G. PULLAIAH COLLEGE OF ENGINEERING AND TECHNOLOGY

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Nandikotkur Road, Venkayapalli, Kurnool – 518452

Department of Civil Engineering

Bridge Course
On
Structural Analysis-II

Three Hinged Arches

An arch can be defined as a humped or curved beam subjected to transverse and other loads as well as the horizontal thrust at the supports."

- It is simplest type of arch, consists of two section hinged at the crown and a hinge at support.
- The hinges at the support makes the ends of the arch to be fixed in position but not in direction.
- It is statically determinate structure.

Arch Vs Beam

- In case of beams supporting UDL, the maximum bending moment increases with the square of the span and hence they become uneconomical for long span structures. In such situations arches could be advantageously employed, as they would develop horizontal reactions, which reduce the design bending moment.
- Arches are mainly used in bridge construction and doorways. In earlier days arches were constructed using stones and bricks. In modern times they are being constructed of reinforced concrete and steel.

Parabolic arches are preferable to carry distributed loads. Because, both, the shape of the arch and the shape of the bending moment diagram are parabolic. Hence the intercept between the theoretical arch and actual arch is zero everywhere. Hence, the bending moment at every section of the arch will be zero. The arch will be under pure compression that will be economical.

Two hinged arches

- Statically indeterminate to first degree
- Might develop temperature stresses.
- Structurally more efficient.
- Will develop stresses due to sinking of supports

Three hinged arches

- Statically determinate
- Increase in temperature causes increases in central rise. No stresses
- Easy to analyze. But, in construction, the central hinge may involve additional expenditure.
- Since this is determinate, no stresses due to support sinking

Fixed vs Hinged Arch

An arch without hinges is called as fixed arch. The fixed arch is most often used in reinforced concrete bridge and tunnel construction, where the spans are short. Because it is subject to additional internal stress caused by thermal expansion and contraction, this type of arch is considered to be statically indeterminate.

The two-hinged arch is most often used to bridge long spans. This type of arch has pinned connections at the base. Unlike the fixed arch, the pinned base is able to rotate allowing the structure to move freely and compensate for the thermal expansion and contraction caused by changes in outdoor temperature. However, this can result in additional stresses, so the two-hinged arch is also statically indeterminate, although not to the degree of the fixed arch.

The three-hinged arch is not only hinged at its base, like the two-hinged arch, but at the mid-span as well. The additional connection at the mid-span allows the three-hinged arch to move in two opposite directions and compensate for any expansion and contraction. This type of arch is thus not subject to additional stress caused by thermal change. The three-hinged arch is therefore said to be statically determinate. It is most often used for medium-span structures, such as large building roofs.

Another advantage of the three-hinged arch is that the pinned bases are more easily developed than fixed ones, allowing for shallow, bearing-type foundations in medium-span structures. In the three-hinged arch, "thermal expansion and contraction of the arch will cause vertical movements at the peak pin joint but will have no appreciable effect on the bases," further simplifying the foundation design.

Slope Deflection and Moment Distribution Method

We have two distinct method of analysis for statically indeterminate structure depending upon how the above equations are satisfied:

- Force method of analysis
- Displacement method of analysis

In the force method of analysis, primary unknown are forces. In this method compatibility equations are written for displacement and rotations (which are calculated by force displacement equations). Solving these equations, redundant forces are calculated. Once the redundant forces are calculated, the remaining reactions are evaluated by equations of equilibrium.

In the displacement method of analysis, the primary unknowns are the displacements. In this method, first force -displacement relations are computed and subsequently equations are written satisfying the equilibrium conditions of the structure.

After determining the unknown displacements, the other forces are calculated satisfying the compatibility conditions and force displacement relations. The displacement-based method is amenable to computer programming and hence the method is being widely used in the modern day structural analysis.

FORCE METHODS	DISPLACEMENT METHODS
Method of consistent deformation	Slope deflection method
Theorem of least work	Moment distribution method
Column analogy method	Kani's method
Flexibility matrix method	Stiffness matrix method

The slope deflection method is a structural analysis method for beams and frames introduced in 1914 by George A. Maney. The slope deflection method was widely used for more than a decade until the moment distribution method was developed.

