
- Fix the first peg at the arrow point.
- Set the vernier A on the second deflection angle (a2) to direct the line of sight to the second peg.
- With the zero of the tape pinned at T, and keeping an arrow at the mark corresponding to the total length of the first and second chords, the assistant will swing an arc till the arrow is bisected by the line of sight.
- Fix the second peg at the arrow point. It should be remembered that the distance is measured from the point T and not from the preceding point.
- Repeat steps (6) and (7) till the last point C on the transition curve is reached.: For setting out the circular curve CC', shift the theodolite to junction point C.
- Orient the theodolite with reference to the common tangent CC1 by directing the line of sight towards CT with the vernier. A set at a reading equal to $\left(360^\circ - \frac{2}{3}\varphi_s\right)$, and swinging the telescope clockwise in azimuth by $\frac{2}{3}$ fs .
- Now the line of sight is directed along the common tangent CC1 and the vernier reads zero.
- Plunge the telescope. The line of sight is now directed along the tangent C1C produced.

- The deflection angles D1, D2, etc. have been calculated with reference to the tangent C1C produced at C (The line of sight is now correctly oriented, and the reading of the vernier A is zero.
- Set the vernier A to the first deflection angle D1, and locate the first peg on the circular curve at a distance of c' from C, where c' is length of the first sub-chord.
- Likewise, locate the second peg on the circular curve at the distance c equal to the normal chord from the first peg with the deflection angle D2 at C.
- Continue the above process till the junction point C' is reached.
- Set out the transition curve T'C' from T' using the same procedure as that for the transition curve TC.

Reverse Curve: A reverse curve (or "S" curve) is a section of the horizontal alignment of a highway or railroad route in which a curve to the left or right is followed immediately by a curve in the opposite direction.

A reverse curve is composed of two or more simple curves turning in opposite directions. Their points of intersection lie on opposite ends of a common tangent, and the PT of the first curve is coincident with the PC of the second. This point is called the point of reverse curvature (PRC). Reverse curves are useful when laying out such things as pipelines, flumes, and levees. The surveyor may also use them on low-speed roads and railroads. They cannot be used on high-speed roads or railroads since they cannot be properly super elevated at the PRC. They are sometimes used on canals, but only with extreme caution, since they make the

The computation of reverse curves presents three basic problems. The first is where the Reverse curve is to be laid out between two successive surveyor performs the computations inexactly the same manner as a compound curve between successive the second is where the curve is to be laid out so it connects two parallel tangents

A reverse curve consists of two simple curves of opposite direction that join at a common tangent point called the point of reverse curvature (P.R.C.). They are used when the straights are parallel or include a very small angle of intersection and are frequently encountered in mountaineous countries, in cities, and in the layout of railway spur tracks and cross-over. The use of reverse curve should be avoided on highways and main railway lines where speeds are high for the following reasons :

(1) Sudden change of cant is required from one side of P.R.C. to the other.

(2) There is no opportunity to elevate the outer bank at P.R.C.

(3) The sudden change of direction is uncomfortable to passengers and is objectionable.

(4) Steering is dangerous in the case of highways and the driver has to be very cautious.

It is definitely an advantage to separate to curves by either a short length of straight or a reversed spiral. The elements of a reverse curve are not directly determinate unless some condition or dimension is specified as, for example, equal radii ($R_1 = R_2$) or equal central angle ($\Delta_1 = \Delta_2$). Frequently, a common or equal radius is used for both parts of the curve in order to use largest radius possible.

Fig. 2.32 shows the general case of a reverse curve in which VA and VC are the two straights and $T_1 E T_2$ is reverse curve. T_1 is the point of curvature (P.C.), E is the point of reverse curvature (P.R.C.) and T_2 is the point of tangency (P.T.). O_1 and O_2 are the centres of the two branches. BD is the common tangent.

