G. PULLAIAH COLLEGE OF ENGINEERING AND TECHNOLOGY
Accredited by NAAC with 'A' Grade of UGC, Approved by AICTE, New Delhi
Permanently Affiliated to JNTUA, Ananthapuramu
(Recognized by UGC under 2(f) and 12(B) & ISO 9001:2008 Certified Institution)
Nandikotkur Road, Venkayapalli, Kurnool – 518452

Department of Mechanical Engineering

Bridge Course
On
Engineering Mechanics

Define rigid body

The bodies which will not deform or the body in which deformation can be neglected in the analysis are called as rigid bodies.

Statics: The mechanics of the rigid bodies dealing with the bodies at rest is termed as Statics

Dynamics: The mechanics of the rigid bodies dealing with the bodies at motion is termed as Dynamics.

Mass

The quantity of the matter possessed by a body is called mass. The mass of a body will not change unless the body is damaged and part of it is physically separated. When a body is taken out in a spacecraft, the mass will not change but its weight may change due to change in gravitational force. Even the body may become weightless when gravitational force vanishes but the mass remain the same.

Time

Time is the measure of succession of events.

Space

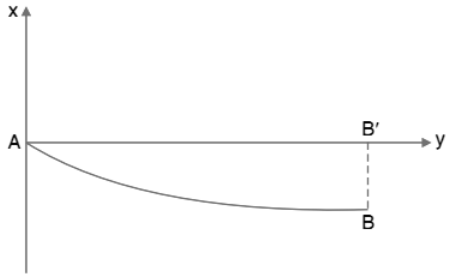
The geometric region in which study of body is involved is called space. A point in the space may be referred with respect to a predetermined point by a set of linear and angular measurements. The reference point is called the origin and set of measurements as 'coordinates'. If coordinates involve only in mutually perpendicular directions they are known as Cartesian coordinates. If the coordinates involve angle and distances, it is termed as polar coordinate system.

Length

It is a concept to measure linear distances.

Displacement

Displacement is defined as the distance moved by a body/particle in the specified direction. Referring to Fig. if a body moves from position A to position B in the x-y plane shown, its displacement in x-direction is AB′ and its displacement in y-direction is B′B.



Velocity

The rate of change of displacement is defined as velocity.

Acceleration

Acceleration is the rate of change of velocity with respect to time. Thus

$a = \frac{dv}{dt}$ where v is velocity

Momentum

The product of mass and velocity is called momentum. Thus

Momentum = Mass × Velocity

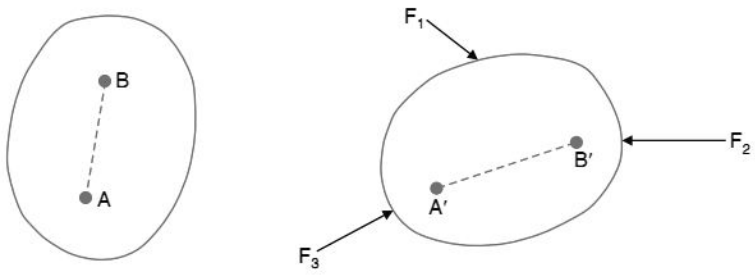
Continuum

A body consists of several matters. It is a well known fact that each particle can be subdivided into molecules, atoms and electrons. It is not possible to solve any engineering problem by treating a body as a conglomeration of such discrete particles. The body is assumed to consist of a continuous distribution of matter. In other words, the body is treated as continuum.

Rigid Body

A body is said to be rigid, if the relative positions of any two particles in it do not change under the action of the forces. In Fig. (a) points A and B are the original position in a body. After application of a system of forces F1, F2, F3, the body takes the position as shown in Fig. (b). A' and B' are the new positions of A and B. If the body is treated as rigid, the relative position of A'B' and AB are the same i.e.,

A'B' = AB.



Many engineering problems can be solved satisfactorily by assuming bodies rigid.

Particle

A particle may be defined as an object which has only mass and no size.

Newton's First Law

It states that everybody continues in its state of rest or of uniform motion in a straight line unless it is compelled by external agency acting on it. This leads to the definition of force as the external agency which changes or tends to change the state of rest or uniform linear motion of the body.

