

## MATHEMATICS – II

(Common to all)

Time: 3 hours

Max. Marks: 70

PART – A  
(Compulsory Question)

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- 1 Answer the following: (10 X 02 = 20 Marks)
- Find  $L [t^2 \cdot e^t \cdot \cos 4t]$
  - Find the Laplace Transform of  $\frac{\sin 2t}{t}$ .
  - What are Dirichlet's conditions?
  - Express  $f(x) = x$  as a Fourier series from  $-\pi$  to  $\pi$ .
  - Write the formula of the Fourier cosine integral of  $f(x)$ .
  - Write the formula for the inverse Fourier transform of  $F(s)$  in  $(-\infty, \infty)$
  - Find the value of  $Z(a^n \cos nt)$
  - Find the Z-transform of the sequence  $\{x(n)\}$  where  $x(n)$  is  $n \cdot 2^n$
  - Derive a partial differential equation by eliminating the arbitrary function  $f$  from the relation:  $f(x^2 + y^2, x^2 - z^2) = 0$
  - Form the PDE from the relation  $z = f(x + it) + g(x - it)$ .

## PART – B

(Answer all five units, 5 X 10 = 50 Marks)

## UNIT – I

- 2 Find the inverse Laplace Transform of  $\frac{s}{(s^2 + a^2)^2}$  by using Convolution theorem.

OR

- 3 Solve  $(D^2 - D - 2)y = 20 \sin 2t$  where  $y(0) = 1, y'(0) = 2$ .

## UNIT – II

- 4 Find a Fourier series to represent  $x - x^2$  from  $x = -\pi$  to  $x = \pi$  and deduce that  $\frac{\pi^2}{12} = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \dots$

OR

- 5 If  $f(x) = \frac{\pi}{3}, 0 \leq x \leq \pi/3$   
 $= 0, \pi/3 \leq x \leq 2\pi/3$   
 $= -\pi/3, 2\pi/3 \leq x \leq \pi$

$$\text{Then } f(x) = \frac{2}{\sqrt{3}} \left[ \cos x - \frac{1}{5} \cos 5x + \frac{1}{7} \cos 7x + \dots \right]$$

## UNIT – III

- 6 Show that  $\int_0^{\infty} \frac{\sin \pi \lambda \sin \lambda x}{1 - \lambda^2} d\lambda = \frac{\pi}{2} \sin x$ , for  $0 \leq x \leq \pi$

$$= 0 \text{ for } x > \pi$$

OR

- 7 Find Fourier transform of  $f(x) = 1 - x^2$  for  $|x| \leq 1 = 0$  for  $|x| > 1$  and hence find  $\int_0^{\infty} \frac{x \cos x - \sin x}{x^3} \cos \frac{x}{2} dx$

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## UNIT - IV

- 8 Find the partial differential equation of all spheres whose centre lie on Z-axis and given by equation  $x^2 + y^2 + (z-a)^2 = b^2$ , a and b being constants

OR

- 9 A string is stretched and fastened to two points  $l$  apart. Motion is started by displacing the string in the form  $y = a \sin \frac{\pi x}{l}$  from which it is released at a time  $t=0$ . Show that the displacement of any point at a distance  $x$  from one end at time  $t$  is given by  $y(x,t) = a \sin\left(\frac{\pi x}{l}\right) \cos\left(\frac{\pi ct}{l}\right)$ .

## UNIT - V

- 10 Solve the difference equation, using Z-transform  $u_{n+2} - u_n = 2^n$ , where  $u_0 = 0$  and  $u_1 = 1$

OR

- 11 If  $f(z) = \frac{2z^2 + 3z + 4}{(z-3)^3}$ ,  $|z| > 3$ , then find the values of  $f(1)$ ,  $f(2)$ ,  $f(3)$ .

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Code: 15A54201

R15

B.Tech I Year II Semester (R15) Supplementary Examinations December 2016

**MATHEMATICS – II**

(Common to all)

Time: 3 hours

Max. Marks: 70

**PART – A**  
(Compulsory Question)

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- 1 Answer the following: (10 X 02 = 20 Marks)
- (a) Write the conditions for existence of Laplace transform of a function.
  - (b) Define Unit Impulse function.
  - (c) Write Dirichlet conditions for Fourier series.
  - (d) Write the Parseval's formula for Fourier series.
  - (e) Write the complex form of Fourier integral.
  - (f) Write any two properties of Fourier transform.
  - (g) What are the assumptions to be made for one dimensional wave equation?
  - (h) What do you mean by steady state and transient state?
  - (i) Find the Z-transform of  $\frac{1}{n}$ .
  - (j) Find  $Z^{-1} \left\{ \frac{z^2 - 2z}{(z-1)^2} \right\}$ .

