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Department of Electronics & Communication Engineering

Bridge Course
On
DIGITAL IMAGE PROCESSING

By

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Signals and Systems Introduction

Signals

In electrical engineering, the fundamental quantity of representing some information is called a signal. It does not matter what the information is i.e.: Analog or digital information. In mathematics, a signal is a function that conveys some information. In fact any quantity measurable through time over space or any higher dimension can be taken as a signal. A signal could be of any dimension and could be of any form.

Analog signals

A signal could be an analog quantity that means it is defined with respect to the time. It is a continuous signal. These signals are defined over continuous independent variables. They are difficult to analyze, as they carry a huge number of values. They are very much accurate due to a large sample of values. In order to store these signals, infinite memory is required because it can achieve infinite values on a real line. Analog signals are denoted by sine waves.

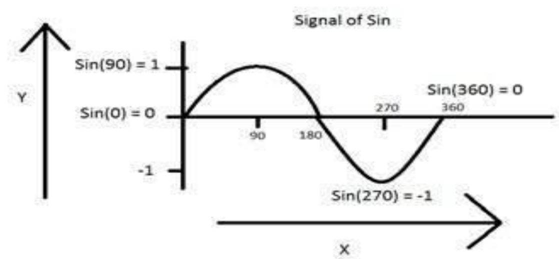
For example:

Human voice

Human voice is an example of analog signals. In a speech signal, the voice that is produced travel through air in the form of pressure waves and thus belongs to a mathematical function, having independent variables of space and time and a value corresponding to air pressure.

Another example is of sine wave which is shown in the figure below.

Y = sin(x) where x is independent



Digital signals

As compared to analog signals, digital signals are very easy to analyze. They are discontinuous signals. They are the appropriation of analog signal. The word digital stands for discrete values and hence it means that they use specific values to represent any information.

Representation of digital signal is done with only two values i.e.: 1 and 0 (binary values). Digital signals are less accurate then analog signals because they are the discrete samples of an analog signal taken over some period of time. However digital signals are not subjected to noise. So they last long and are easy to interpret. Digital signals are denoted by square waves.

For example:

Computer keyboard

Whenever a key is pressed from the keyboard, the appropriate electrical signal is sent to keyboard controller containing the ASCII value that particular key. For example the electrical signal that is generated when keyboard key a is pressed, carry information of digit 97 in the form of 0 and 1, which is the ASCII value of character a.

Difference between analog and digital signals

Comparison Element	Analog Signal	Digital Signal
Analysis	Difficult	Possible to analyze
Representation	Continuous	Discontinuous
Accuracy	More accurate	Less accurate
Storage	Infinite memory	Easily stored
Subject to Noise	Yes	No
Recording Technique	Original signal is preserved	Samples of the signal are taken and preserved
Examples	Human voice, Thermometer, Analog phones etc.	Computers, Digital phones, Digital pens etc.

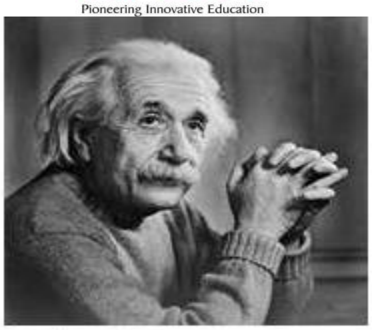
Concept of Pixel

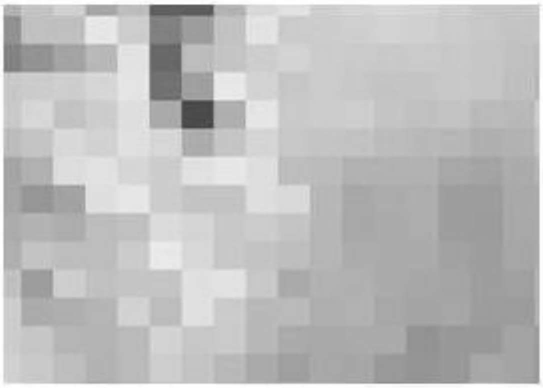
Pixel

Pixel is the smallest element of an image. Each pixel corresponds to any one value. In an 8-bit gray scale image, the value of the pixel is between 0 and 255. The value of a pixel at any point corresponds to the intensity of the light photons striking at that point. Each pixel stores a value proportional to the light intensity at that particular location.

