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G. PULLAIAH COLLEGE OF ENGINEERING AND TECHNOLOGY

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Department of Electronics and Communication Engineering

Bridge Course
On
Antennas& Wave Propagation

CONTENTS

UNIT-I

Static Electric fields

1. Coulomb's Law
2. Electric Field & Electric Flux Density
3. Gauss's Law with Application
4. Capacitance and Capacitors

UNIT-II

MAGNETOSTATICS

1. Biot-savart law, Ampere’s circuital law & applications
2. Magnetic flux density
3. Maxwell’s equations
4. Magnetic potential(vector & scalar)
5. Forces due to magnetic fields & Ampere’s force law
6. Inductance & magnetic energy

UNIT-III

Maxwell's equations (Time varying fields)

1. Faraday’s law, transformer emf &inconsistency of ampere’s law
2. Displacement current density
3. Maxwell’s equations in final form

UNIT-1

STATIC ELECTRIC FIELDS

Introduction

In the previous chapter we have covered the essential mathematical tools needed to study EM fields. We have already mentioned in the previous chapter that electric charge is a fundamental property of matter and charge exist in integral multiple of electronic charge. Electrostatics can be defined as the study of electric charges at rest. Electric fields have their sources in electric charges.

(Note: Almost all real electric fields vary to some extent with time. However, for many problems, the field variation is slow and the field may be considered as static. For some other cases spatial distribution is nearly same as for the static case even though the actual field may vary with time. Such cases are termed as quasi-static.)

In this chapter we first study two fundamental laws governing the electrostatic fields, viz, (1) Coulomb's Law and (2) Gauss's Law. Both these law have experimental basis. Coulomb's law is applicable in finding electric field due to any charge distribution, Gauss's law is easier to use when the distribution is symmetrical.

Coulomb's Law

Coulomb's Law states that the force between two point charges Q1and Q2 is directly proportional to the product of the charges and inversely proportional to the square of the distance between them.

Point charge is a hypothetical charge located at a single point in space. It is an idealized model of a particle having an electric charge.

$$F = \frac{kQ_1Q_2}{R^2}$$

Mathematically, , where k is the proportionality constant.

In SI units, Q1 and Q2 are expressed in Coulombs(C) and R is in meters.

Force F is in Newtons (N) and $k = \frac{1}{4\pi\varepsilon_0}$, ε_0 is called the permittivity of free space.

(We are assuming the charges are in free space. If the charges are any other dielectric medium, we will use $\varepsilon = \varepsilon_0 \varepsilon_r$ instead where ε_r is called the relative permittivity or the dielectric constant of the medium).

Therefore

$$F = \frac{1}{4\pi\varepsilon_0}\frac{Q_1 Q_2}{R^2} \quad \text{......................} \ (1)$$

As shown in the Figure 1 let the position vectors of the point charges Q1and Q2 are given by $\vec{r_1}$ and $\vec{r_2}$. Let $\overrightarrow{F_{12}}$ represent the force on Q1 due to charge Q2.

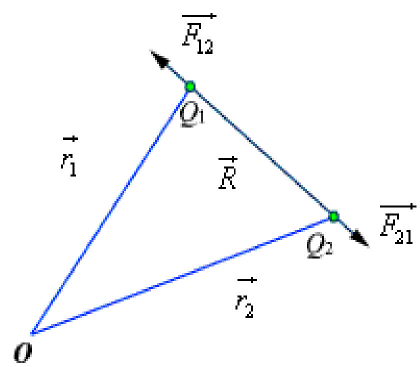

Fig 1: Coulomb's Law

The charges are separated by a distance of $R = \left|\vec{r_1} - \vec{r_2}\right| = \left|\vec{r_2} - \vec{r_1}\right|$. We define the unit vectors as

$$\widehat{a_{12}} = \frac{\left(\vec{r_2} - \vec{r_1}\right)}{R} \text{ and } \widehat{a_{21}} = \frac{\left(\vec{r_1} - \vec{r_2}\right)}{R} \quad \text{.................................} (2)$$

$\overrightarrow{F_{12}}$ can be defined as $\overrightarrow{F_{12}} = \frac{Q_1 Q_2}{4\pi\varepsilon_0 R^2}\widehat{a_{12}} = \frac{Q_1 Q_2}{4\pi\varepsilon_0 R^2}\frac{(\vec{r_2} - \vec{r_1})}{\left|\vec{r_2} - \vec{r_1}\right|^3}$.

Similarly the force on Q_1 due to charge Q_2 can be calculated and if $\overrightarrow{F_{21}}$ represents this force then we can write $\overrightarrow{F_{21}} = -\overrightarrow{F_{12}}$

When we have a number of point charges, to determine the force on a particular charge due to all other charges, we apply principle of superposition. If we have *N* number of charges $Q_1, Q_2, \ldots\ldots\ldots Q_N$ located respectively at the points represented by the position vectors $\vec{r_1}, \vec{r_2}, \ldots\ldots \vec{r_N}$, the force experienced by a charge *Q* located at $\vec{r}$ is given by,

$$\vec{F} = \frac{Q}{4\pi\varepsilon_0} \sum_{i=1}^{N} \frac{Q_i(\vec{r} - \vec{r_i})}{\left|\vec{r} - \vec{r_i}\right|^3} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3)$$

Electric Field :