Let R_1 = the smaller radius

R_2 = the greater radius

Δ_1 = central angle for the curve having smaller radius

Δ_2 = central angle for the curve having greater radius (Δ_1 is greater than Δ_2)

Δ = total deviation between the tangents

δ_1 = angle between tangent AV and the line T_1T_2 joining the tangent points

δ_2 = angle between tangent VC and the line T_2T_1 joining the tangent points.

Since E is the point of reverse curvature, the line O_1O_2 is perpendicular to the common tangent BD at E. Join T_1 and T_2 and drop perpendicular O_1F and O_2G on it from O_1 and O_2 respectively. Through O_1, draw O_1H parallel to T_1T_2 to cut the line O_2G produced in H.

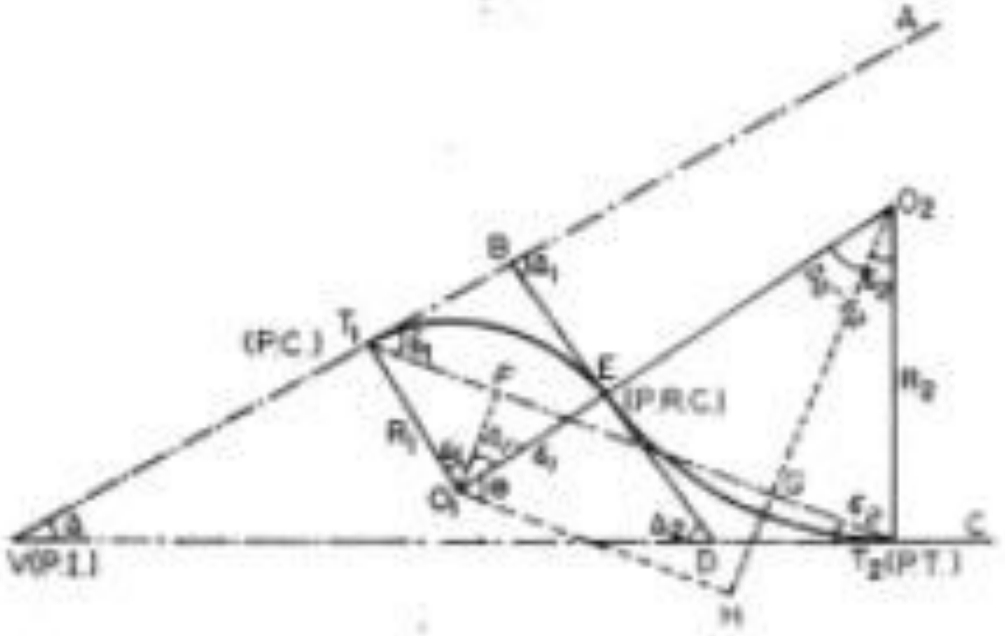


FIG. 2.32. REVERSE CURVE ($\Delta_1 > \Delta_2$)

Since T_1B and BE are tangents to the first arc, $\angle ABE = \Delta_1$. Similarly, since ED and DT_2 are tangents to the second arc, $\angle EDV = \Delta_2$.

From triangle BVD, $\Delta_1 = \Delta + \Delta_2$

or $\Delta = \Delta_1 - \Delta_2$ (1) ...(2.27)

From triangle T_1VT_2, $\delta_1 = \Delta + \delta_2$

$\Delta = \delta_1 - \delta_2$ (2)

From (1) and (2), $\Delta_1 - \Delta_2 = \delta_1 - \delta_2$ (3) ...(2.28)

Since T_1O_1 is $\perp$ to T_1B and O_1F is $\perp$ to T_1T_2 we have

$\angle T_1O_1F = \angle BT_1F = \delta_1$

Similarly, $\angle T_1O_1G = \angle FT_1D = \delta_1$

Hence $\angle FO_1E = \Delta_1 - \delta_1$ and $\angle EO_2G = \Delta_2 - \delta_2$

Since O_1F and O_2G are parallel, we have

$\angle FO_1E = EO_2G$

or $(\Delta_1 - \delta_1) = (\Delta_2 - \delta_2)$ (3a)

which is the same as obtained in (3).

