GPCET
Pioneering Innovative Education

G. PULLAIAH COLLEGE OF ENGINEERING AND TECHNOLOGY
Accredited by NAAC with 'A' Grade of UGC, Approved by AICTE, New Delhi
Permanently Affiliated to JNTUA, Ananthapuramu
(Recognized by UGC under 2(f) and 12(B) & ISO 9001:2008 Certified Institution)
Nandikotkur Road, Venkayapalli, Kurnool – 518452

Department of Mechanical Engineering

Bridge Course
On
Finite Element Method

Finite Element Method

Basics of Heat Transfer

The science of thermodynamics deals with the amount of heat transfer as a system undergoes a process from one equilibrium state to another, and makes no reference to how long the process will take. But in engineering, we are often interested in the rate of heat transfer, which is the topic of the science of heat transfer.

Application Areas of Heat Transfer

Heat transfer is commonly encountered in engineering systems and other aspects of life, and one does not need to go very far to see some application areas of heat transfer. In fact, one does not need to go anywhere. The human body is constantly rejecting heat to its surroundings, and human comfort is closely tied to the rate of this heat rejection. We try to control this heat transfer rate by adjusting our clothing to the environmental conditions. Many ordinary household appliances are designed, in whole or in part, by using the principles of heat transfer. Some examples include the electric or gas range, the heating and air-conditioning system, the refrigerator and freezer, the water heater, the iron, and even the computer, the TV, and the DVD player. Of course, energy-efficient homes are designed on the basis of minimizing heat loss in winter and heat gain in summer. Heat transfer plays a major role in the design of many other devices, such as car radiators, solar collectors, various components of power plants, and even spacecraft (Fig. 1–3). The optimal insulation thickness in the walls and roofs of the houses, on hot water or steam pipes, or on water heaters is again determined on the basis of a heat transfer analysis with economic consideration.


The human body


Air conditioning systems


Airplanes


Car radiators


Power plants


Refrigeration systems

HEAT TRANSFER MECHANISMS

A thermodynamic analysis is concerned with the amount of heat transfer as a system undergoes a process from one equilibrium state to another. The science that deals with the determination of the rates of such energy transfers is the heat transfer. The transfer of energy as heat is always from the higher-temperature medium to the lower-temperature one, and heat transfer stops when the two mediums reach the same temperature.

Heat can be transferred in three different modes: conduction, convection,and radiation. All modes of heat transfer require the existence of a temperature difference, and all modes are from the high-temperature medium to a lower-temperature one.

Conduction is the transfer of energy from the more energetic particles of a substance to the adjacent less energetic ones as a result of interactions between the particles.

- Conduction can take place in solids, liquids, or gases.
- In gases and liquids, conduction is due to the *collisions* and *diffusion* of the molecules during their random motion.
- In solids, it is due to the combination of *vibrations* of the molecules in a lattice and the energy transport by *free electrons.*
- A cold canned drink in a warm room, for example, eventually warms up to the room temperature as a result of heat transfer from the room to the drink through the aluminum can by conduction.

Finally, It is concluded that the rate of heat conduction through a plane layer is proportional to the temperature difference across the layer and the heat transfer area, but is inversely proportional to the thickness of the layer.

Rate of heat conduction α {(Area)(Temperature Difference)/Thickness}

$$Q_{CON} = -KA \{dt/dx\}$$

Convection is the mode of energy transfer between a solid surface and the adjacent liquid or gas that is in motion, and it involves the combined effects of conduction and fluid motion.

- The faster the fluid motion, the greater the convection heat transfer. In the absence of any bulk fluid motion, heat transfer between a solid surface and the adjacent fluid is by pure conduction.
- The presence of bulk motion of the fluid enhances the heat transfer between the solid surface and the fluid, but it also complicates the determination of heat transfer rates.