Newton's Second Law

It states that the rate of change of momentum of a body is directly proportional to the impressed force and it takes place in the direction of the force acting on it. Thus according to this law,

Force ∝ rate of change of momentum. But momentum = mass × velocity

As mass do not change,

Force ∝ mass × rate of change of velocity

i.e., Force ∝ mass × acceleration

F ∝ m × a

Newton's Third Law

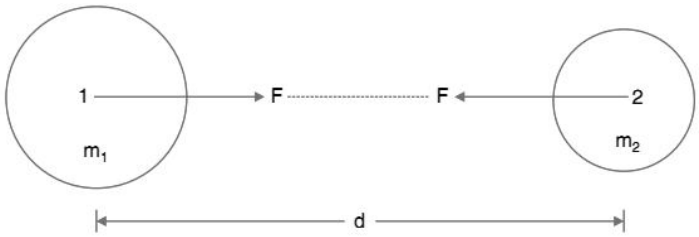
It states that for every action there is an equal and opposite reaction. Consider the two bodies in contact with each other. Let one body applies a force F on another. According to this law the second body develops a reactive force R which is equal in magnitude to force F and acts in the line same as F but in the opposite direction. Figure. shows the action of the ball and the reaction from the floor. In Fig. 1.4 the action of the ladder on the wall and the floor and the reactions from the wall and floor are shown.

Newton's Law of Gravitation

Everybody attracts the other body. The force of attraction between any two bodies is directly proportional to their masses and inversely proportional to the square of the distance between them. According to this law the force of attraction between the bodies of mass m1 and mass m2 at a distance d as shown in Fig. is

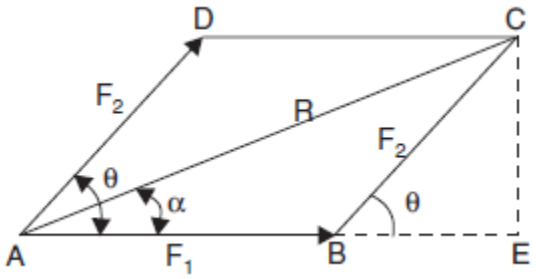
$$F = G \frac{m_1 m_2}{d^2}$$

where G is the constant of proportionality and is known as constant of gravitation.



Parallelogram Law of Forces

If two forces acting simultaneously on a body at a point are presented in magnitude and direction by the two adjacent sides of a parallelogram, their resultant is represented in magnitude and direction by the diagonal of the parallelogram which passes through the point of intersection of the two sides representing the forces.



Triangle Law of Forces:

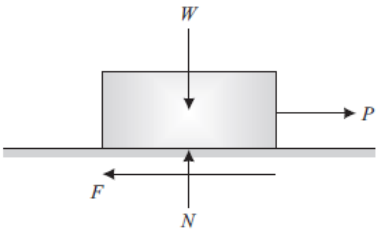
If two forces acting simultaneously on a body are represented by the sides of a triangle taken in order, their resultant is represented by the closing side of the triangle taken in the opposite order.

Moment

Moment is defined as the product of the magnitude of the force and the perpendicular distance of the point from the line of action of the force.

FRICTION: When a body moves or tends to move over another body, a force opposing the motion develops at the contact surfaces. The force which opposes the movement or the tendency of movement is called the frictional force or simply friction

Limiting Friction.



If the magnitude of force goes on increasing, a stage comes, when the body is on the point of motion. At this stage, the force of friction acting on the body is called limiting friction.

Static friction: It is the friction experienced by a body when it is at rest.

Dynamic friction: It is the friction experienced by a body when it is at motion. It is also called kinetic friction.

Sliding Friction: It is the friction experienced by a body when it slides over the other body.

Rolling Friction: It is the friction experienced by a body when it rolls over another body.