PEL

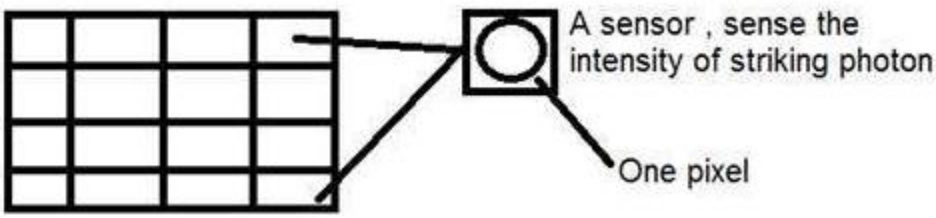
A pixel is also known as PEL. For more understandability of the pixel the below picture can be considered where there may be thousands of pixels that together make up the image. Some of the pixels division can be clearly identified if the image is zoomed to an extent as shown in the image below.





Relationship with CCD array

The image formed in the CCD array is seen. So a pixel can also be defined as the smallest division the CCD array is also known as pixel. Each division of CCD array contains the value against the intensity of the photon striking to it. This value can also be called as a pixel.



Calculation of total number of pixels

An image can be defined as a two dimensional signal or matrix. Then in that case the number of PEL would be equal to the number of rows multiply with number of columns.

This can be mathematically represented as below:

Total number of pixels = number of rows (X) number of columns

Or it can be said that the number of (x,y) coordinate pairs make up the total number of pixels. Calculation of Pixels in a color image can be detailed using the tutorial of image types.

Gray level

The value of the pixel at any point denotes the intensity of image at that location, and that is also known as gray level. The value of the pixels in the image storage and bits per pixel tutorial is shown with only one pixel value.

Pixel value. (0)

Each pixel can have only one value and each value denotes the intensity of light at that point of the image. Here a very unique value 0 is considered. The value 0 means absence of light. It means that 0 denotes dark, and it further means that whenever a pixel has a value of 0, it means at that point, black color would be formed.

Image matrix

0	0	0
0	0	0
0	0	0

The image matrix is all filled up with 0 and all the pixels have a value of 0. The total number of pixels from the matrix is calculated as mentioned below.

Total no of pixels = total no. of rows X total no. of columns

= 3 X 3

= 9.

It means that an image would be formed with 9 pixels, and that image would have a dimension of 3 rows and 3 columns and most importantly that image would be black.

Concept of Bits per Pixel

Bpp or bits per pixel denotes the number of bits per pixel. The number of different colors in an image is depends on the depth of color or bits per pixel.

Bits in mathematics:

It’s just like playing with binary bits.

How many numbers can be represented by one bit.

0 & 1

How many two bits combinations can be made?

00, 01, 10 & 11

To devise a formula for the calculation of total number of combinations that can be made from bit, it would be as,

$$(2)^{bpp}$$

Where bpp denotes bits per pixel. For 1 in the formula 2 combinations are obtained, for 2 in the formula, 4 combinations are obtained and it grows exponentially.

Number of different colors:

The number of different colors depends on the number of bits per pixel.

The table for some of the bits and their color is given below.

Bits per pixel	Number of Colors
1 bpp	2 colors
2 bpp	4 colors
3 bpp	8 colors
4 bpp	16 colors
5 bpp	32 colors
6 bpp	64 colors
7 bpp	128 colors
8 bpp	256 colors
10 bpp	1024 colors
16 bpp	65536 colors
24 bpp	16777216 colors (16.7 million colors)
32 bpp	42949672962 colors (4294 million colors)

This table shows different bits per pixel and the amount of color they contain.

Shades

The pattern of the exponential growth can be easily noticed. The famous gray scale image is of 8 bpp, which means it has 256 different colors in it or 256 shades.

