G. PULLAIAH COLLEGE OF ENGINEERING AND TECHNOLOGY

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Department of Computer Science and Engineering

Bridge Course

On

Discrete Mathematics

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GPCET

Pioneering Innovative Education

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Pioneering Innovative Education

1. INTRODUCTION

Discrete mathematics is the study of mathematical structures that are fundamentally discrete rather than continuous.

In contrast to real numbers that have the property of varying smoothly, the objects studied in discrete mathematics are statements in logic, integers and graphs.

Discrete mathematics therefore excludes topics in "continuous mathematics" such as calculus and analysis. Discrete objects can often be enumerated by integers. More formally, discrete mathematics has been characterized as the branch of mathematics dealing with countable sets.

However, there is no exact definition of the term "discrete mathematics. Indeed, discrete mathematics is described less by what is included than by what is excluded: continuously varying quantities and related notions.

The set of objects studied in discrete mathematics can be finite or infinite. The term **finite mathematics** is sometimes applied to parts of the field of discrete mathematics that deals with finite sets, particularly those areas relevant to business.

Concepts and notations from discrete mathematics are useful in all branches of engineering. In computer science, such as computer algorithms, programming languages, cryptography, automated theorem proving, and software development. Conversely, computer implementations are significant in applying ideas from discrete mathematics to real-world problems, such as in operations research.

The main objects of study in discrete mathematics are discrete objects; analytic methods from continuous mathematics are often employed as well.

2. GOALS OF DISCRETE MATHEMATICS

1: The learner will use matrices and graphs to model relationships and solve problems.

a) Use matrices to model and solve problems.

b) Display and interpret data.

c) Write and evaluate matrix expressions to solve problems.

d)Use graph theory to model relationships and solve problems.

2: **Use theoretical and experimental probability to model and solve problems.**

a) Use addition and multiplication principles.

b) Calculate and apply permutations and combinations.

c) Create and use simulations for probability models.

d) Find expected values and determine fairness.

e) Identify and use discrete random variables to solve problems.

f) Apply the Binomial Theorem.

3. The learner will describe and use recursively-defined relationships to solve problems.

a) Find the sum of a finite sequence.

b) Find the sum of an infinite sequence.

c) Determine whether a given series converges or diverges.

d) Write explicit definitions using iterative processes, including finite differences and arithmetic and geometric formulas.

e) Verify an explicit definition with inductive proof.

3. PREREQUISITES:

1. Logic is the Science dealing with the methods of reasoning i.e True or False.
2. Set Theory is the idea of mathematical structure ,learn the concept of objects .
3. Translate among graphic, algebraic, numeric, and verbal representations of relations.
4. Describe graphically, algebraically and verbally phenomena as functions; identify independent and dependent quantities, domain, and range, input/output, mapping.
5. Define and use linear and exponential functions to model and solve problems.
6. Operate with matrices to model and solve problems.
7. Define complex numbers and perform basic operations with them.
8. To acquire knowledge in discrete mathematical structures as applied to the respective branches of Engineering

4. INSTRUCTIONAL OBJECTIVES

1. To understand logic and mathematical reasoning to count or enumerate objects in systematic way.

2 To understand set theory, relations and functions to read , understand and construct mathematical arguments.

3 To understand the concept of lattices and Boolean algebra.

4 To understand how to apply the knowledge of graph theory to solve real world problems like minimum spanning tree - traversal of binary tree.