The methods of three moment equation, and consistent deformation method are represent the force method of structural analysis, the slope deflection method use displacements as unknowns, hence this method is the displacement method. In this method, if the slopes at the ends and the relative displacement of the ends are known, the end moment can be found in terms of slopes, deflection, stiffness and length of the members. In- the slope-deflection method the rotations of the joints are treated as unknowns. For any one member bounded by two joints the end moments can be expressed in terms of rotations. In this method all joints are considered rigid; i.e. the angle between members at the joints is considered not-to change in value as loads are applied.

Slope deflection method and Moment distribution method are both stiffness methods. That is, node displacements are treated as the unknowns, after solving the stiffness equation for displacements, member forces and reactions are obtained. However, Moment distribution method does not explicitly calculate the node displacements, but obtains member end forces directly.

Moment distribution method is an iterative method. That is, a fixed set of calculations are repeated several times until the answers converge. The user can determine when the solution has converged depending on the level of accuracy required.

Slope deflection method explicitly identifies the node displacements, sets up the linear simultaneous equations which can be solved to determine these displacements and knowing the displacements, member end forces can be determined. It is the classical stiffness method approach.

Assumptions in the slope deflection method

This method is based on the following simplified assumptions.

- All the joints of the frame are rigid, i.e., the angle between the members at the joints does not change, when the members of frame are loaded.
- Distortion, due to axial and shear stresses, being very small, are neglected.

Degree of freedom

The number of joints rotation and independent joint translation in a structure is called the degrees of freedom. The slope defection method is applicable for beams and frames. It is useful for the analysis of highly statically indeterminate structures which have a low degree of kinematical indeterminacy.

Moment Distribution Method

The moment distribution method is a structural analysis method for statically indeterminate beams and frames developed by Hardy Cross. The methodology accounts for flexural effects and ignores axial and shear effects. In the moment distribution method, every joint of the structure to be analyzed is fixed so as to develop the fixed-end moments. Then each fixed joint is sequentially released and the fixed-end moments (which by the time of release are not in equilibrium) are distributed to adjacent members until equilibrium is achieved. The moment distribution method in mathematical terms can be demonstrated as the process of solving a set of simultaneous equations by means of iteration. The moment distribution method falls into the category of displacement method of structural analysis.

In order to apply the moment distribution method to analyze a structure, the following things must be considered.

- **Fixed end moments**: Fixed end moments are the moments produced at member ends by external loads when the joints are fixed.
- **Flexural stiffness**: The flexural stiffness (EI/L) of a member is represented as the product of the modulus of elasticity (E) and the second moment of area (I) divided by the length (L) of the member. What is needed in the moment distribution method is not the exact value but the ratio of flexural stiffness of all members.
- **Distribution factors**: When a joint is released and begins to rotate under the unbalanced moment, resisting forces develop at each member framed together at the joint. Although the total resistance is equal to the unbalanced moment, the magnitudes of resisting forces

developed at each member differ by the members' flexural stiffness. Distribution factors can be defined as the proportions of the unbalanced moments carried by each of the members.

- **Carryover factors:** When a joint is released, balancing moment occurs to counterbalance the unbalanced moment which is initially the same as the fixed-end moment. This balancing moment is then carried over to the member's other end. The ratio of the carried-over moment at the other end to the fixed-end moment of the initial end is the carryover factor.

Rotation Contribution Method or Kani's Method

This method may be considered as a further simplification of moment distribution method wherein the problems involving sway were attempted in a tabular form thrice (for double story frames) and two shear co-efficient had to be determined which when inserted in end moments give the final end moments. All this effort can be cut short very considerably by using this method.-Frame analysis is carried out by solving the slope - deflection equations by successive approximations. Useful in case of side sway as well.

Operation is simple, as it is carried out in a specific direction. If some error is committed, it will be eliminated in subsequent cycles if the restraining moments and distribution factors have been determined correctly. This method does not give realistic results in cases of columns of unequal heights within a storey and for pin ended columns both of these cases are in fact extremely rare even in actual practice. Even codes suggest that RC columns framing into footings or members above may be considered as fixed for analysis and design purposes.