**PART – B**

(Answer all five units, 5 X 10 = 50 Marks)

**UNIT – I**

- 2 (a) Find the Laplace transform of  $f(t) = |t-1| + |t+1|, t \geq 0$ .
- (b) Use Laplace transform to evaluate  $L \left\{ \int_0^{\infty} \frac{e^{-t} \sin t}{t} dt \right\}$ .

OR

- 3 (a) Apply Convolution theorem to evaluate  $L^{-1} \left\{ \frac{s}{(s^2 + a^2)^2} \right\}$ .
- (b) Solve  $ty'' + 2y' + y = \cos t, y(0) = 1$ .

**UNIT – II**

- 4 Find the Fourier series for  $f(x) = 1 + x + x^2$  in  $(-\pi, \pi)$ . Hence deduce that  $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \dots = \frac{\pi^2}{6}$ .

OR

- 5 (a) Expand  $f(x) = \cos x, 0 < x < \pi$  in a Fourier Sine series.
- (b) Find the complex form of the Fourier series of  $f(x) = e^{-x}$  in  $[-1, 1]$ .

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## UNIT - III

- 6 (a) Find Fourier cosine transform of  $e^{-x^2}$ .
- (b) Find Fourier transform of  $f(x) = \begin{cases} 1-x^2, & |x| \leq 1 \\ 0, & |x| > 1 \end{cases}$ .

OR

- 7 (a) Find Fourier sine transform of  $\frac{e^{-ax}}{x}$ .
- (b) Find the Finite Fourier sine and cosine transform of  $f(x) = 2x, 0 < x < 4$ .

## UNIT - IV

- 8 (a) Form the partial differential equation by eliminating the arbitrary functions  $f$  and  $g$  from:  
 $Z = f(2x + y) + g(3x - y)$ .

- (b) Solve by using the method of separation of variables the equation  $2x \frac{\partial z}{\partial x} - 3y \frac{\partial z}{\partial y} = 0$ .

OR

- 9 A rod of length 20 cm has its ends A and B kept at temperature  $30^\circ\text{C}$  and  $90^\circ\text{C}$  respectively until steady state conditions prevail. If the temperature at each end is then suddenly reduced to  $0^\circ\text{C}$  and maintained so, find the temperature distribution at a distance  $x$  from A at time  $t$ .

## UNIT - V

- 10 (a) If  $U(Z) = \frac{2z^2 + 5z + 14}{(z-1)^4}$  then find  $U_2$  and  $U_3$ .

- (b) Use convolution theorem to evaluate  $Z^{-1} \left\{ \frac{z^2}{(z-a)(z-b)} \right\}$ .

OR

- 11 Use Z-transform to solve:  $y_{n+2} - 2y_{n+1} + y_n = 3n + 5$ .

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## MATHEMATICS - I

(Common to CE, EEE, CSE, ECE, ME, EIE and IT)

Time: 3 hours

Max. Marks: 70

PART - A  
(Compulsory Question)

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1 Answer the following: (10 X 02 = 20 Marks)

- (a) If  $x = r \cos \theta$ ,  $y = r \sin \theta$  find  $\frac{\partial(x, y)}{\partial(r, \theta)}$ .
- (b) Find Particular Integral of  $(D^2 + 1)y = \cosh 2x$
- (c) Find the orthogonal trajectories of the family of curve  $ay^2 = x^3$
- (d) Solve  $y'' + 6y' + 9y = 0$ ,  $y(0) = -4$  and  $y'(0) = 14$
- (e) Solve  $\frac{dy}{dx} + y \tan x = \cos^3 x$
- (f) State Newton's law of cooling.
- (g) State Stokes theorem.
- (h) In what direction from  $(3, 1, -2)$ , direction derivative of  $f = x^2y^2z^4$  is maximum. Find the Maximum value.
- (i) Evaluate  $\int_1^a \int_1^b \frac{dy dx}{xy}$
- (j) Find the unit normal to the surface  $x^3 + y^3 + 3xyz = 3$  at the point  $(1, 2, -1)$ .

## PART - B

(Answer all five units, 5 X 10 = 50 Marks)

## UNIT - I

- 2 (a) Solve  $(1 + y^2) + (x - e^{\tan^{-1} y}) \frac{dy}{dx} = 0$
- (b) The number  $N$  of bacteria in culture grew at a rate proportional to  $N$ . The value of  $N$  was initially 100 and increases to 332 in one hour. What was value of  $N$  after  $1\frac{1}{2}$  hours.