Shades can be represented as:

$$\text{Shades} = \text{number of colors} = (2)^{bpp}$$

Color images are usually of the 24 bpp format, or 16 bpp.

The detailed explanation of other color formats and image types are given in the tutorial of image types.

Color values:

In the tutorial concept of pixel, 0 pixel value denotes black color.

Black color:

The pixel value ‘0’ always denotes black color. But there is no fixed value that denotes white color.

White color:

The value that denotes white color can be calculated as:

White color = $(2)^{bpp} - 1$

In case of 1 bpp, 0 denotes black, and 1 denotes white.

In case 8 bpp, 0 denotes black, and 255 denotes white.

Gray color:

To calculate the black and white color value, the pixel value of gray color is to be calculated. Gray color is actually the midpoint of black and white. In case of 8bpp, the pixel value that denotes gray color is 127 or 128bpp (if you count from 1, not from 0).

Image storage requirements

After the discussion of bits per pixel, the size of an image is to be calculated.

Image size

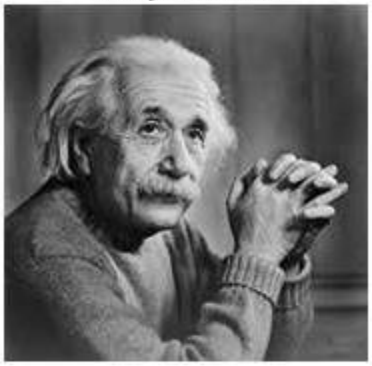
The size of an image depends upon three things.

- Number of rows
- Number of columns
- Number of bits per pixel

The formula for calculating the size is given below.

Size of an image = rows * cols * bpp

To calculate image size the below mentioned picture is taken as an example.



Assuming it has 1024 rows and it has 1024 columns. And since it is a gray scale image, it has 256 different shades of gray or it has bits per pixel. By putting these values in the formula, the image size is represented as,

Size of an image = rows * cols * bpp

= 1024 * 1024 * 8

= 8388608 bits.

But since it's not a standard answer that can be generally recognized it is converted into a format.

Converting it into bytes = 8388608 / 8 = 1048576 bytes.

Converting into kilo bytes = 1048576 / 1024 = 1024kb.

Converting into Mega bytes = 1024 / 1024 = 1 Mb.

This shows that the image size is calculated and it is stored. Now in the formula, if the size of image and the bits per pixel are given, the rows and columns of the image can be calculated, provided the image is square (same rows and same column).

Types of Images

There are different types of images and their color distribution is discussed in details below.

The binary image

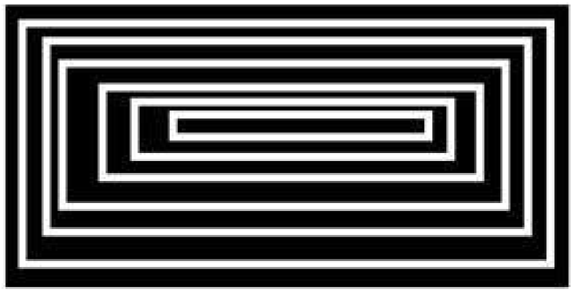
The binary image as it name states, contain only two pixel values.

0 and 1.

In the previous tutorial of bits per pixel, the representation of pixel values to their respective colors is explained in detail. Here 0 refers to black color and 1 refers to white color. It is also known as Monochrome.

Black and white image:

The resulting image that is formed hence consists of only black and white color and thus can also be called as Black and White image.



No gray level

One of the interesting observations about this binary image is that there is no gray level in it. Only two colors that are black and white are found in it.

Format

Binary images have a format of PBM (Portable bit map)

2, 3, 4, 5, 6 bit color format

The images with a color format of 2, 3, 4, 5 and 6 bit are not widely used today. They were used in old times for old TV displays, or monitor displays.

