G. PULLAIAH COLLEGE OF ENGINEERING AND TECHNOLOGY

Accredited by NAAC with 'A' Grade of UGC, Approved by AICTE, New Delhi Permanently Affiliated to JNTUA, Ananthapuramu (Recognized by UGC under 2(f) and 12(B) & ISO 9001:2008 Certified Institution) Nandikotkur Road, Venkayapalli, Kurnool – 518452

Department of Mechanical Engineering

Bridge Course

On

Mechanics of Solids

Strength of materials is the science which deals with the relations between externally applied loads and their internal effects on bodies. The bodies are no longer assumed to be rigid and deformations are of major interest. During deformation the external forces acting upon the body do work. This work is transferred completely or partially into potential energy of strain. If the forces which produce the deformation of the body are gradually removed, the body returns or try to return to its original shape. During this return the stored potential energy can be recovered in form of external work. The main concern of the subject is regarding three S's, namely strength, stiffness and stability of various load carrying members.

Stress

When an external force acts on a body, it undergoes deformation. At the same time the body resists deformation. The magnitude of the resisting force is numerically equal to the applied force. This internal resisting force per unit area is called stress.

Stress = Force/Area = P/A unit is N/mm^2

Strain

When a body is subjected to an external force, there is some change of dimension in the body. Numerically the strain is equal to the ratio of change in length to the original length of the body

Strain = Change in length/Original length

$e = \delta L/L$

Hooke's law

It states that when a material is loaded, within its elastic limit, the stress is directly proportional to the strain.

Stress α Strain

$\sigma \alpha e$

$\sigma = Ee$

$E = \sigma/e$ unit is N/mm^2

Where,

E - Young's modulus

σ- Stress

e - Strain

Shear stress and shear strain

The two equal and opposite force act tangentially on any cross sectional plane of the body tending to slide one part of the body over the other part. The stress induced is called shear stress and the corresponding strain is known as shear strain. When a body is stressed, within its elastic limit, the ratio of lateral strain to the longitudinal strain is constant for a given material.

Poisson' ratio (μ or 1/m) = Lateral strain /Longitudinal strain

Relationship between Young's Modulus and Modulus of Rigidity

$E = 2G (1+1/m)$

Where,

E - Young's Modulus

K - Bulk Modulus

1/m - Poisson's ratio

Whenever a body is strained, some amount of energy is absorbed in the body. The energy that is absorbed in the body due to straining effect is known as strain energy.

Resilience

The total strain energy stored in the body is generally known as resilience.

Proof resilience

The maximum strain energy that can be stored in a material within elastic limit is known as proof resilience.

Modulus of resilience

It is the proof resilience of the material per unit volume.

Relationship between Bulk Modulus and Young's Modulus

$E = 3K (1-2/m)$

Where,

E - Young's Modulus

K - Bulk Modulus

1/m - Poisson's ratio

Compound bar

A composite bar composed of two or more different materials joined together such that system is elongated or compressed in a single unit.

Thermal stresses

If the body is allowed to expand or contract freely, with the rise or fall of temperature no stress is developed but if free expansion is prevented the stress developed is called temperature stress or strain.

Elastic limit

Some external force is acting on the body, the body tends to deformation. If the force is released from the body its regain to the original position. This is called elastic limit.

Young's modulus

The ratio of stress and strain is constant with in the elastic limit.

Bulk-modulus

The ratio of direct stress to volumetric strain.

Lateral strain

When a body is subjected to axial load P. The length of the body is increased. The axial deformation of the length of the body is called lateral strain.

Longitudinal strain

The strain right angle to the direction of the applied load is called lateral strain.

Rigidity modulus

The shear stress is directly proportional to shear strain.

Beam

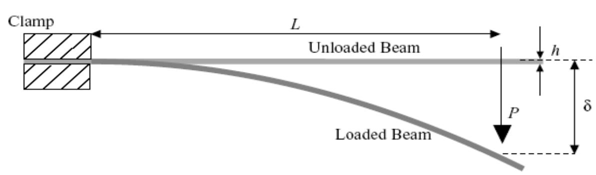
Structural member which is acted upon a system of external loads at right angles to its axis is known as beam.