COEFFICIENT OF FRICTION

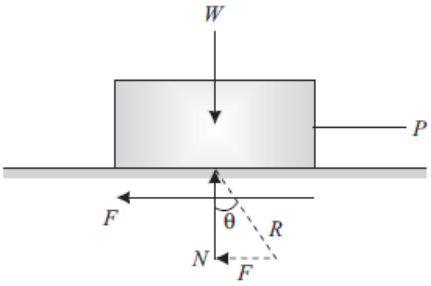
It is defined as the ratio of the limiting friction (F) to the normal reaction (N) between two bodies. It is denoted by the symbol μ.

$$\mu = \frac{\text{Limiting Friction}}{\text{normal Reaction}} = \frac{F}{N}$$

$$F = \mu N$$

Angle of friction:

Consider the block shown in Figsubject to pull P. Let F be the frictional force developed and N the normal reaction. Thus, at contact surface, the reactions are F and N. They can be combined to get the resultant reaction R which acts at angle to normal reaction. This angle is given by



$$\text{Tan } \theta = \frac{F}{N}$$

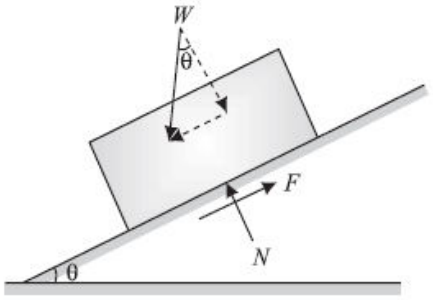
As frictional force increases the angle θ increases and it can reach maximum value Φ when limiting value of friction is reached. Thus, when motion is impending

$$\text{Tan } \Phi = \frac{F}{N} = \mu$$

and this value of Φ is called *angle of limiting friction*. Hence, the angle of limiting friction can be defined as the angle between the resultant reaction and the normal to the plane on which the motion of the body is impending.

Angle of repose

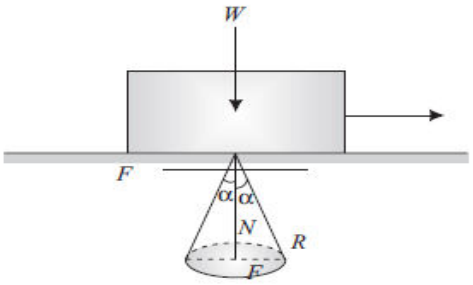
It is very well-known that when grains (food grains, sand, cement, soil etc.) are heaped, there exists a limit for the inclination of the heap. Beyond that the grains start rolling down. The limiting angle up to which the grains repose (sleep) is called angle of repose.



Now consider the block of weight *W* shown in which is resting on an inclined plane that makes angle θ with the horizontal. When θ is a small, block rests on the plane. If θ is increased gradually a stage is reached at which the block starts sliding. The angle made by the plane with the horizontal is called angle of friction for the contacting surfaces. Thus, the maximum inclination of the plane on which the body, free from external forces, can repose is called *angle of repose*. The value of angle of repose is the same as the value of limiting angle of friction.

Cone of friction

When a body is having impending motion in the direction of *P*, the frictional force will be the limiting friction and the resultant reaction *R* will make limiting frictional angle Φ with the normal as shown in fig. If the body is having impending motion in some other direction, the resultant reaction makes limiting frictional angle Φ with the normal. Thus, if the direction of force *P* is gradually changed through 360°, the resultant *R* generates a right circular cone with semi central angle equal to Φ.



If the resultant reaction lies on the surface of this inverted right circular cone whose semi central angle is limiting frictional angle Φ, the motion of the body is impending. If the resultant is within this cone the body is stationary. This inverted cone with semicentral

angle, equal to limiting frictional angle _, is called *cone of friction*.

Wedge friction

Wedges are small pieces of hard materials with two of their opposite surfaces not parallel to each other. They are used to slightly lift heavy blocks, machinery, precast beams etc. for making final alignment or to make place for inserting lifting devices.

State the laws of dry friction

The principles discussed in a previous article are mainly due to the experimental studies by Coulomb (1781) and by Mozin (1831). These principles constitute the laws of dry friction and may be called as *Coulomb's laws of dry friction*. These laws are listed below:

(*i*) The frictional force always acts in a direction opposite to that in which the body tends to move.

(*ii*) Till the limiting value is reached, the magnitude of frictional force is exactly equal to the tangential force which tends to move the body.

(*iii*) The magnitude of the limiting friction bears a constant ratio to the normal reaction between the two contacting surfaces.

(*iv*) The force of friction depends upon the roughness/smoothness of the surfaces.