Again, $T_1F = R_1 \sin \delta_1$

$T_2G = R_2 \sin \delta_2$

curve.

RELATIONSHIPS BETWEEN VARIOUS PARTS OF A REVERSE CURVE

The various quantities involved in a reverse curve are Δ, Δ_1, Δ_2, δ_1, δ_2, R_1 and R_2. In order to co-relate these, three quantities and one condition equation (of either equal radius or equal central angle) must be known. We shall consider variour cases of common occurrence.

CASE 1. NON-PARALLEL STRAIGHTS

Given. The central angles Δ_1 and Δ_2, ($\Delta_2 > \Delta_1$) and the length of the common tangent.

Required. To find length of the common radius R and the chainages of T_1, E and T_2 if that of V is given.

Condition equation. $R_1 = R_2 = R$.

In Fig. 2.33, BD = common tangent of length d

$$OE_1 = EO_2 = R$$

Other notations are the same in Fig. 2.32.

Since T_1B and BE are tangents to the first arc, they are equal in length and $\angle VBE = \Delta_1$

Similarly, the tangents T_2D and DE are equal in length and

$$\angle EDC = \Delta_2$$

$$\therefore \qquad BT_1 = BE = R \tan \frac{\Delta_1}{2}$$

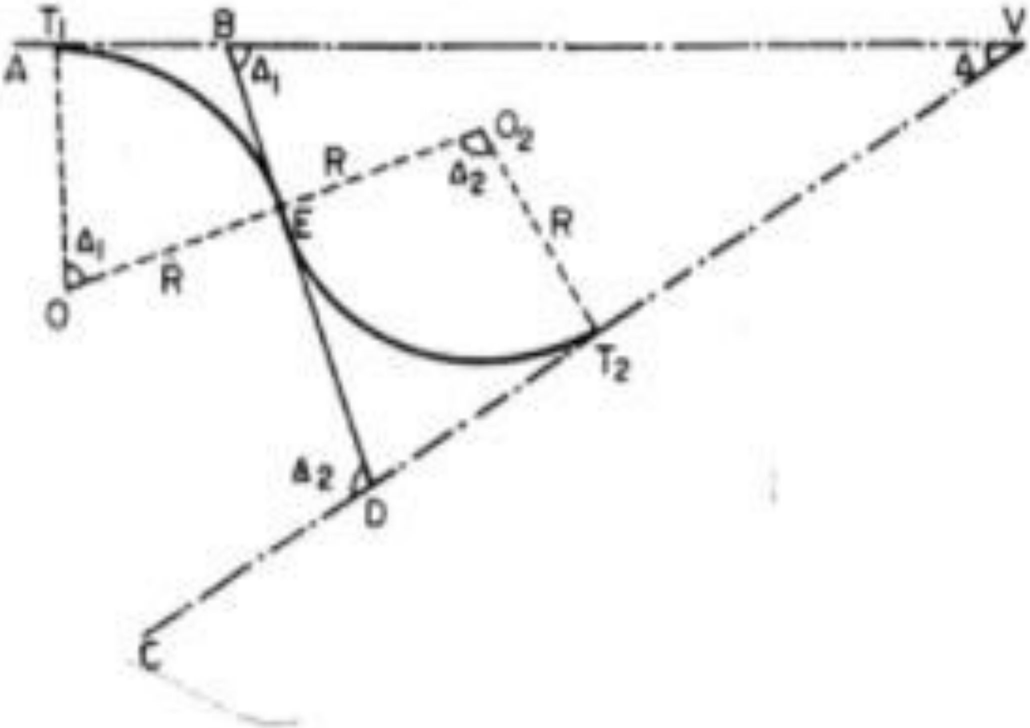


FIG. 2.33.

$$DT_2 = DE = R \tan \frac{\Delta_2}{2}$$

$\therefore$
$$BD = d = R \tan \frac{\Delta_1}{2} + R \tan \frac{\Delta_1}{2}$$

Hence
$$R = \frac{d}{(\tan \frac{1}{2} \Delta_1 + \tan \frac{1}{2} \Delta_2)} \qquad ...(2.31)$$

Knowing R_1, Δ_1 and Δ_2, lengths of the two arcs can be calculated.