The electric field intensity or the electric field strength at a point is defined as the force per unit charge. That is

$$\vec{E} = \lim_{Q \to 0} \frac{\vec{F}}{Q} \quad \text{or,} \quad \vec{E} = \frac{\vec{F}}{Q} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4)$$

The electric field intensity *E* at a point *r* (observation point) due a point charge *Q* located at $\vec{r'}$ (source point) is given by:

$$\vec{E} = \frac{Q(\vec{r} - \vec{r'})}{4\pi\varepsilon_0 \left|\vec{r} - \vec{r'}\right|^3} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(5)$$

For a collection of *N* point charges $Q_1, Q_2, \ldots\ldots\ldots Q_N$ located at $\vec{r_1}, \vec{r_2}, \ldots\ldots \vec{r_N}$, the electric field intensity at point $\vec{r'}$ is obtained as

$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \sum_{i=1}^{N} \frac{Q_k(\vec{r} - \vec{r_i})}{\left|\vec{r} - \vec{r_i}\right|^3} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(6)$$

The expression (6) can be modified suitably to compute the electric filed due to a continuous distribution of charges. In figure 2 we consider a continuous volume distribution of charge *(t)* in the region denoted as the source region.

For an elementary charge $dQ = \rho(\vec{r'})dv'$, i.e. considering this charge as point charge, we can write the field expression as:

$$d\vec{E} = \frac{dQ(\vec{r}-\vec{r}')}{4\pi\varepsilon_0 \left|\vec{r}-\vec{r}'\right|^3} = \frac{\rho(\vec{r}')dv'(\vec{r}-\vec{r}')}{4\pi\varepsilon_0 \left|\vec{r}-\vec{r}'\right|^3}$$ ....(7)

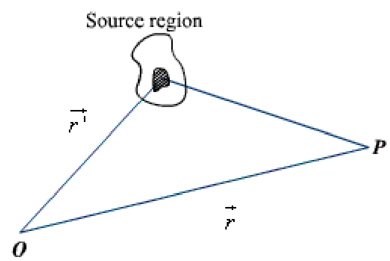

Fig 2: Continuous Volume Distribution of Charge

When this expression is integrated over the source region, we get the electric field at the point *P* due to this distribution of charges. Thus the expression for the electric field at *P* can be written as:

$$\overrightarrow{E(r)} = \int_V \frac{\rho(\vec{r}')(\vec{r}-\vec{r}')}{4\pi\varepsilon_0 \left|\vec{r}-\vec{r}'\right|^3} dv'$$ ....(8)

Similar technique can be adopted when the charge distribution is in the form of a line charge density or a surface charge density.

$$\overrightarrow{E(r)} = \int_L \frac{\rho_L(\vec{r}')(\vec{r}-\vec{r}')}{4\pi\varepsilon_0 \left|\vec{r}-\vec{r}'\right|^3} dl'$$ ....(9)

$$\overrightarrow{E(r)} = \int_S \frac{\rho_s(\vec{r}')(\vec{r}-\vec{r}')}{4\pi\varepsilon_0 \left|\vec{r}-\vec{r}'\right|^3} ds'$$ ....(10)

Electric flux density:

As stated earlier electric field intensity or simply 'Electric field' gives the strength of the field at a particular point. The electric field depends on the material media in which the field is being considered. The flux density vector is defined to be independent of the material media (as we'll see that it relates to the charge that is producing it).For a linear isotropic medium under consideration; the flux density vector is defined as:

$$\vec{D} = \varepsilon\vec{E}$$ ....(11)

We define the electric flux as

$$\psi = \int_S \vec{D}.d\vec{s} \qquad \text{....................................(12)}$$

Gauss's Law: Gauss's law is one of the fundamental laws of electromagnetism and it states that the total electric flux through a closed surface is equal to the total charge enclosed by the surface.

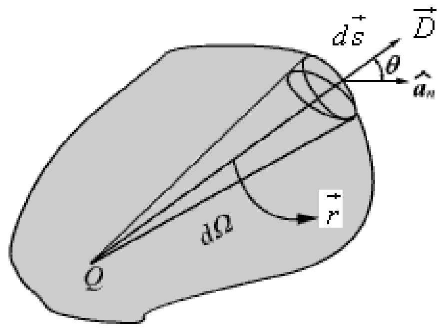

Fig 3: Gauss's Law

Let us consider a point charge *Q* located in an isotropic homogeneous medium of dielectric constant .
The flux density at a distance *r* on a surface enclosing the charge is given by

$$\vec{D} = \varepsilon\vec{E} = \frac{Q}{4\pi r^2}\hat{a}_r \qquad \text{..(13)}$$

If we consider an elementary area *d**s***, the amount of flux passing through the elementary area is given by

$$d\psi = \vec{D}.ds = \frac{Q}{4\pi r^2} ds\cos\theta \qquad \text{....................................(14)}$$

But $\frac{ds\cos\theta}{r^2} = d\Omega$, is the elementary solid angle subtended by the area $d\vec{s}$ at the location of *Q*.

Therefore we can write $d\psi = \frac{Q}{4\pi}d\Omega$

For a closed surface enclosing the charge, we can write $\psi = \oint_S d\psi = \frac{Q}{4\pi}\oint_S d\Omega = Q$

which can seen to be same as what we have stated in the definition of Gauss's Law.