Despite the complexity of convection, the rate of *convection heat transfer* is observed to be proportional to the temperature difference, and is conveniently expressed by **Newton's law of cooling** as

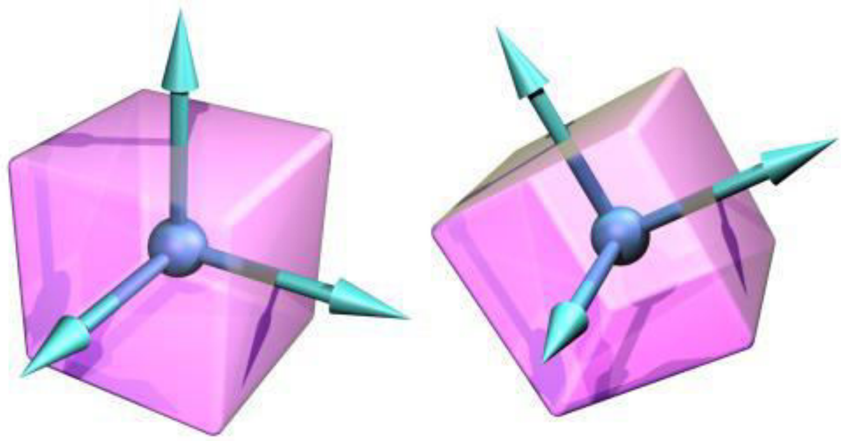
$$Q_{conv} = hA_s\,(T_s - T\infty\}$$

Radiation is the energy emitted by matter in the form of electromagnetic waves (or photons) as a result of the changes in the electronic configurations of the atoms or molecules.

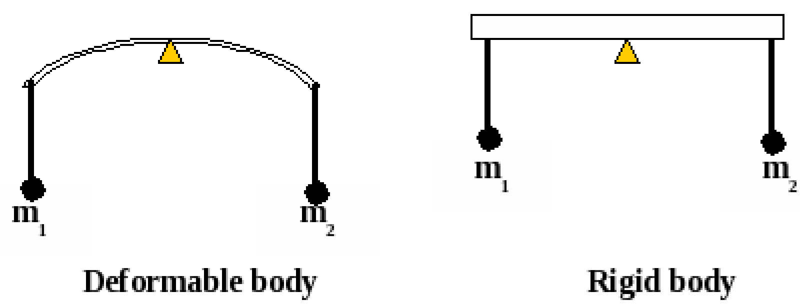
- Unlike conduction and convection, the transfer of heat by radiation does not require the presence of an intervening medium.
- In fact, heat transfer by radiation is fastest (at the speed of light) and it suffers no attenuation in a vacuum. This is how the energy of the sun reaches the earth.
- In heat transfer studies we are interested in thermal radiation, which is the form of radiation emitted by bodies because of their temperature.
- It differs from other forms of electromagnetic radiation such as x-rays, gamma rays, microwaves, radio waves, and television waves that are not related to temperature.
- All bodies at a temperature above absolute zero emit thermal radiation.
- Radiation is a volumetric phenomenon, and all solids, liquids, and gases emit, absorb, or transmit radiation to varying degrees.

BASIC CONCEPTS AND DEFINITIONS of Engineering Mechanics

- The state of rest and state of motion of the bodies under the action of different forces has engaged the attention of philosophers, mathematicians and scientists for many centuries. The branch of physical science that deals with the state of rest or the state of motion is termed as **Mechanics**.
- The bodies which will not deform or the body in which deformation can be neglected in the analysis are called as **Rigid Bodies**.