Again, $\Delta = \Delta_2 - \Delta_1$

From triangle BDV,

$$BV = BD \,.\, \frac{\sin \Delta_2}{\sin \Delta} = d \frac{\sin \Delta_2}{\sin \Delta}$$

$\therefore$
$$T_1V = BT_1 + BV = R \tan \frac{\Delta_1}{2} + d \frac{\sin \Delta_2}{\sin \Delta}$$

Chainage of T_1 = chainage of $V - T_1V$.

Chainage of E = chainage of T_1 + length of first arc.

Chainage of T_2 = chainage of E + length of second arc.

The first branch of the curve can be set out from T_1 and the second branch from E by method of tangential angles.

CASE 2. NON-PARALLEL STRAIGHTS

Given. Length L of the line joining the tangents T_1 and T_2 and angles δ_1 and δ_2 which the line joining the tangent points makes with the two tangents.

Required. To find the common radius R.

Condition equation. $R_1 = R_2 = R$

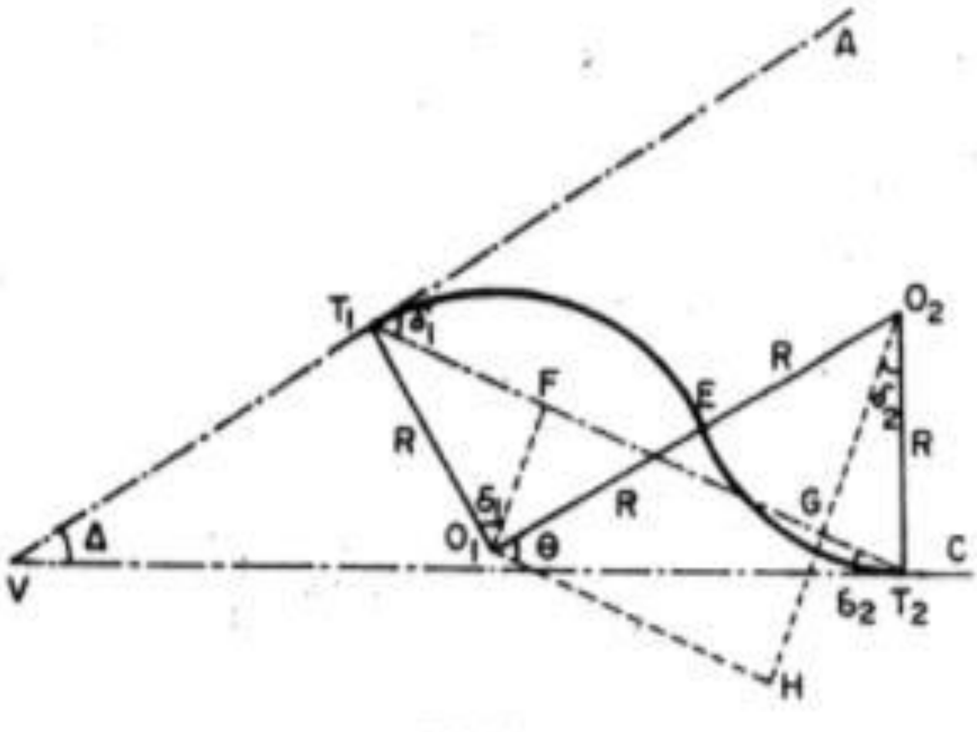


FIG. 2.34

In Fig. 2.34, let T_1 and T_2 be the two tangent points, the distance T_1T_2 being equal to L. The notations etc. are the same in Fig. 2.32. Draw O_1F and O_2G perpendicular to T_1T_2. Through O_1 draw O_1H parallel to T_1T_2, meeting O_2G produced in H. Let $\angle O_2O_1H = \theta$