Capacitance and Capacitors

We have already stated that a conductor in an electrostatic field is an Equipotential body and any charge given to such conductor will distribute themselves in such a manner that electric field inside the conductor vanishes. If an additional amount of charge is supplied to an isolated conductor at a given potential, this additional charge will increase the surface charge density ρ_s . Since the potential of the conductor is given by $V = \frac{1}{4\pi\varepsilon_0}\int_{S'} \frac{\rho_S ds'}{r}$, the potential of the conductor will also increase maintaining the ratio same $\frac{Q}{V}$. Thus we can write $C = \frac{Q}{V}$ where the constant of proportionality C is called the capacitance of the isolated conductor. SI unit of capacitance is Coulomb/ Volt also called Farad denoted by F. It can It can be seen that if V=1, C = Q. Thus capacity of an isolated conductor can also be defined as the amount of charge in Coulomb required to raise the potential of the conductor by 1 Volt.

Of considerable interest in practice is a capacitor that consists of two (or more) conductors carrying equal and opposite charges and separated by some dielectric media or free space. The conductors may have arbitrary shapes. A two-conductor capacitor is shown in figure below.

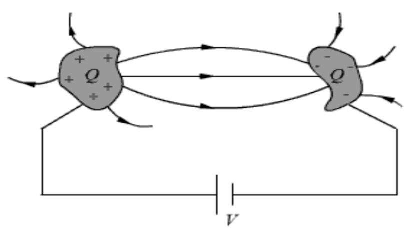

Fig : Capacitance and Capacitors

When a d-c voltage source is connected between the conductors, a charge transfer occurs which results into a positive charge on one conductor and negative charge on the other conductor. The conductors are equipotential surfaces and the field lines are perpendicular to the conductor surface. If V is the mean potential difference between the conductors, the capacitance is given by $C = \frac{Q}{V}$. Capacitance of a capacitor depends on the geometry of the conductor and the permittivity of the medium between them and does not depend on the charge or potential difference between conductors. The capacitance can be computed by assuming Q(at the same time -Q on the other conductor), first determining $\vec{E}$

using Gauss's theorem and then determining $V = -\int \vec{E}.d\vec{l}$. We illustrate this procedure by taking the example of a parallel plate capacitor.

Example: Parallel plate capacitor

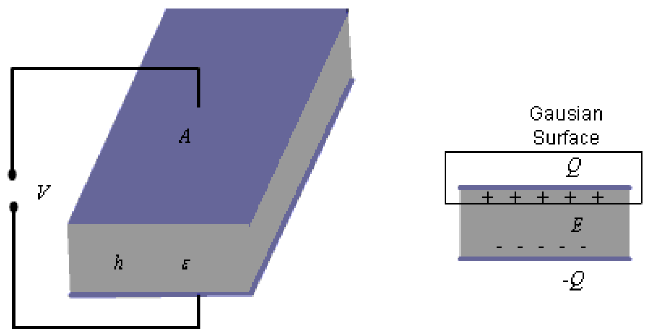

Fig : Parallel Plate Capacitor

For the parallel plate capacitor shown in the figure about, let each plate has area A and a distance h separates the plates. A dielectric of permittivity ε fills the region between the plates. The electric field lines are confined between the plates. We ignore the flux fringing at the edges of the plates and charges are assumed to be uniformly distributed over the conducting plates with densities ρ_s and - ρ_s,

$\rho_s = \frac{Q}{A}$.

By Gauss's theorem we can write

$$E = \frac{\rho_s}{\varepsilon} = \frac{Q}{A\varepsilon} \quad ....(1)$$

As we have assumed ρ_s to be uniform and fringing of field is neglected, we see that E is constant in the region between the plates and therefore, we can write $V = Eh = \frac{hQ}{\varepsilon A}$. Thus, for a parallel plate capacitor we have,

$$C = \frac{Q}{V} = \varepsilon \frac{A}{h} \quad ....(2)$$

Series and parallel Connection of capacitors

Capacitors are connected in various manners in electrical circuits; series and parallel connections are the two basic ways of connecting capacitors. We compute the equivalent capacitance for such connections.

Series Case: Series connection of two capacitors is shown in the figure 1. For this case we can write,

$$V = V_1 + V_2 = \frac{Q}{C_1} + \frac{Q}{C_2}$$

$$\frac{V}{Q} = \frac{1}{C_{eqs}} = \frac{1}{C_1} + \frac{1}{C_2} \quad \text{......................(1)}$$

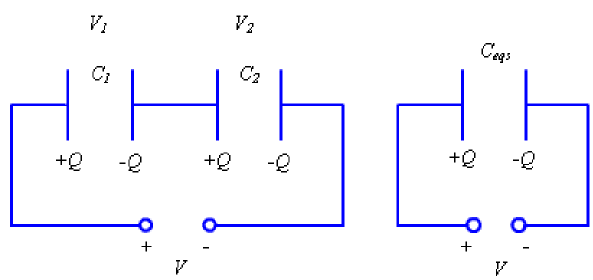

Fig 1.: Series Connection of Capacitors

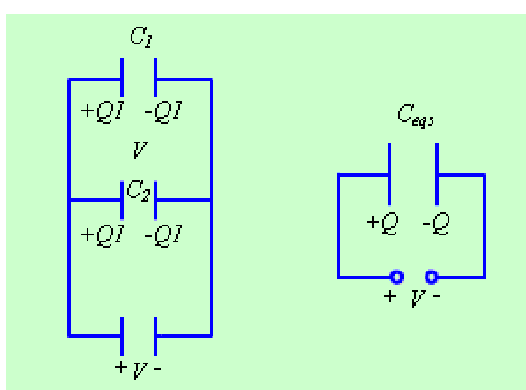

Fig 2: Parallel Connection of Capacitors

The same approach may be extended to more than two capacitors connected in series.