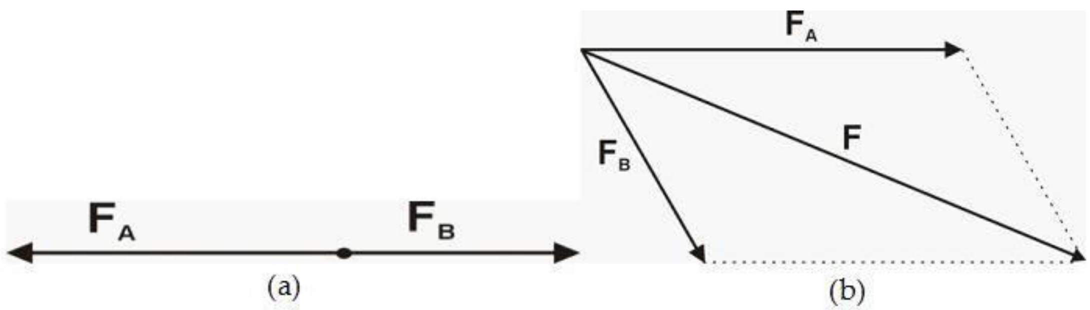


- The mechanics of the rigid bodies dealing with the bodies at rest is termed as **Statics**.
- The mechanics of the rigid bodies dealing with the bodies in motion is termed as **Dynamics.**
- The dynamics dealing with the problems with out referring to the forces causing the motion of the body is termed as **Kinematics.**
- The dynamics dealing with the problems with the forces causing the motion of the body is termed as **Kinetics.**
- If the internal stresses developed in a body are to be studied, the deformation of the body should be considered. This field of mechanics is called **Mechanics of Deformable Bodies.**