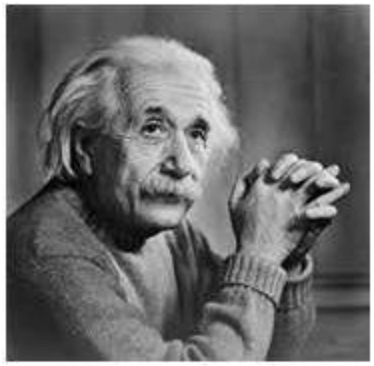
But each of these colors have more than two gray levels, and hence has gray color unlike the binary image.In a 2 bit 4, in a 3 bit 8, in a 4 bit 16, in a 5 bit 32, in a 6 bit 64 different colors are present.

8 bit color format

8 bit color format is one of the most famous image formats. It has 256 different shades of colors in it. It is commonly known as Grayscale image. The range of the colors in 8 bit vary from 0-255 where 0 stands for black, and 255 stands for white, and 127 stands for gray color.

This format was used initially by early models of the operating systems UNIX and the early color Macintoshes.

A gray scale image of Einstein is shown below:



Format

The format of these images is PGM (Portable Gray Map).

This format is not supported by default from windows. In order to see gray scale image, an image viewer or image processing toolbox such as Matlab is needed.

Behind gray scale image:

As explained several times in the previous tutorials, that an image is nothing but a two dimensional function, and can be represented by a two dimensional arrays or matrix. So in the case of the image of Einstein shown above, there would be two dimensional matrix in behind with values ranging between 0 and 255.But that's not the case with the color images.

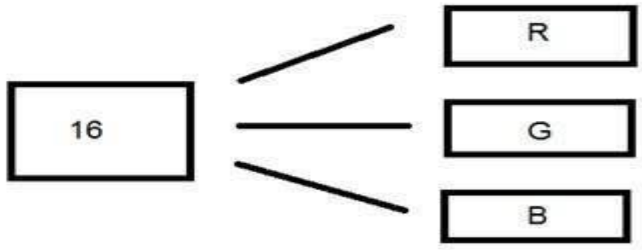
16 bit color format

It is a color image format. It has 65,536 different colors in it. It is also known as High color format.

It has been used by Microsoft in their systems that support more than 8 bit color format. Now in this 16 bit format and the next format we are going to discuss which is a 24 bit format are both color formats. The distribution of color in a color image is not as simple as it was in gray scale image.

A 16 bit format is actually divided into three further formats which are Red, Green and Blue. The famous (RGB) format.

It is pictorially represented in the image below.



Now the question arises, that how 16 is distributed into three. If it is done like this as,

5 bits for R, 5 bits for G, 5 bits for B then one bit remains in the end. So the distribution of 16 bit has been done like this as 5 bits for R, 6 bits for G, 5 bits for B.

The additional bit that was left behind is added into the green bit. Because green is the color which is most soothing to eyes in all of these three colors.

Note this distribution is not followed by all the systems. Some have introduced an alpha channel in the 16 bit.

Another distribution of 16 bit format is like this:

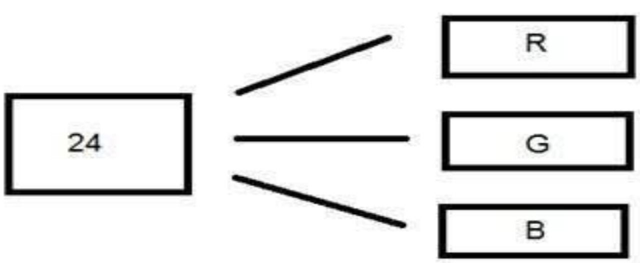
4 bits for R, 4 bits for G, 4 bits for B, 4 bits for alpha channel.

Or some distribute it like this

5 bits for R, 5 bits for G, 5 bits for B, 1 bits for alpha channel.

24 bit color format

24 bit color format also known as true color format. Like 16 bit color format, in a 24 bit color format, the 24 bits are again distributed in three different formats of Red, Green and Blue.



Since 24 are equally divided on 8, so it has been distributed equally between three different color channels.

Their distribution is like this.
8 bits for R, 8 bits for G, 8 bits for B.