5 To understand recurrence relation, generating functions and algebraic systems.

5. SYMBOLS REQUIRED IN DISCRETE MATHEMATICS

Symbol	**Meaning**	**Example**
{ }	Set: a collection of elements	{1,2,3,4}
$A \cup B$	Union: in A or B (or both)	$C \cup D = \{1,2,3,4,5\}$
$A \cap B$	Intersection: in both A and B	$C \cap D = \{3,4\}$
$A \subseteq B$	Subset: A has some (or all) elements of B	$\{3,4,5\} \subseteq D$
$A \subset B$	Proper Subset: A has some elements of B	$\{3,5\} \subset D$
$A \not\subset B$	Not a Subset: A is not a subset of B	$\{1,6\} \not\subset C$
$A \supseteq B$	Superset: A has same elements as B, or more	$\{1,2,3\} \supseteq \{1,2,3\}$
$A \supset B$	Proper Superset: A has B's elements and more	$\{1,2,3,4\} \supset \{1,2,3\}$
$A \not\supset B$	Not a Superset: A is not a superset of B	$\{1,2,6\} \not\supset \{1,9\}$
A^c	Complement: elements not in A	$D^c = \{1,2,6,7\}$ When $= \{1,2,3,4,5,6,7\}$
$A - B$	Difference: in A but not in B	$\{1,2,3,4\} - \{3,4\} = \{1,2\}$
$a \in A$	Element of: *a* is in A	$3 \in \{1,2,3,4\}$
$b \notin A$	Not element of: *b* is not in A	$6 \notin \{1,2,3,4\}$
$\varnothing$	Empty set = {}	$\{1,2\} \cap \{3,4\} = \emptyset$
$\mathbb{U}$	Universal Set: set of all possible values (in the area of interest)	
P(A)	Power Set: all subsets of A	P({1,2}) = { {}, {1}, {2}, {1,2} }
A = B	Equality: both sets have the same members	{3,4,5} = {3,4,5}
$A \times B$	Cartesian Product: set of ordered pairs from A and B	$\{1,2\} \times \{3,4\}$ $= \{(1,3), (1,4), (2,3), (2,4)\}$

|A|	Cardinality: the number of elements of set A	|{3,4}| = 2
∀	For All	∀x>1, x^2>x
∃	There Exists	∃ x | x^2>x
∴	Therefore	*a=b ∴ b=a*
ℕ	Natural Numbers	{1,2,3,...} or {0,1,2,3,...}
ℤ	Integers	{..., -3, -2, -1, 0, 1, 2, 3, ...}
ℚ	Rational Numbers	
𝔸	Algebraic Numbers	
ℝ	Real Numbers	
𝕀	Imaginary Numbers	3i
ℂ	Complex Numbers	

6. FUTURE DIRECTION AND CHALLENGES IN DISCRETE MATHEMATICS

Discrete Mathematics, as well as some thoughts about the future direction and challenges in this area are described as follows.

1. The originators of the basic concepts of Discrete Mathematics, the mathematics of finite structures, who knew the formulae for the number of permutations of a set of n elements, and for the number of subsets of cardinality k in a set of n elements already in the sixth century.
2. Algebraic and topological techniques play a crucial role in the modern theory, and Polyhedral Combinatorics, Linear Programming and constructions of designs have been developed extensively
3. The beginning of Combinatorics as know it today started with the work of Pascal and De Moivre in the 17th century, and continued in the 18th century with the similar ideas of Euler in Graph Theory, with his work on partitions and their enumeration, and with his interest in latin squares.
4. These old results are among the roots of the study of formal methods of enumeration, the development of configurations and designs, and the extensive work on Graph Theory in the last two centuries.
5. The tight connection between Discrete Mathematics and Theoretical Computer Science, and the rapid development of the latter in recent years, led to an increased interest in Combinatorial techniques and to an impressive development of the subject. It also stimulated the study and development of algorithmic combinatorics and combinatorial optimization.
6. While many of the basic combinatorial results were obtained mainly by ingenuity and detailed reasoning, the modern theory has evolved into a much deeper theory with a systematic and powerful toolkit.
7. Most of the new significant results obtained in the area are inevitably based on the knowledge of these well developed concepts and techniques, and while there is, of course, still a great deal room for pure ingenuity in Discrete Mathematics, much progress is obtained with the aid of our accumulated body of knowledge.

7. TOPICS IN DISCRETE MATHEMATICS

1. MATHEMATICAL LOGIC

Logic is the study of the principles of valid reasoning and inference, as well as of consistency, soundness, and completeness, for classical logic, it can be easily verified with a truth table. The study of mathematical proof is particularly important in logic, and has applications to automated theorem proving and formal verification of software.

Logical formulas are discrete structures, as are proofs, which form finite trees or, more generally, directed acyclic graph structures. The truth values of logical formulas usually form a finite set, generally restricted to two values are true and false.

2. SET THEORY

Set theory is the branch of mathematics that studies sets, which are collections of objects, such as {blue, white, red} or the (infinite) set of all prime numbers. Partially ordered sets and sets with other relations have applications in several areas.