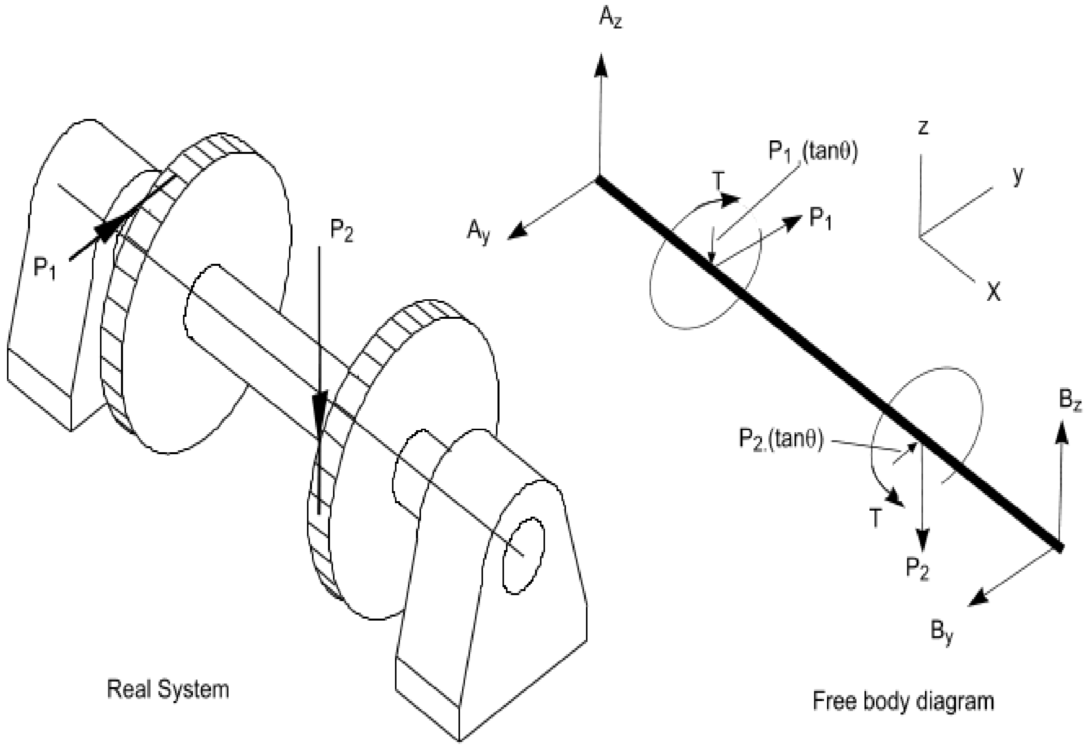


- Body may become weightless when gravitational force vanishes but the mass remains the same.
- **Second** is defined as the duration of 9192631770 period of radiation of the cesium -133 atom.
- **Meter** is defined as 1690763.73 wave length of kyrpton-86 atom.
- The rate of change of displacement with respect to time is defined as **Velocity.**
- The rate of change of velocity with respect to time is called as **Acceleration.**
- The product of mass and velocity is called **Momentum.**
- A particle may be defined as an object which as **Only Mass** and **No Size.**
- **Newton's First Law** states that every body continues in its state of rest or uniform motion in a straight line unless it is compelled by external agency acting on it.

- **Newton's Second Law** states that the rate of change of momentum of a body is directly proportional to the impressed force and it takes place in the direction of force acting on it.
- **Newton's Third Law** states that for every action there is an equal and opposite reaction.
- **Newton's Law of Gravitation** states that the force of attraction between any two bodies is directly proportional to their masses an inversely proportional to the square of the distance between them.
- **Moment of a Force** is defined as the product of the magnitude of the force and the perpendicular distance of the point from the line of action of the force.
- The moment of the force about moment centre is **Zero.**
- Two parallel forces equal in magnitude and opposite in direction and separated by a definite distance are said to form a **Couple**.
- The single force which will have the same effects the system of forces is called as **Resultant Force.**

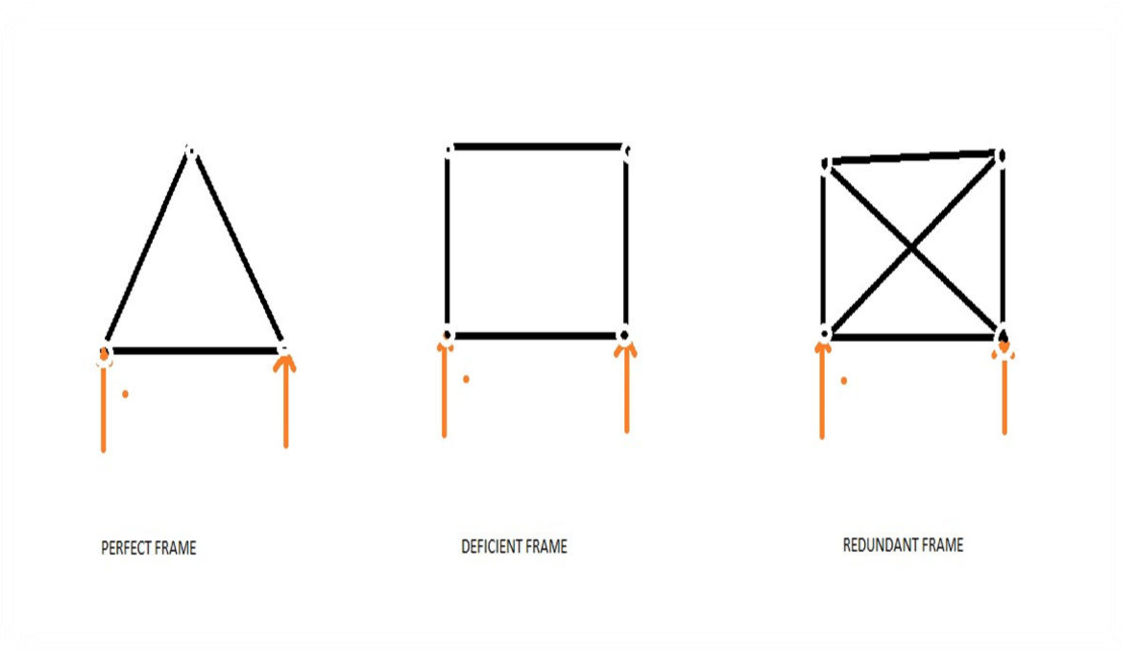


- The diagram in which the body is shown with all the forces acting on it including self weight and the reactions from the adjacent bodies is called as **Free Body Diagram.**