OR

- 3 (a) Solve  $(D^2 - 1)y = xe^x \sin x$
- (b) Prove that the system of parabolas  $y^2 = 4a(x + a)$  is self orthogonal

## UNIT - II

- 4 Solve  $(D^2 + a^2)y = \tan ax$  by method of variation of parameter.

OR

- 5 Solve  $x^2 \frac{d^2 y}{dx^2} - 3x \frac{dy}{dx} - 5y = \sin(\log x)$

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## UNIT - III

- 6 (a) Verify whether the following functions are functionally dependence, if so, find the relation between them  $u = \frac{x+y}{1-xy}$ ,  $v = \tan^{-1}x + \tan^{-1}y$ .

- (b) Examine for Maxima and Minima of  $\sin x + \sin y + \sin(x+y)$

OR

- 7 Find a point at the plane  $3x + 2y + z - 12 = 0$  which is nearest to the origin.

## UNIT - IV

- 8 Evaluate the following integral by changing the order of integration  $\int_0^1 \int_{x^2}^{2-x} xy dx dy$

OR

- 9 (a) Show that the double integration, the area between the parabolas  $y^2 = 4ax$  and  $x^2 = 4ay$  is  $\frac{16}{3}a^2$ .

- (b) Evaluate the  $\int_0^1 \int_0^{1-x} \int_0^{1-x-y} x dz dx dy$

## UNIT - V

- 10 (a) Prove that  $\text{div}(\text{grad } r^m) = m(m+1)r^{m-2}$

- (b) Find the directional derivative of  $f = xy + yz + zx$  in the direction of vector  $i + 2j + 2k$  at the point  $(1, 2, 0)$ .

OR

- 11 Verify Green's theorem in the plane for  $\oint_C (3x^2 - 8y^2)dx + (4y - 6xy)dy$  where C is the region by  $y = \sqrt{x}$  and  $y = x^2$ .

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B.Tech II Year I Semester (R15) Regular Examinations November/December 2016

**MATHEMATICS – III**

(Common to CE, CSE, IT, ME, EEE, ECE &amp; EIE)

Time: 3 hours

Max. Marks: 70

**PART – A**

(Compulsory Question)

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1 Answer the following: (10 X 02 = 20 Marks)

$$(a) \text{ What is the rank of the matrix } \begin{pmatrix} 1 & 2 & 0 & 3 \\ 1 & -2 & 3 & 0 \\ 0 & 0 & 4 & 8 \\ 2 & 4 & 0 & 6 \end{pmatrix}$$

- (b) Explain Unitary matrix with proper example.  
 (c) What are the merits of Newton's method of iteration?  
 (d) Write the sufficient condition for Gauss Seidel method to converge.  
 (e) Write the formula of gauss forward formula.  
 (f) Write the formula of Stirling's formula.  
 (g) What is the use of method of least squares?  
 (h) Write about simpson's 3/8 rule.  
 (i) Write the formula of Taylor's method.  
 (j) What are the advantages of finite difference method?

**PART – B**

(Answer all five units, 5 X 10 = 50 Marks)

**UNIT – I**

2 Find the characteristic equation of the matrix  $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}$  and hence find its inverse. Use Cayley-Hamilton theorem.

OR

3 Find a matrix P which transforms the matrix,  $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$  to diagonal form.

**UNIT – II**

4 Determine the approximate root of the equation  $x^2 - 3x + 1 = 0$ , using Regula-Falsi method, up to 3-stages.

OR

5 Solve by Gauss Seidel method  $x - 2y = -3$ ,  $2x + 25y = 15$  correct to four decimal places.

**UNIT – III**

6 The table gives the distances in nautical miles of the visible horizon for the given heights in feet above the earth's surface:

x = height	100	150	200	250	300	350	400
y = distance	10.63	13.03	15.04	16.81	18.42	19.90	21.27

Find the values of y when: (i)  $x = 218$  ft. (ii)  $x = 410$  ft. Use Newton's formula.

OR

7 Given the values:

x	8	9	9.5	11
f(x)	150	392	1452	2366

Evaluate f (9.4), using Newton's divided difference formula.

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## UNIT - IV

- 8 Using Simpson's one third rule evaluate  $\int_0^6 xe^x dx$  taking 4 intervals. Compare your result with actual value.