In discrete mathematics, countable sets are the main focus. The beginning of set theory as a branch of mathematics is usually marked by Georg Cantor's work distinguishing between different kinds of infinite set, motivated by the study of trigonometric series, and further development of the theory of infinite sets is outside the scope of discrete mathematics.

In set Theory the collection of objects can be represented either graphical or non graphical representation like venn - Diagrams & Relations.

A function defined as an interval of the integers is usually called a sequence. A sequence could be a finite sequence from a data source or an infinite sequence from a discrete dynamical system. Such a discrete function could be defined explicitly by a list, or by a formula for its general term, or it could be given implicitly by a recurrence relation or difference equation.

3. **ALGEBRAIC STRUCTURES**

Algebraic structures occur as both discrete examples and continuous examples. Discrete algebras include: Boolean algebra used in logic gates and programming; relational algebra used in databases; discrete and finite versions of groups, rings and fields are important in algebraic coding theory; Discrete semi groups and monoids appear in the theory of formal languages.

4. **GRAPH THEORY**

Graph theory has close links to group theory. Graph theory, the study of graphs and networks, is often considered part of combinatorics, but has grown large enough and distinct enough, with its own kind of problems, to be regarded as a subject in its own right. Graphs are one of the prime objects of study in discrete mathematics. They are among the most ubiquitous models of both natural and human-made structures. They can model many types of relations and process dynamics in physical, biological and social systems.

A. In computer science Graph Theory can represents
B. Networks of communication
C. Data organization
D. Computational devices, the flow of computation, etc.

In mathematics, they are useful in geometry and certain parts of topology. Algebraic graph theory has close links with group theory. There are also continuous graphs, however for the most part research in graph theory falls within the domain of discrete mathematics.

5. COMBINATORICS & NUMBER SYSTEMS

Combinatorics studies the way in which discrete structures can be combined or arranged. Enumerative combinatorics concentrates on counting the number of certain combinatorial objects. Analytic combinatorics concerns the enumeration of combinatorial structures using tools from complex analysis and probability theory. In contrast with enumerative combinatorics which uses explicit combinatorial formulae and generating functions to describe the results, analytic combinatorics aims at obtaining asymptotic formulae.

Design theory is a study of combinatorial designs, which are collections of subsets with certain intersection properties. Partition theory studies various enumeration and asymptotic problems related to integer partitions, and is closely related to q-series, special functions and orthogonal polynomials. Originally a part of number theory and analysis, partition theory is now considered a part of combinatorics or an independent field. Order theory is the study of partially ordered sets, both finite and infinite.

Number theory is concerned with the properties of numbers in general, particularly integers. It has applications to cryptography, cryptanalysis, and cryptology, particularly with regard to modular arithmetic, linear and quadratic congruence's, prime numbers and primarily testing. Other discrete aspects of number theory include geometry of numbers. In analytic number theory, techniques from continuous mathematics are also used. Topics that go beyond discrete objects include transcendental numbers, and function fields.

8. APPLICATIONS OF DISCRETE MATHEMATICS

1. Concepts and questions of Discrete Mathematics appear naturally in many branches of mathematics, and the area has found applications in other disciplines.

2. These areas include applications in Information Theory and Electrical Engineering, in Statistical Physics, in Chemistry and Molecular Biology, and, of course, in Computer Science. Combinatorial topics such as Ramsey Theory, Combinatorial Set Theory, Graph Theory.

3. Combinatorial Geometry and Discrepancy Theory are related to a large part of the mathematical and scientific world, and these topics have already found numerous applications in other fields.

4. It seems safe to predict that in the future Discrete Mathematics will be continue to incorporate methods from other mathematical areas. However, such methods usually provide non-constructive proof techniques, and the conversion of these to algorithmic ones may well be one of the main future challenges of the area

5. Another interesting recent development is the increased appearance of computer-aided proofs in Combinatorics, starting with the proof of the Four Color Theorem. One hopes that automated approaches will continue to bear fruit hopefully without destroying the special beauty and appeal of the field to human mathematicians.

6. The fundamental nature of Discrete Mathematics, its tight connection to other disciplines, and its many fascinating open problems ensure that this field will continue to play an essential role in the general development of science.