Behind a 24 bit image.

Unlike an 8 bit gray scale image, which has one matrix behind it, a 24 bit image has three different matrices of R, G and B.

Format

It is the most common used format. Its format is PPM (PortablepixMap) which is supported by Linux operating system. The famous windows have its own format for it which is BMP (Bitmap).

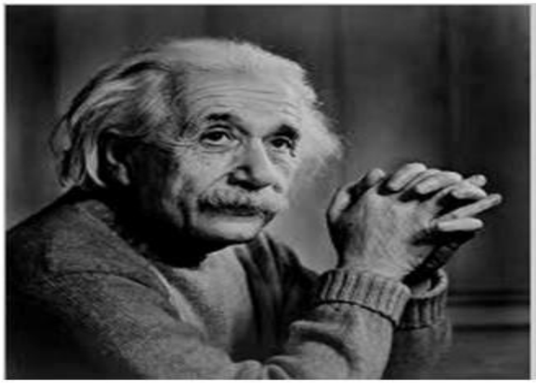
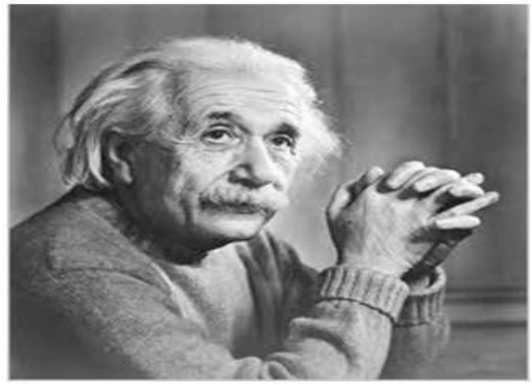
Brightness and Contrast

Brightness

Brightness is a relative term. It depends on your visual perception. Since brightness is a relative term, so brightness can be defined as the amount of energy output by a source of light relative to the source that is to be compared. In some cases when the image is bright it can be easily perceived, and in some cases, it’s not easy to perceive.

For example

If these images are compared based on their brightness,

It can be easily seen, that the image on the right side is brighter as compared to the image on the left. But if the image on the right is made darker than the first one, then the image on the left is brighter than the right.

How to make an image brighter.

Brightness can be simply increased or decreased by simple addition or subtraction, to the image matrix.

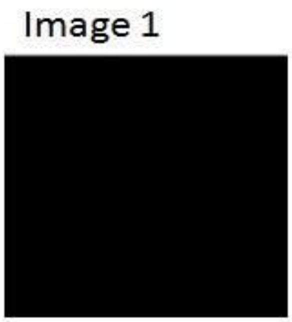
Consider this black image of 5 rows and 5 columns

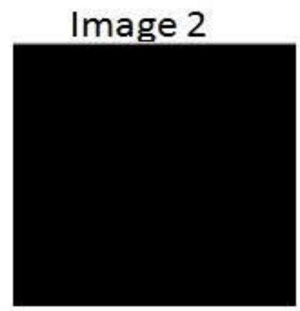
As it is already known, that each image has a matrix at its behind that contains the pixel values. This image matrix is given below.

0	0	0	0	0
0	0	0	0	0
0	0	0	0	0
0	0	0	0	0
0	0	0	0	0

Since the whole matrix is filled with zero, and the image is very much darker.

Now if this image is compared with another same black image to see this image got brighter or not.

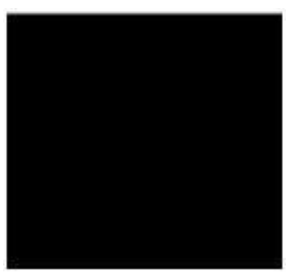




Still both the images are same, now if the same operation is performed cfcon image1 , due to which it becomes brighter than the second one.

If a value of 1 is simply added to each of the matrix value of image 1 then the image is shown the below mentioned way.

If it is again compared with image 2, and see any difference.



It can be noticed that still both images looks the same and it is difficult to identify which image is brighter.