Now
$$O_1F = R\cos\delta_1 = GH$$
$$O_2G = R\cos\delta_2$$
$$O_1O_2 = 2R$$

$$\sin\theta = \frac{O_2H}{O_1O_2} = \frac{O_2G+GH}{O_1O_2} = \frac{R\cos\delta_1+R\cos\delta_2}{2R}$$

$$\theta = \sin^{-1}\frac{\cos\delta_1+\cos\delta_2}{2} \qquad ...(2.32)$$

$$T_1F = R\sin\delta_1$$

$$FG = O_1H = 2R\cos\theta$$

and $$GT_2 = R\sin\delta_1$$

$$\therefore \quad T_1T_2 = T_1F + FG + GT_2 = L$$

or $$R\sin\delta_1 + 2R\cos\theta + R\sin\delta_2 = L$$

$$\therefore \quad R = \frac{L}{\sin\delta_1 + 2\cos\theta + \sin\delta_2} \qquad ...(2.33)$$

where θ is given by the equation 2.32.

The central angle for the first branch

$$= \Delta_1 = \delta_1 + (90° - \theta)$$

The central angle for the second branch

$$= \Delta_2 = \delta_2 + (90° - \theta)$$

Knowing R, Δ_1 and Δ_2 the lengths of the arcs can be calculated.

CASE 3. NON-PARALLEL STRAIGHTS

Given. Length L of the line joining the tangent points T_1 and T_2, the angles δ_1 and δ_2 which the line joining the tangent points makes with the two tangents, *and any one of the two radii.*

Required. To find the other radius.

Refer Fig. 2.32 in which R_1 is smaller radius and R_2 is the greater radius.

$$T_1F = R_1\sin\delta_1$$

$$T_2G = R_2\sin\delta_2$$

$$FG = O_1H = \sqrt{(O_1O_2)^2 - (O_2H)^2}$$

$$= \sqrt{(R_1+R_2)^2 - (R_1\cos\delta_1 + R_2\cos\delta_2)^2}$$

Now $\qquad T_1T_2 = L = T_1F + FG + T_2G$

or $\qquad L = R_1 \sin\delta_1 + \sqrt{(R_1+R_2)^2 - (R_1\cos\delta_1 + R_2\cos\delta_2)^2} + R_2\sin\delta_2$

or $\qquad \{L - (R_1\sin\delta_1 + R_2\sin\delta_2)\}^2 = \{(R_1+R_2)^2 - (R_1\cos\delta_1 + R_2\cos\delta_2)^2\}$

or $\qquad L^2 + R_1^2\sin^2\delta_1 + R_2^2\sin^2\delta_2 + 2R_1R_2\sin\delta_1\sin\delta_2$

$$- 2L(R_1\sin\delta_1 + R_2\sin\delta_2)$$

$$= R_1^2 + R_2^2 + 2R_1R_2 - (R_1^2\cos^2\delta_1$$

$$+ R_2^2\cos^2\delta_2 + 2R_1R_2\cos\delta_1\cos\delta_2)$$

or $\qquad L^2 - 2L(R_1\sin\delta_1 + R_2\sin\delta_2) + R_1^2(\sin^2\delta_1 + \cos^2\delta_1)$

$$+ R_2^2(\sin^2\delta_2 + \cos^2\delta_2)$$

$$= R_1^2 + R_2^2 + 2R_1R_2 - 2R_1R_2\cos\delta_1\cos\delta_2$$

$$- 2R_1R_2\sin\delta_1\sin\delta_2$$

or $\qquad L^2 - 2L(R_1\sin\delta_1 + R_2\sin\delta_2) = 2R_1R_2 - 2R_1R_2\cos(\delta_1 - \delta_2)$

or $\qquad L^2 - 2L(R_1\sin\delta_1 + R_2\sin\delta_2) = 4R_1R_2\sin^2\left(\frac{\delta_1 - \delta_2}{2}\right) \quad ...(2.34)$

Knowing R_1 (or R_2), we can calculate R_2 (or R_1) from the above equation. The angle O_2O_1H ($= \theta$) and hence Δ_1 and Δ_2 can then

CASE 4. PARALLEL STRAIGHTS

Given. The two radius R_1 and R_2 and the central angles.