Moment Of Inertia

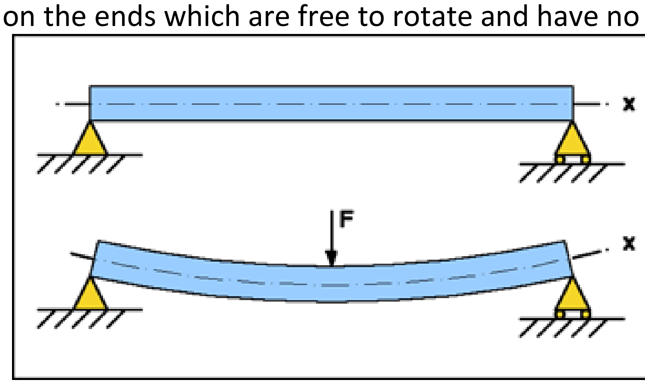
The moment of inertia is a physical quantity which tells how easily a body can be rotated about a given axis. It is a rotational analogue of mass. It plays the same role in rotational motion as ‘inertia’ does in translational motion. *Inertia* is the property of matter which resists change in its state of motion. Inertia is a measure of the force that keeps a stationary object stationary, or a moving object moving at its current speed. The larger the inertia, the greater the force that is required to bring some change in its velocity in a given amount of time. Suppose a heavy truck and a light car are both at rest, then intuitively we know that more force will be required to push the truck to a certain speed in a given amount of time than will be needed to push the car to that same speed in the same amount of time

Types of beams

1. Cantilever beam

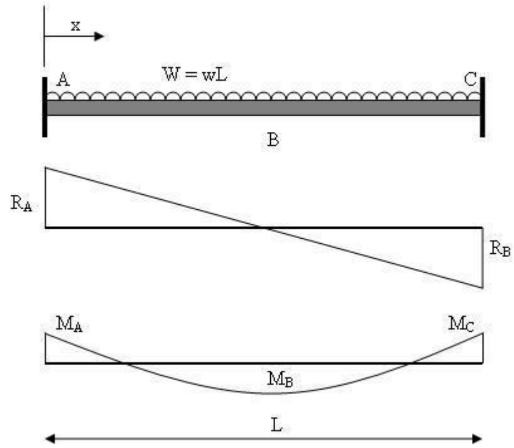


2. Simply supported beam
   A beam supported on the ends which are free to rotate and have no moment resistance.

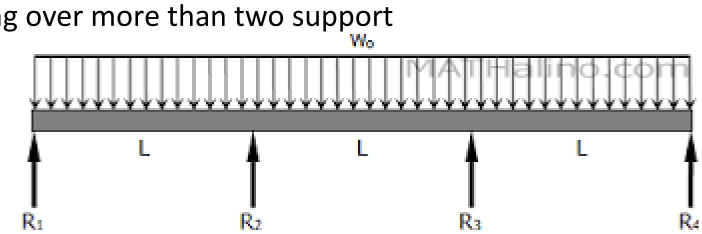


3. Fixed beam

   A beam supported on both ends and restrained from rotation



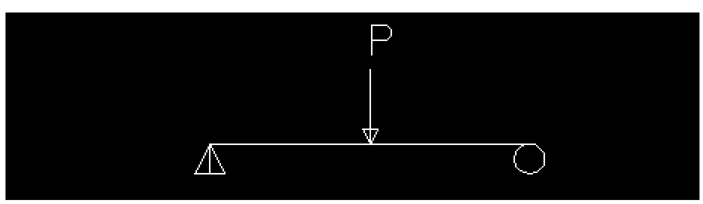
4. Continuous beam
   A beam extending over more than two support



Types of loads

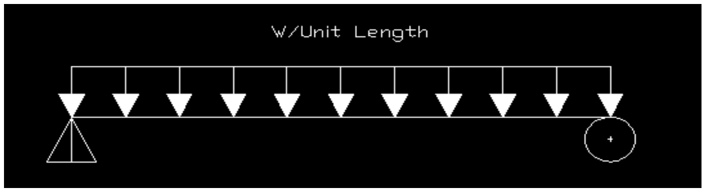
1. Concentrated load or point load

Point load is that load which acts over a **small distance**. Because of concentration over small distance this load can may be considered as acting on **a point**. Point load is denoted by **P** and symbol of point load is arrow heading downward **(↓)**



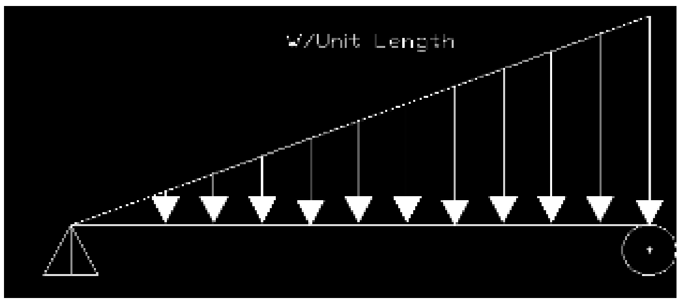
2. Uniform distributed load

Uniformly distributed load is that whose magnitude remains uniform throughout the length. **For Example:** If 10k/ft load is acting on a beam whose length is 15ft. Then 10k/ft is acting **throughout the length** of 15ft.Uniformly distributed load is usually represented by **W** and is pronounced as **intensity of udl** over the beam, slab etc.