Now the image is added a value of 50 to each of the matrix value of the image 1 and the output is given below.

Now again, the image 1 is compared with image 2.

Image 1

Image 2

Now it can be seen that the image 1 is slightly brighter than the image 2. If the same procedure continues, and add another 45 value to its matrix of image 1, and also both images are compared again and can be identified that image1 is clearly brighter than the image 2..

Image 1

Image 2

Now in this comparison, it is even observed that it is brighter than the old image1. At this point the matrix of the image1 contains 100 at each index as first add 5, then 50, then 45. So 5 + 50 + 45 = 100.

Contrast

Contrast can be simply explained as the difference between maximum and minimum pixel intensity in an image.

For example.

Consider the final image1 in brightness.

The matrix of this image is:

100	100	100	100	100
100	100	100	100	100
100	100	100	100	100
100	100	100	100	100
100	100	100	100	100

The maximum value in this matrix is 100.

The minimum value in this matrix is 100.

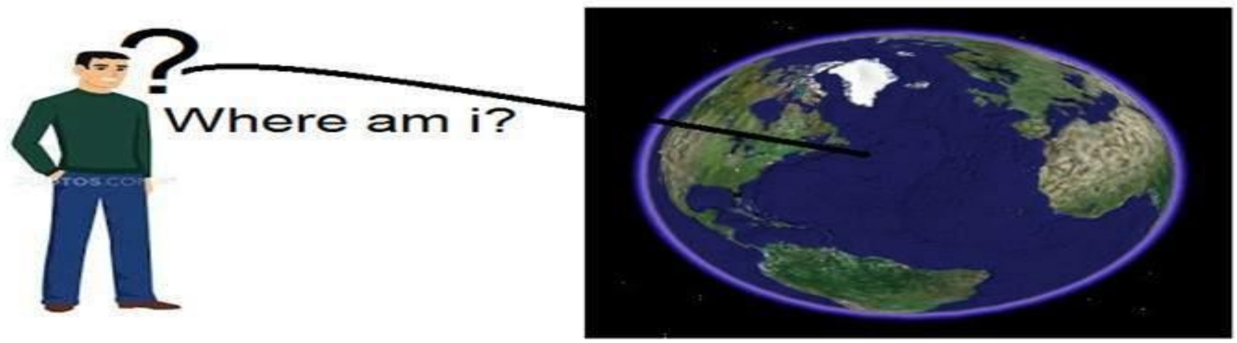
Contrast = maximum pixel intensity (subtracted by) minimum pixel intensity

= 100 (subtracted by) 100 = 0

0 means that this image has 0 contrasts.

Concept of Dimensions

Consider this example in order to understand the concept of dimension.



Consider a person(X) who lives on moon, and he wants to send a gift to another person(Y) on earth for his birthday. X asks about Y residence on earth. The only problem is that the courier service on moon doesn't understand the alphabetical address, rather it only understand the numerical co-ordinates. So how does Y send his position on earth? That's where the concept of dimensions comes. Dimensions define the minimum number of points required to point a position of any particular object within a space.

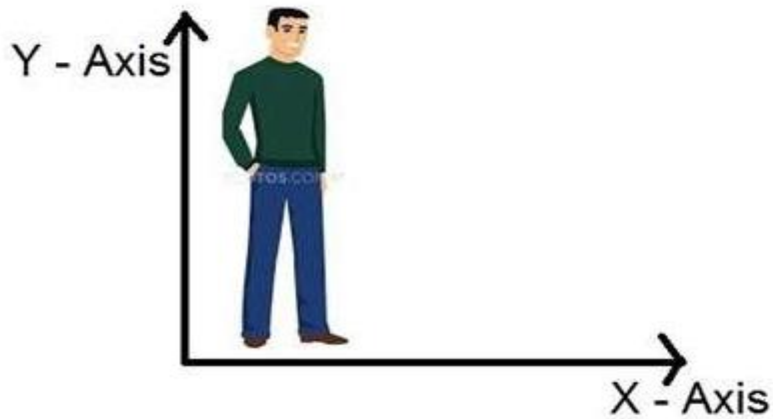
So going back to the example again in which Y have to send his position on earth to his friend(X) on moon. Y sends X, three pair of co-ordinates. The first one is called longitude, the second one is called latitude, and the third one is called altitude.