Required. To calculate various elements.

Condition Equation $\Delta_1 = \Delta_2$

In Fig. 2.35, let AT_1 and T_2C be two straights parallel to each other so that there is no point of intersection.

Let R_1 = smaller radius

R_2 = larger radius

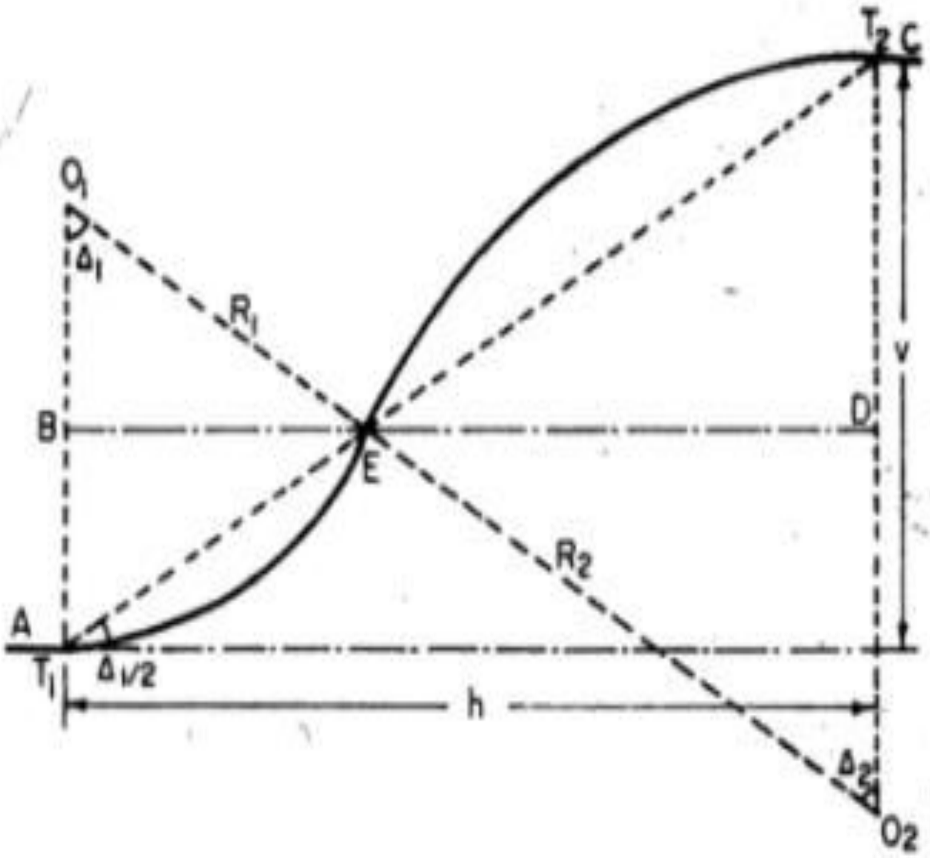


FIG. 2.35. REVERSE CURVE : PARALLEL TANGENTS.

Δ_1 = central angle corresponding to R_1

Δ_2 = central angle corresponding to R_2

L = distance T_1T_2

v = perpendicular distance between the two straights

h = distance between the perpendicular at T_1 and T_2

E = point of reverse curvature.

Through E, draw a line BD parallel to the two tangents. Since O_1T_1 and O_2T_2 are parallel to each other, we have

$$\Delta_1 = \Delta_2$$

$$T_1B = O_1T_1 - O_1B = R_1 - R_1 \cos \Delta_1$$
$$= R_1(1 - \cos \Delta_1) = R_1 \text{ versin } \Delta_1$$
$$T_2D = O_2T_2 - O_2D = R_2 - R_2 \cos \Delta_2 = R_2 - R_2 \cos \Delta_1$$
$$= R_2(1 - \cos \Delta_1) = R_2 \text{ versin } \Delta_1$$
$$v = T_1B + DT_2 = R_1 \text{ versin } \Delta_1 + R_2 \text{ versin } \Delta_1$$
$$= (R_1 + R_2) \text{ versin } \Delta_1 = (R_1 + R_2)(1 - \cos \Delta_1) \quad ...(2.35)$$