3. Uniform varying load

   Triangular load is that whose magnitude is **zero** at **one end** of span and increases constantly till the **2nd end** of the span. As shown in the diagram



Shear force and bending moment

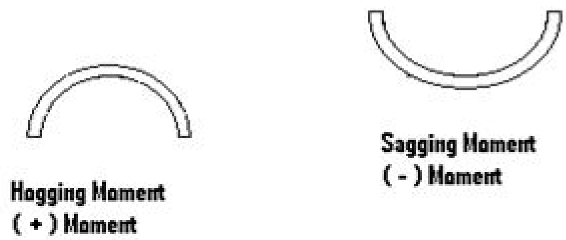
SF at any cross section is defined as algebraic sum of all the forces acting either side of beam. BM at any cross section is defined as algebraic sum of the moments of all the forces which are placed either side from that point.

Sagging BM & Hogging BM

Sagging and Hogging are the terms used to define the sign of a bending moment. The bending moment which causes a beam to bend with the concave side upwards, is called a Sagging Bending Moment. This kind of bending moment is treated as a positive bending moment.

On the other hand, the bending moment which causes a beam to bend with the concave side upwards is called a Hogging Bending Moment. This kind of a bending moment is treated as a negative bending moment.

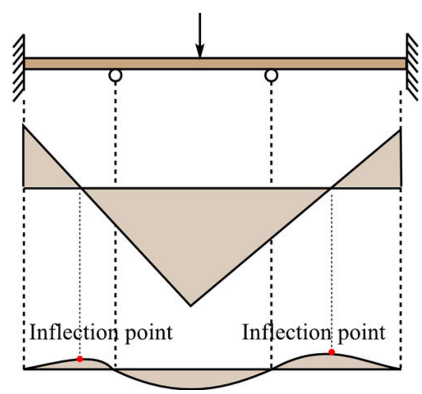
Alternatively, if the bending moment is in the anticlockwise direction on the R.H.S of the beam, then it is considered as a positive bending moment. Similarly, if it is in the clockwise direction on the L.H.S. of the beam, it is taken as a positive bending moment



BM is said to positive if moment on left side of beam is clockwise or right side of the beam is counter clockwise.

Define point of contra flexure? In which beam it occurs?

Point of the Contraflexure is the point where Bending moment value is 0. or other way where Bending moments begins to change. Flexural reinforcement may be reduced at this point. However, to omit reinforcement at the point of contra flexure entirely is inadvisable as the actual location is unlikely to realistically be defined with confidence. Additionally, an adequate quantity of reinforcement should extend beyond the point of contra flexure to develop bond strength and to facilitate shear force transfer Point at which BM changes to zero is point of contra flexure. It occurs in overhanging beam.



Assumptions in the theory of simple bending

1. The material of the beam is homogeneous and isotropic.

2. The beam material is stressed within the elastic limit and thus obeys hooke's law.

3. The transverse section which was plane before bending remains plains after bending also.

4. Each layer of the beam is free to expand or contract independently about the layer, above or below.

5. The value of E is the same in both compression and tension.

Theory of simple bending Equation

M/ I = F/Y = E/R

M - Maximum bending moment

I - Moment of inertia

F - Maximum stress induced

Y - Distance from the neutral axis

E - Young's modulus

R - Constant.

Shear stress distribution

The variation of shear stress along the depth of the beam is called shear stress distribution.

Section Modulus

It can be defined s the ratio of moment of inertia of a section about the n eutral axis to the distance of the outermost layer from the neutral axis. It is denoted by Z.

$Z = I/y_{max}$

Since, $M/I = \sigma/y$

$\Rightarrow M/I = M/I = \sigma_{max}/Y_{max}$ $Z = I/y_{max} = M/\sigma_{max}$

Moment Area method Cantilever

1. Draw Bending moment Diagram.

2. The total area of the BM diagram will give the slope at free end.

3. To find the slope at the other point in the beam. Find the area of the BM diagram from the support to that point, that area would give the slope at that point. A/EI.

4. The total area of the BM diagram multiplied by centroid from free end will give the deflection at the free end. Ax/EI x is centroid from point of deflection to be found.

5. To find the deflection at the other point in the beam. Find the area of the BM diagram from the support to that point multiplied by centroid from that point. That would give the deflection at that point.