Parallel Case: For the parallel case, the voltages across the capacitors are the same.

The total charge

$$Q = Q_1 + Q_2 = C_1V + C_2V$$

Therefore,

$$C_{eq} = \frac{Q}{V} = C_1 + C_2 \qquad \text{......................(2)}$$

UNIT-II

MAGNETOSTATICS

Introduction :

In previous chapters we have seen that an electrostatic field is produced by static or stationary charges. The relationship of the steady magnetic field to its sources is much more complicated. The source of steady magnetic field may be a permanent magnet, a direct current or an electric field changing with time. In this chapter we shall mainly consider the magnetic field produced by a direct current. The magnetic field produced due to time varying electric field will be discussed later. Historically, the link between the electric and magnetic field was established Oersted in 1820. Ampere and others extended the investigation of magnetic effect of electricity . There are two major laws governing the magnetostatic fields are:

Biot-Savart Law, (*Ampere's Law*)

Usually, the magnetic field intensity is represented by the vector . It is customary to represent the direction of the magnetic field intensity (or current) by a small circle with a dot or cross sign depending on whether the field (or current) is out of or into the page as shown in Fig. 1.

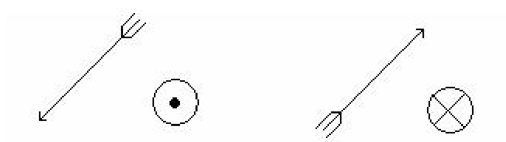

Fig. 1: Representation of magnetic field (or current)

Biot- Savart Law

This law relates the magnetic field intensity dH produced at a point due to a differential current element $Id\vec{l}$ as shown in Fig. 2.

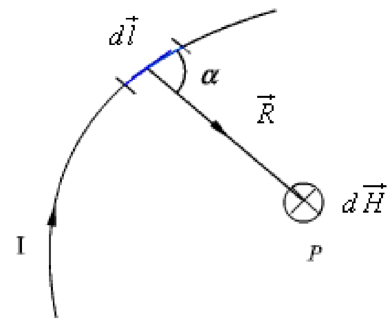

Fig. 2: Magnetic field intensity due to a current element

The magnetic field intensity $d\vec{H}$ at P can be written as,

$$d\vec{H} = \frac{Id\vec{l} \times \hat{a}_R}{4\pi R^2} = \frac{Id\vec{l} \times \vec{R}}{4\pi R^3} \quad \text{..........................(1a)}$$

$$dH = \frac{IdlSin\alpha}{4\pi R^2} \quad \text{...(1b)}$$

Where $R = |\vec{R}|$ is the distance of the current element from the point P.

Similar to different charge distributions, we can have different current distribution such as line current, surface current and volume current. These different types of current densities are shown in Fig. 3.

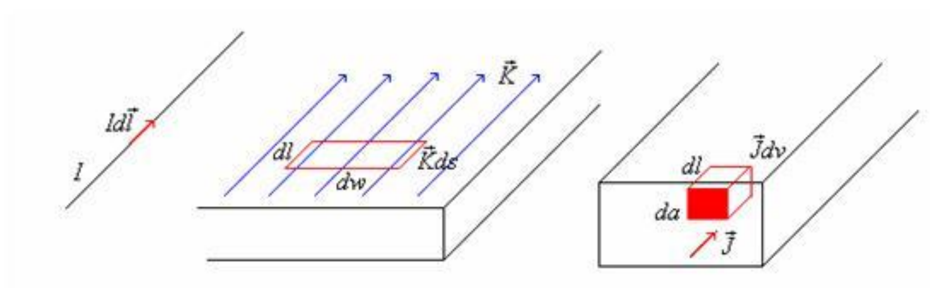

Fig. 3: Different types of current distributions

By denoting the surface current density as K (in amp/m) and volume current density as J (in amp/m2) we can write:

$$Id\vec{l} = \vec{K}ds = \vec{J}dv \quad \text{....................................(2)}$$

(It may be noted that

$$I = Kdw = Jda$$)

Employing Biot-Savart Law, we can now express the magnetic field intensity H. In terms of these current distributions.

$$\vec{H} = \int_l \frac{Id\vec{l} \times \vec{R}}{4\pi R^3}$$ .... for line current............................(3a)

$$\vec{H} = \int_S \frac{Kd\vec{s} \times \vec{R}}{4\pi R^3}$$ .... for surface current ....(3b)

$$\vec{H} = \int_v \frac{\vec{J}dv \times \vec{R}}{4\pi R^3}$$ .... for volume current......................(3c)

Ampere's Circuital Law:

Ampere's circuital law states that the line integral of the magnetic field $\vec{H}$ (circulation of H) around a closed path is the net current enclosed by this path. Mathematically,

$$\oint \vec{H}.d\vec{l} = I_{enc}$$ ....(4)

The total current I enc can be written as,

$$I_{enc} = \int_S \vec{J}.d\vec{s}$$ ....(5)

By applying Stoke's theorem, we can write

$$\oint \vec{H}.d\vec{l} = \int_S \nabla \times \vec{H}.d\vec{s}$$

$$\therefore \int_S \nabla \times \vec{H}.d\vec{s} = \int_S \vec{J}.d\vec{s}$$ ....(6)

which is the Ampere's law in the point form.