Again, $T_1E = 2R_1 \sin \frac{\Delta_1}{2}$

$$T_2E = 2R_2 \sin \frac{\Delta_2}{2} = 2R_1 \sin \frac{\Delta_1}{2}$$

$\therefore \quad T_1T_2 = L = T_1E + ET_2$

$$= 2R_1 \sin\frac{\Delta_1}{2} + 2R_2 \sin\frac{\Delta_1}{2}$$

$$= 2(R_1 + R_2)\sin\frac{\Delta_1}{2} \quad ...(2.36)$$

But $\sin\frac{\Delta_1}{2} = \frac{v}{L}$

$$L = 2(R_1 + R_2)\frac{v}{L}$$

From which, $L = \sqrt{2v(R_1 + R_2)}$ ...(2.37)

$BE = R_1 \sin\Delta_1, ED = R_2 \sin\Delta_2 = R_2 \sin\Delta_1$

$\therefore$ $BD = h = (R_1 \sin\Delta_1 + R_2 \sin\Delta_1)$ ...(2.38)

$$= (R_1 + R_2)\sin\Delta_1$$

Special case :

If $R_1 = R_2 = R$, we have

$v = 2R(1 - \cos\Delta_1)$ ...(2.35 *a*)

$L = 4R\sin\frac{\Delta_1}{2}$ ...(2.36 *a*)

$L = \sqrt{4Rv}$ ...(2.37*a*)

$h = 2R\sin\Delta_1$ ...(2.38 *a*)

Example 2.19. *Two parallel railway lines are to be* **connected** *by a reverse curve, each section having the same radius.* ***If the lines*** *are 12 meters apart and the maximum distance between tan***gent points** *measured parallel to the straights is 48 metres, find the* ***maximum*** *allowable radius.*

If however, both the radii are to be different, calculate the radius of the second branch if that of the first branch is 60 metres. Also, calculate the lengths of both the branches.

Solution. (Fig. 2.35)

(*a*) *Given* : $h = 48$ m and $v = 12$ m

$$\tan\frac{\Delta_1}{2} = \frac{v}{h} = \frac{12}{48} = 0.25 \text{ m}$$

$$\therefore \quad \frac{\Delta_1}{2} = 14°\,2' \text{ or } \Delta_1 = 28°\,4'$$

$$\sin\Delta_1 = 0.47049$$

Now $\quad BE = R\sin\Delta_1$ and $ED = R\sin\Delta_1$

$$\therefore \quad BE + ED = h = R\sin\Delta_1 + R\sin\Delta_1 = 2R\sin\Delta_1$$

or
$$R = \frac{h}{2\sin\Delta_1} = \frac{48}{2 \times 0.47049} = \mathbf{51.1\ m.}$$

(*b*) Let R_1 and R_2 be the radii.

As calculated above, $\Delta_1 = 28°\,4'$ and $\sin\Delta_1 = 0.47079$

Now, $\quad h = (R_1 + R_2)\sin\Delta_1$

$$\therefore \quad (R_1 + R_2) = \frac{h}{\sin\Delta_1} = \frac{48}{0.47049} = 102.2 \qquad ...(i)$$

If $\quad R_1 = 60$ m,

$$R_2 = 102.2 - R_1 = 102.20 - 60 = \mathbf{42.2\ m.}$$

Length of the first branch

$$= \frac{\pi R_1 \Delta_1}{180°} = \frac{\pi \times 60 \times 28°\,4'}{180°} = 29.38 \text{ m}$$

Length of the second branch

$$= \frac{\pi R_2 \Delta_1}{180°} = \frac{\pi \times 42.2 \times 28°\,4'}{180°} = \mathbf{20.67\ m.}$$