(*v*) The force of friction is independent of the area of contact between the two surfaces.

(*vi*) After the body starts moving, the dynamic friction comes into play, the magnitude of which is less than that of limiting friction and it bears a constant ratio to the normal force. This ratio is called coefficient of dynamic friction.

Define Centroid: The plane figures (like rectangle, circle, triangle etc,.) have only areas but no mass. The center of area of such figures is known as Centroid.

Define Centroid of gravity.

Centroid is the geometrical center of the body whereas center of gravity is the point through which weight of the body acts.

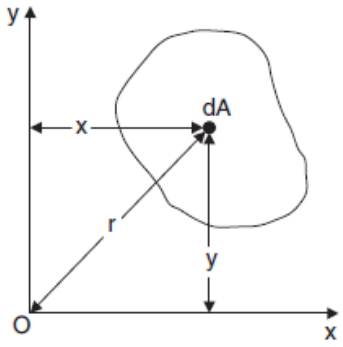
Define line of symmetry.

Line of symmetry is line a about which the area on one side is a mirror image of the area of the side

Perpendicular Axis Theorem (or) polar moment of inertia

The moment of inertia of an area about an axis perpendicular to its plane (polar moment of inertia) at any point O is equal to the sum of moments of inertia about any two mutually perpendicular axis through the same point O and lying in the plane of the area.

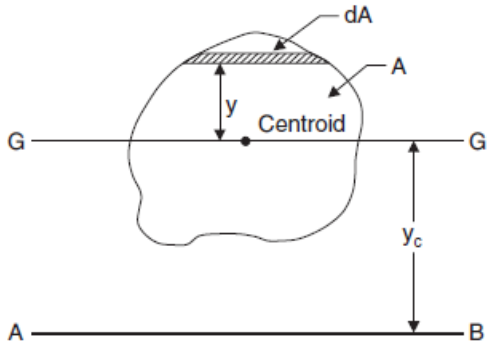
$$Izz = Ixx + Iyy$$



State parallel axis theorem.

Moment of inertia of an area about an axis is equal to the sum of (a) moment of inertia an axis passing through the Centroid parallel to the given axis and (b) the product of area and square of the distance between the two parallel axes.

$$I_{AB} = I_{GG} + A\, y_c^2$$



Difference between Centre of Gravity and Centroid

From the above discussion we can draw the following differences between centre of gravity and Centroid:

(1) The term centre of gravity applies to bodies with weight, and Centroid applies to lines, plane areas and volumes.

(2) Centre of gravity of a body is a point through which the resultant gravitational force (weight) acts for any orientation of the body whereas Centroid is a point in a line plane area volume such that the moment of area about any axis through that point is zero.

Define first moment of an area about of an axis.

The first moment of an area about of an axis is the product of an area and the perpendicular distance of its Centroid from axis

First moment or moment, $M = A * d$

Where, M = moment

A = force

d = perpendicular

Define second moment of an area about of an axis.

The second moment of an area about of an axis is the product of an area and the perpendicular square distance of its Centroid from axis

First moment or moment, $M = A * d^2$

Where, M = moment

A = force

d = perpendicular

Radius of Gyration: The distance from an axis at which the mass of a body may be assumed to be concentrated and at which the moment of inertia will be equal to the moment of inertia of the actual mass about the axis, equal to the square root of the quotient of the moment of inertia and the mass.

$$k = \sqrt{\frac{I}{A}}$$

Where k = radius of gyration,

I = moment of inertia,

A = the cross-sectional area

Kinematics: The dynamics dealing with the problems without referring to the forces causing the motion of the body is termed as Kinematics.

Rectilinear:

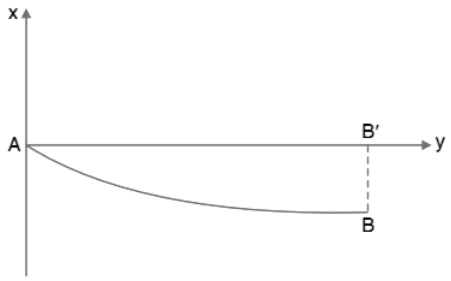
The motion of a body in a straight line is called rectilinear motion.