Magnetic Flux Density:

In simple matter, the magnetic flux density $\vec{B}$ related to the magnetic field intensity $\vec{H}$ as $\vec{B} = \mu\vec{H}$ where μ called the permeability. In particular when we consider the free space $\vec{B} = \mu_0\vec{H}$ where $\mu_0 = 4\pi\times10^{-7}$ H/m is the permeability of the free space. Magnetic flux density is measured in terms of Wb/m 2 .

The magnetic flux density through a surface is given by:

$$\psi = \int_S \vec{B}.d\vec{s} \text{ Wb} \qquad \text{.....................................(7)}$$

In the case of electrostatic field, we have seen that if the surface is a closed surface, the net flux passing through the surface is equal to the charge enclosed by the surface. In case of magnetic field isolated magnetic charge (i. e. pole) does not exist. Magnetic poles always occur in pair (as N-S). For example, if we desire to have an isolated magnetic pole by dividing the magnetic bar successively into two, we end up with pieces each having north (N) and south (S) pole as shown in Fig. 6 (a). This process could be continued until the magnets are of atomic dimensions; still we will have N-S pair occurring together. This means that the magnetic poles cannot be isolated.

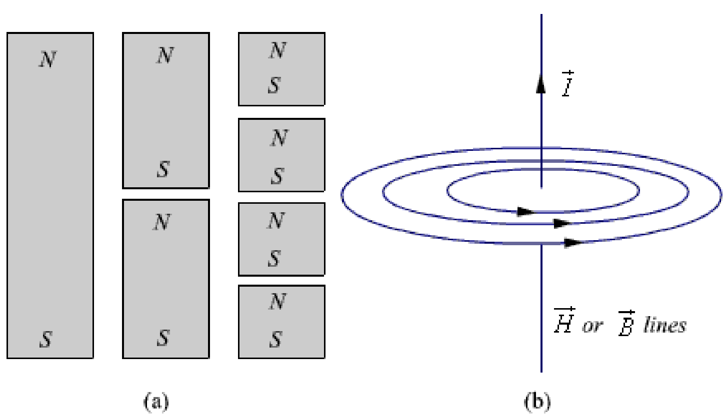

Fig. 6: (a) Subdivision of a magnet (b) Magnetic field/ flux lines of a straight current carrying conductor

Similarly if we consider the field/flux lines of a current carrying conductor as shown in Fig. 6 (b), we find that these lines are closed lines, that is, if we consider a closed surface, the number of flux lines that would leave the surface would be same as the number of flux lines that would enter the surface.

From our discussions above, it is evident that for magnetic field,

$$\oint_S \vec{B}.d\vec{s} = 0 \quad \text{.....................................(8)}$$

which is the Gauss's law for the magnetic field.

By applying divergence theorem, we can write:

$$\oint_S \vec{B}.d\vec{s} = \int_V \nabla.\vec{B}dv = 0$$

Hence, $\nabla.\vec{B} = 0$ ....(9)

which is the Gauss's law for the magnetic field in point form.

Magnetic Scalar and Vector Potentials:

In studying electric field problems, we introduced the concept of electric potential that simplified the computation of electric fields for certain types of problems. In the same manner let us relate the magnetic field intensity to a scalar magnetic potential and write:

$$\vec{H} = -\nabla V_m \quad \text{...................................(10)}$$

From Ampere's law , we know that

$\nabla\times\vec{H} = \vec{J}$ ....(11)

Therefore, $\nabla\times\left(-\nabla V_m\right) = \vec{J}$ ....(12)

But using vector identity, $\nabla\times\left(\nabla V\right) = 0$ we find that $\vec{H} = -\nabla V_m$ is valid only where $\vec{J} = 0$. Thus the scalar magnetic potential is defined only in the region where $\vec{J} = 0$. Moreover, Vm in general is not a single valued function of position.

This point can be illustrated as follows. Let us consider the cross section of a coaxial line as shown in fig 7.

In the region

$a<\rho<b$, $\vec{J}=0$ and $\vec{H}=\frac{I}{2\pi\rho}\hat{a}_\phi$ ....(13)

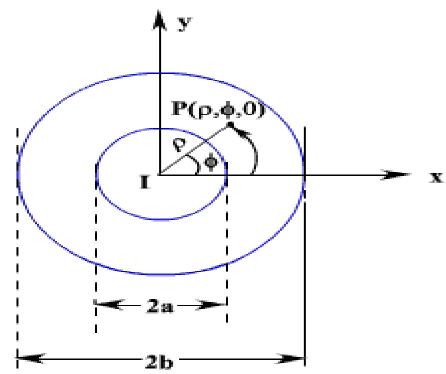

Fig. 7: Cross Section of a Coaxial Line

If Vm is the magnetic potential then,

$$-\nabla V_m = -\frac{1}{\rho}\frac{\partial V_m}{\partial \phi} = \frac{I}{2\pi\rho} \quad ....(14)$$

If we set Vm = 0 at $\phi = 0$ then c=0 and $V_m = -\frac{I}{2\pi}\phi$

$$\therefore \text{At } \phi = \phi_0 \quad V_m = -\frac{I}{2\pi}\phi_0 \quad ....(15)$$

We observe that as we make a complete lap around the current carrying conductor , we reach ϕ_0 again but Vm this time becomes

$$V_m = -\frac{I}{2\pi}(\phi_0 + 2\pi) \quad ......(16)$$

We observe that value of Vm keeps changing as we complete additional laps to pass through the same point. We introduced Vm analogous to electostatic potential V. But for static electric fields, $\nabla\times\vec{E}=0$

and $\oint \vec{E}\cdot d\vec{l} = 0$ $\nabla\times\vec{H} = 0$, whereas for steady magnetic field $\nabla\times\vec{H} = 0$ wherever $\vec{J} = 0$ but $\oint \vec{H}\cdot d\vec{l} = I$ even if $\vec{J} = 0$ along the path of integration.