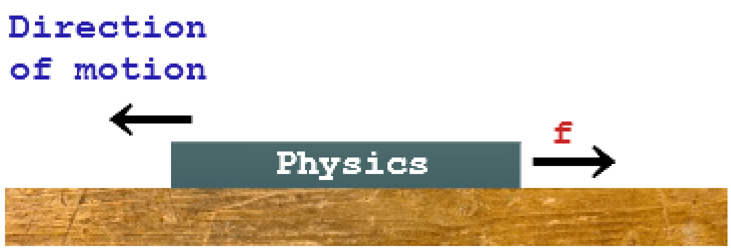
V_z V_y

- **Lamis Theorem** states that if a body is in equilibrium under the action of three forces only then each force is proportional to the sign of the angle between the other two forces.
- Pin Jointed truss which has got just sufficient number of members to resist the loads with out under going appreciable deformation in shape is called **A Perfect Truss.**
    - ➢ A truss is said to be **Deficient** if number of members in it are less than that required for a perfect truss.
    - ➢ A truss is said to be **redundant** if number of members in it are more than that required for a perfect truss.



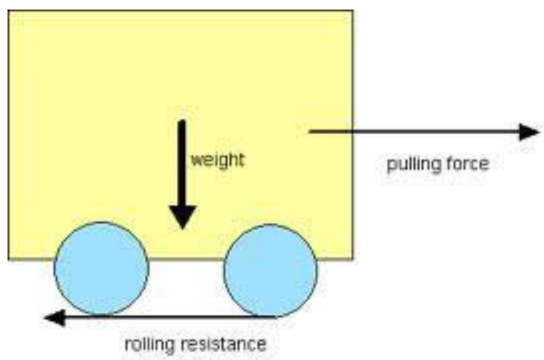
- **Centre of gravity** can be defined as the point through which resultant of force of gravity (weight) of the body acts.
- The term centre of gravity applies to bodies with **Mass and Weight,** and Centroid applies to **plane areas.**
- Centre of gravity of a body is a point through which the resultant gravitational force (Weight) acts for any orientation of the body where as centroid is point in a plane area such that the moment of area about any axis through that point is zero.
- **Centroid** is the point in a plane section such that for any axis through that point moment of area is **Zero.**
- Moment of inertia about an axis Perpendicular to the plane of area is known as **Polar Moment of Inertia.**
- The moment of Inertia of an area about an axis Perpendicular to its plane at any point is equal to the sum of moments of inertia about any two mutually perpendicular axes through the same point and lying in the plane of area is called as **Perpendicular axis theorem.**
- **Parallel Axis Theorem** states that the moment of inertia about any axis in the plane of area is equal to the sum of the moment of inertia about parallel centroidal axis and the product of area and square of the distance between the two parallel axes.
- **Mass moment of inertia** of a body about an axis is defined as the sum total of product of its elemental masses and square of their distances from the axis.
- **Radius of gyration** is that distance which when squared and multiplied with total mass of the body gives the mass moment of inertia of the body.

- A motion is said to be **Translation,** if a straight line drawn on the moving bodies remains parallel to its original position at any time. During translation if the path traced by a point is a straight line it is called **Rectilinear motion** and if the path is curve one it is called **Curve linear motion.**
- **Energy** is defined as capacity to do the work and hence as the same unit as that for the work done.
- **Power** is defined as the time rate of doing the work.
- The opposing force which acts in the opposite direction of the movement of the block is called **Force of Friction** or simply **Friction**.
- **Sliding friction** is the friction, experienced by a body when it slides over another body.



Sliding Friction

- **Rolling Friction** is the friction, experienced by a body when it rolls over another body.