This method was introduced by Gasper Kani in 1940's. It involves distributing the unknown fixed end moments of structural members to adjacent joints, in order to satisfy the conditions of continuity of slopes and displacements.

Advantages of Rotation Contribution Method:

1. Hardy Cross method distributed only the unbalanced moments at joints, whereas Kani's method distributes the total joint moment at any stage of iteration.

2. The more significant feature of Kani's method is that the process is self corrective. Any error at any stage of iteration is corrected in subsequent steps.

Framed structures are rarely symmetric and subjected to side sway, hence Kani's method is best and much simpler than other methods like moment distribution method and slope displacement method.

Flexibility and Stiffness Method

The flexibility of a structure is defined as the displacement caused by a unit force. The stiffness is defined as the force required for producing a unit displacement.

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Plastic Analysis

Plastic analysis is defined as the analysis in which the criterion for the design of structures is the ultimate load. We can define it as the analysis inelastic material is studied beyond the elastic limit (which can be observed in stress strain diagram). Plastic analysis derives from a simple mode failure in which plastic hinges form. Actually the ultimate load is found from the strength of steel in plastic range. This method of analysis is quite rapid and has rational approach for analysis of structure. It controls the economy regarding to weight of steel since the sections required by this method are smaller than those required by the method of elastic analysis. Plastic analysis has its application in the analysis and design of indeterminate structures.

Plastic analysis is usually based on the idealization of stress strain curve as perfectly plastic. In this analysis it is assumed that width thickness ratio of plate elements is small so the local buckling does not occur. Broadly speaking the section will be declared as perfectly plastic. Keeping in mind these assumptions, it can be said that section will reach its plastic moment capacity and after that will be subjected to considerable moment at applied moments.

Principles of Plastic Analysis

There are following conditions for plastic analysis

- Mechanism condition
- Equilibrium condition
- Plastic moment condition

Plastic moment condition

The bending moment at any section in the structure should not be more than the full plastic moment (moment at which plastic hinges form and structure moves to failure) of the section.

Plastic moment

In a simply supported beam, when the load is gradually applied on it, bending moment and stresses increases. As the load is increased, the stresses in fibers of beam reach to yield stress. At this stage the moment which has converted the stresses into the yield stress is said to be as Plastic moment. it is usually denoted by Mp. At this stage the beam member cannot take up any additional moment but may maintain this moment for some amount of rotation and acts like a plastic hinge (hinge means having no capacity to resist moment). Plastic hinge behaves like an ordinary hinge allowing free rotation about itself.

Yield moment

It is defined as the moment at which the entire cross section has reached its yield stress. This is theoretically the maximum bending moment that the section can resist - when this point is reached a plastic hinge is formed and any load beyond this point will result in theoretically infinite plastic deformation.

Assumptions for plastic analysis:

Plane transverse sections remain plane and normal to the longitudinal axis before and after bending.

- Effect of shear is neglected.
- The material is homogeneous and isotropic both in the elastic and plastic state.
- Modulus of elasticity has the same value both in tension and compression.
- There is no resultant axial force in the beam.
- The cross-section of the beam is symmetrical about an axis through its centroid and parallel to the plane of bending.

Plastic hinge

When a section attains full plastic moment Mp, it acts as hinge which is called a plastic hinge. It is defined as the yielded zone due to bending at which large rotation s can occur with a constant value of plastic moment Mp.

Difference between Plastic hinge and mechanical hinge

Plastic hinges modify the behavior of structures in the same way as mechanical hinges. The only difference is that plastic hinges permit rotation with a constant resisting moment equal to the plastic moment Mp. At mechanical hinges, the resisting moment is equal to zero.

Collapse Load

The load that causes the $(n+1)^{th}$ hinge to form a mechanism is called collapse load where n is the degree of statically indeterminacy once the structure becomes a mechanism.

The two theorems for the determination of collapse load are:

(a)Static Method [Lower bound Theorem]

(b)Kinematic Method [Upper bound Theorem]

When an indeterminate structure develops n plastic hinges, it becomes determinate and the formation of an additional hinge will reduce the structure to a mechanism. Once a structure becomes a mechanism, it will collapse.