Curvilinear:

The motion of a body along a circular path is called curvilinear motion.

Displacement

Displacement is defined as the distance moved by a body/particle in the specified direction. Referring to Fig., if a body moves from position A to position B in the x-y plane shown, its displacement in x-direction is AB' and its displacement in y-direction is B'B.



Velocity

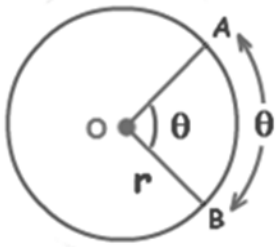
The rate of change of displacement is defined as velocity.

Acceleration

Acceleration is the rate of change of velocity with respect to time. Thus

$a = \frac{dv}{dt}$ where v is velocity

Angular displacement (θ): The angular displacement of an object moving around a circular path is defined as the angle subtended by the radius vector at the centre of the circular path in the given time.



Angular velocity (ω): It is defined as the time rate of change of angular displacement of the object i.e. $\omega = \frac{d\theta}{dt}$. Its S.I unit is rad/s.

Angular Acceleration: It is defined as the time rate of change of angular velocity of the object i.e. $\alpha = \frac{dv}{dt}$. Its S.I unit is rad/s^2.

Derive the equation of displacement in the nth second of the motion

Distance traveled during nth second is equal to the difference between the distance traveled in 'n' seconds and (n-1) seconds.

$S_n = (un + \frac{1}{2} an^2) - [u(n-1) + \frac{1}{2} a(n-1)^2]$

$= un + \frac{1}{2} an^2 - [un - u + \frac{1}{2} a(n^2 - 2n + 1^2)]$

$= un + \frac{1}{2} an^2 - un + u - \frac{1}{2} an^2 + an - \frac{1}{2} a$

$= u + an - \frac{1}{2} a$

Hence, $S_n = u + a(2n - 1)$.

What do you understand Motion under gravity from a height:

Any body dropped, moves towards the earth with nearly constant acceleration. Its motion is an example of one dimensional motion under gravity. This acceleration is the same for all bodies which are dropped and independent of their size, mass or composition and is called "acceleration due to gravity".

It is always directed towards the center of the earth and is denoted by 'g'.

The equation of motion for a freely falling body are obtained by replacing 'a' by 'g'

i.e.

For a body falling freely under gravity,

1. $V = u + gt$
2. $S = ut + \frac{1}{2} gt^2$
3. $V^2 = u^2 + 2gh$

For a body rising vertically against gravity,

1. $V = u - gt$

2 $S = ut - \frac{1}{2} gt^2$

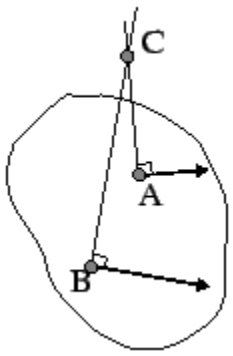
3. $V^2 = u^2 - 2gh$

Plane motion:

A body is said to have plane motion if it possesses either translation and rotation simultaneously.

Instantaneous center:

The instantaneous center method of analyzing the motion in a mechanism is based upon the concept that any displacement of a body having motion in one plane, can be considered as a pure rotational motion of a rigid link as whole about some center, known as instantaneous center or virtual center of motion.



Kinetics: The dynamics dealing with the problems with referring to the forces causing the motion of the body is termed Kinetics.

D'Alembert Principle:

Alternative form of Newton's second law of motion The system of forces acting on a body in motion is in dynamic equilibrium with the internal force of the body.

What is uniform motion?

If the velocity of body does not change with time, then the motion is called as uniform motion.

Work:

The work done by a force on a body is defined as the product of the force and the distance moved in the direction of the force.

Energy:

Energy is defined as the capability to do work.

There are many forms of energy like heat energy, mechanical energy, electrical energy, chemical energy. In engineering mechanics, we are interested in mechanical energy. This energy may be classified into potential energy and kinetic energy.

Potential energy:

It is the capability to do work due to the position of the body. A body of weight 'W' held at a height h possesses an energy Wh.

Kinetic energy:

It is the capability to do work due to the motion of the body.

Power:

Power is defined as time rate of doing work. Unit of power is Watt (W).