These three co-ordinates define Y's position on the earth. The first two defines Y's location, and the third one defines Y's height above the sea level.So that means that only three co-ordinates are required to define Y's position on earth. That means living in the world defines 3 dimensional system. And thus this not only answers the question about dimension, but also answers the reason, that why a 3d world is considered.

Since this concept is studied in reference to the digital image processing, this also relates to the concept of dimension with an image.

Dimensions of image

In the 3d world, means a 3 dimensional world, and then what are the dimensions of an image that are captured. An image is a two dimensional array, that's why an image is defined as a 2 dimensional signal. An image has only height and width. An image does not have depth. Just have a look at this image below.



Consider the above figure, it shows that it has only two axis which are the height and width axis. The depth of this image can't be perceived. That's why an image is two dimensional signals. But human eye is able to perceive three dimensional objects, but this would be more explained in the next tutorial of how the camera works, and image is perceived.

This discussion leads to some other questions that how 3 dimension systems are formed from 2 dimensions.

Different dimensions of signals

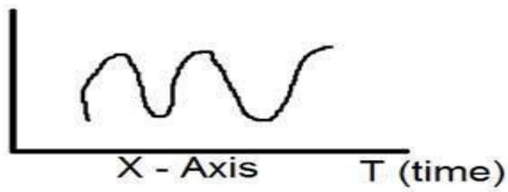
1 dimension signal

The common example of a 1 dimension signal is a waveform. It can be mathematically represented as

F(x) = waveform

Where x is an independent variable. Since it is a one dimension signal, so that's why there is only one variable x that is used.

Pictorial representation of a one dimensional signal is given below:



The above figure shows a one dimensional signal.

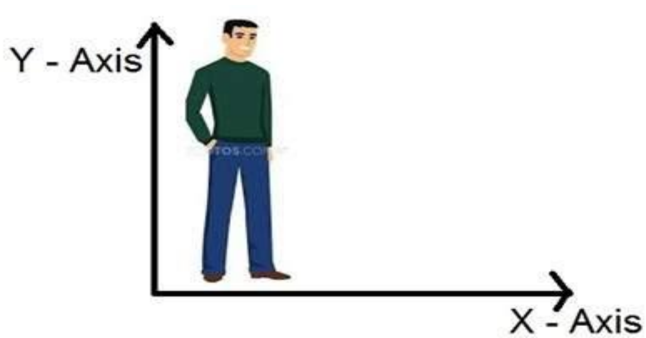
Now this lead to another question, which is, even though it is a one dimensional signal, then why does it have two axes? The answer to this question is that even though it is a one dimensional signal, but it is drawn in a two dimensional space. Or the space in which the signal is represented is two dimensional. That's why it looks like a two dimensional signal.

Perhaps the concept of one dimension can be understood better by looking at the figure below.

Now refer back to the initial discussion on dimension, Consider the above figure is a real line with positive numbers from one point to the other. Now if the location of any point on this line is explained, only one number is needed, which means only one dimension.

2 dimension signal

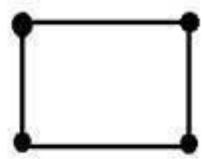
The common example of a two dimensional signal is an image, which has already been discussed above.



As the image is already a two dimensional signal, i.e.: it has two dimensions. It can be mathematically represented as:

F (x, y) = Image

Where x and y are two variables. The concept of two dimensions can also be explained in terms of mathematics as:



Now in the above figure, label the four corners of the square as A, B, C and D respectively. If one line segment in the figure is called as AB and the other CD, then these two parallel segments join up and make a square. Each line segment corresponds to one dimension, so these two line segments correspond to 2 dimensions.