We now introduce the vector magnetic potential which can be used in regions where current density may be zero or nonzero and the same can be easily extended to time varying cases. The use of vector magnetic potential provides elegant ways of solving EM field problems.

Since $\nabla.\vec{B} = 0$ and we have the vector identity that for any vector $\vec{A}$, $\nabla.\left(\nabla\times\vec{A}\right) = 0$, we can write $\vec{B} = \nabla\times\vec{A}$.

Here, the vector field $\vec{A}$ is called the vector magnetic potential. Its SI unit is Wb/m. Thus if can find $\vec{A}$ of a given current distribution, $\vec{B}$ can be found from $\vec{A}$ through a curl operation. We have introduced the vector function $\vec{B}$ and $\vec{A}$ related its curl to $\vec{B}$. A vector function is defined fully in terms of its curl as well as divergence. The choice of $\nabla.\vec{A}$ is made as follows.

$$\nabla\times\nabla\times\vec{A} = \mu\nabla\times\vec{H} = \mu\vec{J} \quad \text{..(17)}$$

By using vector identity, $\nabla\times\nabla\times\vec{A} = \nabla(\nabla.\vec{A}) - \nabla^2\vec{A}$ ....(18)

$$\nabla(\nabla.\vec{A}) - \nabla^2\vec{A} = \mu\vec{J} \quad \text{..(19)}$$

Great deal of simplification can be achieved if we choose $\nabla.\vec{A} = 0$.

Putting $\nabla.\vec{A} = 0$, we get $\nabla^2\vec{A} = -\mu\vec{J}$ which is vector poisson equation.

In Cartesian coordinates, the above equation can be written in terms of the components as

$$\nabla^2 A_x = -\mu J_x \quad \text{.....................................(19)}$$

$$\nabla^2 A_y = -\mu J_y \quad \text{.....................................(20)}$$

$$\nabla^2 A_z = -\mu J_z \quad \text{.....................................(22)}$$

The form of all the above equation is same as that of

$$\nabla^2 V = -\frac{\rho}{\varepsilon} \quad \text{...(23)}$$

for which the solution is

$$V = \frac{1}{4\pi\varepsilon}\int_{V'}\frac{\rho}{R}dv', \qquad R = \left|\vec{r} - \vec{r}'\right| \quad \text{.................(24)}$$

In case of time varying fields we shall see that $\nabla.\vec{A} = \mu\varepsilon\frac{\partial V}{\partial t}$, which is known as Lorentz condition, V being the electric potential. Here we are dealing with static magnetic field, so $\nabla.\vec{A} = 0$.

By comparison, we can write the solution for Ax as

$$A_x = \frac{\mu}{4\pi}\int_{V'}\frac{J_x}{R}dv' \quad \text{...................................(25)}$$

Computing similar solutions for other two components of the vector potential, the vector potential can be written as

$$\vec{A} = \frac{\mu}{4\pi}\int_{V'}\frac{\vec{J}}{R}dv' \quad \text{.....................................(26)}$$

This equation enables us to find the vector potential at a given point because of a volume current density $\vec{J}$. Similarly for line or surface current density we can write

$$\vec{A} = \frac{\mu}{4\pi}\int_{L'}\frac{I}{R}d\vec{l}' \quad \text{...(27)}$$

$$\vec{A} = \frac{\mu}{4\pi}\int_{S'}\frac{\vec{K}}{R}ds' \quad \text{..............................(28)}$$

The magnetic flux ψ through a given area S is given by

$$\psi = \int_S \vec{B}.d\vec{s} \qquad \text{...(29)}$$

Substituting $\vec{B} = \nabla \times \vec{A}$

$$\psi = \int_S \nabla \times \vec{A}.d\vec{s} = \oint_C \vec{A}.d\vec{l} \qquad \text{...(30)}$$

Vector potential thus have the physical significance that its integral around any closed path is equal to the magnetic flux passing through that path.

Inductance and Inductor:

Resistance, capacitance and inductance are the three familiar parameters from circuit theory. We have already discussed about the parameters resistance and capacitance in the earlier chapters. In this section, we discuss about the parameter inductance. Before we start our discussion, let us first introduce the concept of flux linkage. If in a coil with N closely wound turns around where a current I produces a flux ϕ and this flux links or encircles each of the N turns, the flux linkage is defined as Λ. In a linear medium $\Lambda = N\phi$, where the flux is proportional to the current, we define the self inductance L as the ratio of the total flux linkage to the current which they link.

i.e., $$L = \frac{\Lambda}{I} = \frac{N\phi}{I} \qquad \text{...................................(31)}$$

To further illustrate the concept of inductance, let us consider two closed loops C1 and C2 as shown in the figure 8, S1 and S2 are respectively the areas of C1 and C2 .