Conjugate method Cantilever

It is a modification of Moment Area Method. It is effective where the inertia of section is different along the length of the beam. Conjugate method for cantilever is almost same as moment area method of cantilever.

1. Draw Bending moment Diagram of the given load.

2. The total area of the BM diagram will give the slope at free end.

3. The sum of the area of the BM diagram at varying inertia from a point to the support would give the slope at that point. ΣAn/EI.

4. The sum of moment of the BM diagram at varying section taken from a point to the support would give the deflection at that point. ΣAnXn/EI.

5. To find the deflection at the other point in the beam. Find the area of the BM diagram from the support to that point multiplied by centroid from that point. That would give the deflection at that point.

Conjugate method simply supported beam

1. Find the reaction of the given load and draw Bending moment Diagram.

2. Find the reaction of the support assuming the bending moment diagram as the load for varying inertia. This beam is known as conjugate beam.

3. The reaction at the supports will give the slope at the supports.

4. The upward load minus downward load of the conjugate beam will give slope at a point.

5. The moment taken at a point from the conjugate beam will give the deflection.

How to Calculate the Reactions at the Supports of a Beam

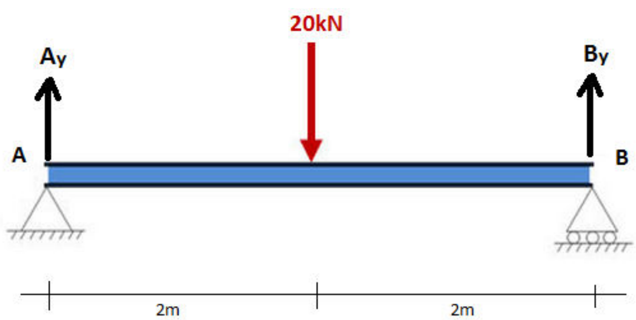
This is always the first step in analyzing a beam structure, and it is generally the easiest. It involves calculating the values of the reaction forces at the supports (supports A and B in the below example) due to the forces acting on the beam. You will need to know this to progress through and calculate bending moment diagrams (BMDs) and shear force diagrams (SFDs); an important part of your statics and structural college/university courses. SkyCiv offers a powerful beam software that allow you to model any beam and show these hand calculations for you, but it is also an important concept to understand.

When solving a problem like this we want to first remember that the beam is static; meaning it is not moving. From simple physics, this means that the sum of the forces in the y direction equals zero (i.e. the total downward forces equal the total upward forces). A second formula to remember is that the sum of the moments about any given point is equal to zero. This is because the beam is static and therefore not rotating.

To find the reactions of a simple beam, follow these simple steps:

1. **Let the sum of moments about a reaction point equal ZERO (ΣM = 0)**

All we need to know about moments at this stage is that they are they are equal to the force multiplied by the distance from a point (i.e. the force x distance from point).Consider a simple example of a 4m beam with a pin support at A and a roller support at B. The free body diagram is shown below where Ay and By are the vertical reactions at the supports:



We firstly want to consider the sum of moments about point B and let it equal zero. We have chosen point B to prove this can be done at either ends of the beam (provided it is pin supported). However you could just as easily work from point A. So, now we sum the moments about point B and let the sum equal 0:

$$\Sigma M_B = 0 = 20(2) - A_y(4)$$

$$A_y = 10kN$$

NOTE: The sign convention we have chosen is that counter-clockwise moments are positive and clockwise moments are negative. This is the most common sign convention but it is up to you. You must ALWAYS use the same sign convention throughout the whole problem. Always use the same sign convention from the start.

We now have our first equation. We need to solve another equation in order to find By (the vertical reaction force at support B).

2. Let the sum of vertical forces equal 0 (ΣFy = 0)
Sum the forces in the y (vertical) direction and let the sum equal zero. Remember to include all forces including reactions and normal loads such as point loads. So if we sum the forces in the y direction for the above example, we get the following equation:

$$\Sigma F_Y = 0 = A_Y + B_Y - 20kN$$

And since we know $A_Y = 10kN$

$$0 = 10kN + B_Y - 20kN$$

$$B_Y = 10kN$$

NOTE: Again we stuck to a sign convention which was to take upward forces (our reactions) as positive and downward forces (the point load) as negative. Remember the sign convention is up to you but you must ALWAYS use the same sign convention throughout the whole problem.

So there we have it, we have used the two above equations (sum of moments equals zero and sum of vertical forces equals zero) and calculated that the reaction at support A is 10 kN and the reaction at support B 10kN. This makes sense as the point load is right in the middle of the beam, meaning both supports should have the same vertical forces (i.e. it is symmetric).