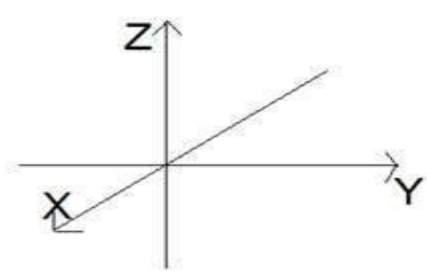
3 dimension signal

Three dimensional signals as it names refers to those signals which has three dimensions. The most common example has been discussed in the beginning is about the world everyone lives which is three dimensional. This example has been discussed very elaborately. Another example of a three dimensional signal is a cube or a volumetric data or the most common example would be animated or 3d cartoon character.

The mathematical representation of three dimensional signals is:

F(x,y,z) = animated character.

Another axis or dimension Z is involved in a three dimension, which gives the illusion of depth. In a Cartesian co-ordinate system it can be viewed as:



4 dimension signal

In a four dimensional signal, four dimensions are involved. The first three are the same as of three dimensional signals which are: (X, Y, Z), and the fourth one which is added to them is T (time). Time is often referred to as temporal dimension which is a way to measure change. Mathematically a four d signal can be stated as:

F(x, y, z, t) = animated movie.

The common example of a 4 dimensional signal can be an animated 3d movie. As each character is a 3d character and then they are moved with respect to the time, due to which an illusion of a three dimensional movie more like a real world is seen.

So that means, in reality the animated movies are 4 dimensional i.e.: movement of 3d characters over the fourth dimension time.

Concept of Transforms

1-D Fourier Transform Of Sampled Function:-

Let $F(\mu)$ denote the Fourier Transform of a continuous function f(t) corresponding sampled function $f(t)$ is product of f(t) and an impulse train. From convolution theorem the Fourier transform of the product of two functions in the spatial domain is the convolution of the transforms of two functions in the frequency domain.

$$F = \Im\{f(t)\}$$

$$= \Im\{f(t)s_{\Delta T}(t)\}$$

$$f(t) = F(\mu) * S(\mu)$$

The Fourier transform of a one-dimensional function f(x) is defined as

$$\Im[f(x)] = F(u) = \int_{-\infty}^{\infty} f(x)\exp\{2\pi iux\}dx \qquad (1)$$

The inverse transform $\Im^{-1}$ is defined as

$$f(x) = \Im^{-1}[\Im\{f(x)\}] = \int_{-\infty}^{\infty} F(u)\exp\{-2\pi iux\}du \qquad (2)$$

Two dimensional Fourier Transform is given by

$$\Im[f(x,y)] = F(u,v) = \int_{-a/2}^{a/2}\int_{-b/2}^{b/2}\exp\{2\pi i(ux+vy)\}dydx$$

1-D Discrete Fourier Transform (DFT) of One Variable:-

The Fourier transform of a sampled , band-limited function extending from -∞ to ∞ is a continuous , periodic function that also extends from -∞ to ∞.in practice we work with a finite number of samples, and the objective of this section is to derive the DFT corresponding to such sample set. The 1-D DFT pair is given below

$$F(\mu) = \int_{-\infty}^{\infty} f(t)e^{-j2\Pi\mu t}dt$$

$$F(u) = \sum_{x=0}^{M-1} f(x)e^{-j2\pi ux/M} \quad u=0,1,2,\ldots..M-1$$

and

$$f(x) = \frac{1}{M}\sum_{u=0}^{M-1} F(u)e^{j2\pi ux/M} \quad x=0,\ 1.2,\ldots.M-1$$

Relationship between the Sampling and Frequency Intervals

If f(x) consists of M samples of a function f(t) taken ΔT units apart, the duration of the record comprising the set {f(x)}, x=0,1,2,…..,M-1, is

$$T = M\Delta T$$

The corresponding spacing, Δu, in the discrete frequency domain follows from

$$\Delta u = \frac{1}{M\Delta T} = \frac{1}{T}$$

The entire frequency range spanned by the M components of the DFT is

$$\Omega = M\Delta u = \frac{1}{\Delta T}$$