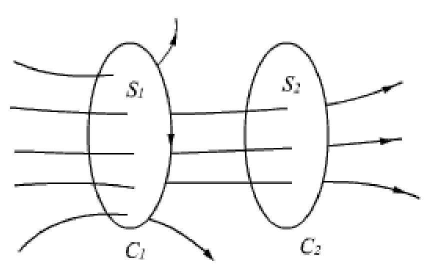

Fig:8

If a current I1 flows in C1 , the magnetic flux B1 will be created part of which will be linked to C2 as shown in Figure 8:

$$\phi_{12} = \int_{S_2} \vec{B}1.d\vec{S}2 \qquad \text{..................................(32)}$$

In a linear medium, ϕ_{12} is proportional to I 1. Therefore, we can write

$$\phi_{12} = L_{12} I_1 \qquad \text{..................................(33)}$$

where L12 is the mutual inductance. For a more general case, if C2 has N2 turns then

$$\Lambda_{12} = N_2 \phi_{12} \qquad \text{..................................(34)}$$

and $\Lambda_{12} = L_{12} I_1$

or
$$L_{12} = \frac{\Lambda_{12}}{I_1} \qquad \text{..................................(35)}$$

i.e., the mutual inductance can be defined as the ratio of the total flux linkage of the second circuit to the current flowing in the first circuit.

As we have already stated, the magnetic flux produced in C1 gets linked to itself and if C1 has N1 turns then $\Lambda_{11} = N_1 \phi_{11}$, where ϕ_{11} is the flux linkage per turn.

Therefore, self inductance

$$L_{11}(\text{or } L \text{ as defined earlier}) = \frac{\Lambda_{11}}{I_1} \qquad \text{..................................(36)}$$

As some of the flux produced by I1 links only to C1 & not C2.

$$\Lambda_{11} = N_1 \phi_{11} > N_2 \phi_{12} = \Lambda_{12} \qquad \text{..................................(37)}$$

Further in general, in a linear medium, $L_{12} = \frac{d\Lambda_{12}}{dI_1}$ and $L_{11} = \frac{d\Lambda_{11}}{dI_1}$

UNIT-III

MAXWELL'S EQUATIONS (TIME VARYING FIELDS)

Introduction:

In our study of static fields so far, we have observed that static electric fields are produced by electric charges, static magnetic fields are produced by charges in motion or by steady current. Further, static electric field is a conservative field and has no curl, the static magnetic field is continuous and its divergence is zero. The fundamental relationships for static electric fields among the field quantities can be summarized as:

$$\nabla \times \vec{E} = 0 \quad (1)$$

$$\nabla \cdot \vec{D} = \rho_v \quad (2)$$

For a linear and isotropic medium,

$$\vec{D} = \varepsilon \vec{E} \quad (3)$$

Similarly for the magnetostatic case

$$\nabla . \vec{B} = 0 \quad (4)$$

$$\nabla \times \vec{H} = \vec{J} \quad (5)$$

$$\nabla \times \vec{H} = \vec{J} \quad (6)$$

It can be seen that for static case, the electric field vectors $\vec{E}$ and $\vec{D}$ and magnetic field vectors $\vec{B}$ and $\vec{H}$ form separate pairs.

In this chapter we will consider the time varying scenario. In the time varying case we will observe that a changing magnetic field will produce a changing electric field and vice versa.

We begin our discussion with Faraday's Law of electromagnetic induction and then present the Maxwell's equations which form the foundation for the electromagnetic theory.

Faraday's Law of electromagnetic Induction

Michael Faraday, in 1831 discovered experimentally that a current was induced in a conducting loop when the magnetic flux linking the loop changed. In terms of fields, we can say that a time varying magnetic field produces an electromotive force (emf) which causes a current in a closed circuit. The quantitative relation between the induced emf (the voltage that arises from conductors moving in a magnetic field or from changing magnetic fields) and the rate of change of flux linkage developed based on experimental observation is known as Faraday's law. Mathematically, the induced emf can be written as

$$\text{Emf} = -\frac{d\phi}{dt} \quad \text{Volts} \qquad (7)$$

where ϕ is the flux linkage over the closed path.

A non zero $\frac{d\phi}{dt}$ may result due to any of the following:

(a) time changing flux linkage a stationary closed path.

(b) relative motion between a steady flux a closed path.

(c) a combination of the above two cases.

The negative sign in equation (7) was introduced by Lenz in order to comply with the polarity of the induced emf. The negative sign implies that the induced emf will cause a current flow in the closed loop in such a direction so as to oppose the change in the linking magnetic flux which produces it. (It may be noted that as far as the induced emf is concerned, the closed path forming a loop does not necessarily have to be conductive).

If the closed path is in the form of N tightly wound turns of a coil, the change in the magnetic flux linking the coil induces an emf in each turn of the coil and total emf is the sum of the induced emfs of the individual turns, i.e.,

$$\text{Emf} = -N\frac{d\phi}{dt} \quad \text{Volts} \qquad (8)$$

By defining the total flux linkage as

$$\lambda = N\phi \qquad (9)$$

The emf can be written as

$$\text{Emf} = -\frac{d\lambda}{dt} \qquad (10)$$

Continuing with equation (3), over a closed contour 'C' we can write

$$\text{Emf} = \oint_C \vec{E}.d\vec{l} \qquad (11)$$

where $\vec{E}$ is the induced electric field on the conductor to sustain the current.

Further, total flux enclosed by the contour 'C ' is given by

$$\phi = \int_S \vec{B}.d\vec{s} \qquad (12)$$

Where S is the surface for which 'C' is the contour.