OR

- 9 The following data related to drying time of a certain varnish and the amount of an additive that is intended to reduce the drying time.

amount of varnish additive(x)	0	1	2	3	4	5	6	7	8
Drying time(hours)y	12.0	10.5	10.0	8.0	7.0	8.0	7.5	8.5	9.0

Fit a second degree polynomial by the method of least square method.

## UNIT - V

- 10 Using Runge-Kutta method of fourth order, solve  $y' = (y^2 - x^2)/(y^2 + x^2)$  with  $y(0) = 1$  at  $x = 0.2, 0.4$ .

OR

- 11 Solve differential equation  $y'(x) = x + y$  satisfying  $y(0) = 1$  by Taylor series method and hence compute  $y(0.2)$  and  $y(0.4)$ . Compare the results with exact solution

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B.Tech I Year I Semester (R15) Regular & Supplementary Examinations December 2016

**MATHEMATICS - I**

(Common to CE, EEE, CSE, ECE, ME, EIE and IT)

Time: 3 hours

Max. Marks: 70

**PART - A**  
(Compulsory Question)

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Answer the following: (10 X 02 = 20 Marks)

- (a) Find the orthogonal trajectories of the family of parabolas through the origin and foci on the y - axis.
- (b) Find the complementary function  $(D^3 + 2D)y = e^{2x} + \cos(3x + 7)$ .
- (c)  $x^2 \frac{d^2y}{dx^2} + 3x \frac{dy}{dx} = 0$  has the general solution \_\_\_\_\_
- (d) Find  $P.I. (\theta^2 - 4\theta + 1)^{-1} \sin z$ .
- (e) If  $u = e^{x+y}, v = e^{-x+y}$ , then find  $J$ .
- (f) Find the radius of curvature at any point of the cardioids  $s = 4a \sin \frac{\psi}{3}$ .
- (g)  $\int_D \int (x^2 + y^2) dx dy = \text{_____}$   $D: y = x, y^2 = x$ .
- (h) Evaluate  $\int_0^1 dx \int_1^2 dy \int_1^3 xyz dz$ .
- (i)  $\nabla \times (\nabla \times \vec{A})$  is \_\_\_\_\_
- (j) Evaluate  $\int_C y^2 dx - 2x^2 dy$  along the parabola  $y = x^2$  from  $(0, 0)$  to  $(2, 4)$ .

**PART - B**

(Answer all five units, 5 X 10 = 50 Marks)

**UNIT - I**

2 Solve:  $x(x-1) \frac{dy}{dx} - y = x^2(x-1)^3$ .

OR

3 Solve:  $(D^3 + 2D^2 - 3D)y = xe^{3x}$ .

**UNIT - II**

4 Solve:  $(D^2 + a^2)y = \tan ax$  by the method of variation of parameters.

OR

5 The deflection  $y$  of a strut of length  $l$  with one end built-in and other end subjected to the end thrust  $P$ , satisfies  $\frac{d^2y}{dx^2} + a^2y = \frac{a^2R}{P}(1-x)$ . Find the deflection  $y$  of the strut at a distance  $x$  from the built-in end.

**UNIT - III**

6 (a) If  $u = \sin^{-1} \left( \frac{x^2y^2}{x+y} \right)$  then show that  $xu_x + yu_y = 3 \tan u$ .

(b) If  $u = x + y + z, uv = y + z, uvw = z$ , then prove  $\frac{\partial(x,y,z)}{\partial(u,v,w)} = u^2v$ .

OR

7 (a) Find the points on the surface  $z^2 = xy + 1$  nearest to the origin.

(b) Find the radius of curvature at  $(3,3)$  on the curve  $x^3 + xy^2 - 6y^2 = 0$ .

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## UNIT - IV

8 Evaluate  $\int_0^1 \int_0^{\sqrt{1-x^2}} y^2 dx dy$  by changing the order of integration.

OR

9 Evaluate  $\iiint xy^2 z dx dy dz$  taken through the positive octant of the sphere:  $x^2 + y^2 + z^2 = a^2$ .

## UNIT - V

10 (a) Find the directional derivative of  $f = xy + yz + zx$  in the direction of vector  $\bar{i} + 2\bar{j} + 2\bar{k}$  at the point  $(1, 2, 0)$ .

(b) Find  $\text{curl } \bar{f}$  where  $\bar{f} = \text{grad}(x^3 + y^3 + z^3 - 3xyz)$ .

OR

11 Evaluate by Green's theorem  $\oint_C (y - \sin x) dx + \cos x dy$  where C is triangle enclosed the lines  $y = 0, x = \frac{\pi}{2}, \pi y = 2x$ .

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