From (11) and using (12) in (3) we can write

$$\oint_C \vec{E}.d\vec{l} = -\frac{\partial}{\partial t}\oint_S \vec{B}.d\vec{s} \qquad (13)$$

By applying stokes theorem

$$\int_S \nabla\times\vec{E}.d\vec{s} = -\int_S \frac{\partial\vec{B}}{\partial t}.d\vec{s} \qquad (14)$$

Therefore, we can write

$$\nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t} \qquad (15)$$

which is the Faraday's law in the point form

We have said that non zero $\frac{d\phi}{dt}$ can be produced in a several ways. One particular case is when a time varying flux linking a stationary closed path induces an emf. The emf induced in a stationary closed path by a time varying magnetic field is called a transformer emf .

Maxwell's Equation

Equation (5.1) and (5.2) gives the relationship among the field quantities in the static field. For time varying case, the relationship among the field vectors written as

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad (1)$$

$$\nabla \times \vec{H} = \vec{J} \quad \text{.............}(2)$$

$$\nabla . \vec{D} = \rho \quad (3)$$

$$\nabla . \vec{B} = 0 \quad (4)$$

In addition, from the principle of conservation of charges we get the equation of continuity

$$\nabla . \vec{J} = -\frac{\partial \rho}{\partial t}$$

The equation must be consistent with equation of continuity

We observe that

$$\nabla . \nabla \times \vec{H} = 0 = \nabla . \vec{J} \quad (5)$$

Since $\nabla . \nabla \times \vec{A}$ is zero for any vector $\vec{A}$.

Thus $\nabla \times \vec{H} = \vec{J}$ applies only for the static case i.e., for the scenario when $\frac{\partial \rho}{\partial t} = 0$.

A classic example for this is given below .

Suppose we are in the process of charging up a capacitor as shown in fig 3.

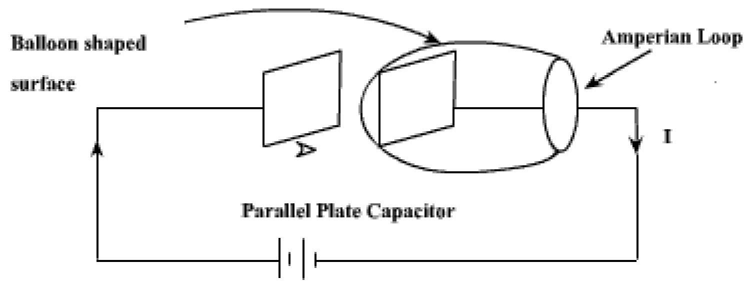

Fig 3

Let us apply the Ampere's Law for the Amperian loop shown in fig 3. Ienc = I is the total current passing through the loop. But if we draw a baloon shaped surface as in fig 5.3, no current passes through this surface and hence Ienc = 0. But for non steady currents such as this one, the concept of current enclosed by a loop is ill-defined since it depends on what surface you use. In fact Ampere's Law should also hold true for time varying case as well, then comes the idea of displacement current which will be introduced in the next few slides.

We can write for time varying case,

$$\nabla.\left(\nabla\times\vec{H}\right) = 0 = \nabla.\vec{J} + \frac{\partial\rho}{\partial t}$$

$$= \nabla.\vec{J} + \frac{\partial}{\partial t}\nabla.\vec{D} \quad \text{............(1)}$$

$$= \nabla.\left(\vec{J} + \frac{\partial\vec{D}}{\partial t}\right) \quad \text{................(2)}$$

$$\therefore \nabla\times\vec{H} = \vec{J} + \frac{\partial\vec{D}}{\partial t} \quad \text{........... (3)}$$

The equation (3) is valid for static as well as for time varying case.Equation (3) indicates that a time varying electric field will give rise to a magnetic field even in the absence of The term $\frac{\partial\vec{D}}{\partial t}$ has a dimension of current densities $\left(A/m^2\right)$ and is called the displacement current density.

Introduction of $\frac{\partial \vec{D}}{\partial t}$ in $\nabla\times\vec{H}$ equation is one of the major contributions of Jame's Clerk Maxwell. The modified set of equations

$$\nabla\times\vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad (4)$$

$$\nabla\times\vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} \quad (5)$$

$$\nabla.\vec{D} = \rho \quad (6)$$

$$\nabla.\vec{B} = 0 \quad (7)$$

is known as the Maxwell's equation and this set of equations apply in the time varying scenario, static fields are being a particular case $\left(\frac{\partial}{\partial t} = 0\right)$.

In the integral form

$$\oint_c \vec{E}.d\vec{l} = -\int_s \frac{\partial \vec{B}}{\partial t}.d\vec{S} \quad (8)$$

$$\oint_c \vec{H}.d\vec{l} = \int_s \left(J + \frac{\partial D}{\partial t}\right).d\vec{S} = I + \int_s \frac{\partial \vec{D}}{\partial t}.d\vec{S} \quad \text{………… (9)}$$

$$\int_V \nabla.\vec{D}\,dv = \oint_s \vec{D}.d\vec{S} = \int_V \rho\,dv \quad (10)$$

$$\oint \vec{B}.d\vec{S} = 0 \quad (11)$$

The modification of Ampere's law by Maxwell has led to the development of a unified electromagnetic field theory. By introducing the displacement current term, Maxwell could predict the propagation of EM waves. Existence of EM waves was later demonstrated by Hertz experimentally which led to the new era of radio communication.