- The maximum value of frictional force which comes into play when, a body just begin to slide over the surface of the other body is known as **Limiting friction.**
- The angle of the inclined plane, at which a body just begins to slide down the plane is called **Angle of Friction.**
- **Vibration** means mechanical oscillations about an equilibrium point. The oscillations may or mayn't be periodic.
- **Free Vibration or Transient Vibration** occurs when a mechanical system is set off with an initial input and then allowed to vibrate freely. When an alternating force or motion is applied to mechanical system **Forced Vibration** or **Steady state vibration** occurs.

- **Damping** is an effect, either deliberately engendered or inherent to a system which tends to reduce the amplitude the oscillations of an oscillatory system. Depending on the presence or absence of damping we have free undamped, free damped, forced undamped and forced damped vibration.
- **FORMULAE**

Centroid of some common figures:

SHAPE	$\bar{X}$	$\bar{Y}$	AREA
TRIANGLE	--	h/3	bh/2
SEMICIRCLE	0	4R/3π	$\pi R^2/2$
QUARTER CIRCLE	4R/3π	4R/3π	$\pi R^2/4$
SECTOR OF THE CIRCLE	2RSinα/3α	0	αR^2
PARABOLA	0	3h/5	4ah/3
SEMI PARABOLA	3a/8	3h/5	2ah/3
PARABOLIC SPANDREL	3a/4	3h/10	ah/3

Centre of Gravity of some common solids:

- Right circular cone lies at a distance 3h/4 from the vertex or h/4 from the base.
- Solid hemisphere at a distance 3R/8 from its diametral axis.

BASICS OF STRENGTH OF MATERIALS

Strength of materials is the science which deals with the relations between externally applied loads and their internal effects on bodies. The bodies are no longer assumed to be rigid and deformations are of major interest. During deformation the external forces acting upon the body do work. This work is transferred completely or partially into potential energy of strain. If the forces which produce the deformation of the body are gradually removed, the body returns or try to return to its original shape. During this return the stored potential energy can be recovered in form of external work. The main concern of the subject is regarding three S's, namely strength, stiffness and stability of various load carrying members.

Stress

When an external force acts on a body, it undergoes deformation. At the same time the body resists deformation. The magnitude of the resisting force is numerically equal to the applied force. This internal resisting force per unit area is called stress.

Stress = Force/Area = P/A unit is N/mm^2

Strain

When a body is subjected to an external force, there is some change of dimension in the body. Numerically the strain is equal to the ratio of change in length to the original length of the body

Strain = Change in length/Original length

$e = \delta L/L$

Hooke's law

It states that when a material is loaded, within its elastic limit, the stress is directly proportional to the strain.

Stress α Strain

$\sigma \propto e$

$\sigma = Ee$

$E = \sigma/e$ unit is N/mm^2

Where,

E - Young's modulus

σ- Stress

e - Strain

Shear stress and shear strain

The two equal and opposite force act tangentially on any cross sectional plane of the body tending to slide one part of the body over the other part. The stress induced is called shear stress and the corresponding strain is known as shear strain. When a body is stressed, within its elastic limit, the ratio of lateral strain to the longitudinal strain is constant for a given material.

Poisson' ratio (μ or 1/m) = Lateral strain /Longitudinal strain

Relationship between Young’s Modulus and Modulus of Rigidity

$E = 2G (1+\{1/m\})$

Where,

E - Young’s Modulus

K - Bulk Modulus

1/m - Poisson’s ratio

Whenever a body is strained, some amount of energy is absorbed in the body. The energy that is absorbed in the body due to straining effect is known as strain energy.

Resilience

The total strain energy stored in the body is generally known as resilience.

Proof resilience

The maximum strain energy that can be stored in a material within elastic limit is known as proof resilience.

Modulus of resilience

It is the proof resilience of the material per unit volume.

Relationship between Bulk Modulus and Young’s Modulus

$E = 3K (1-\{2/m\})$

Where,

E - Young’s Modulus

K - Bulk Modulus

1/m - Poisson’s ratio

Compound bar

A composite bar composed of two or more different materials joined together such that system is elongated or compressed in a single unit.

Thermal stresses

If the body is allowed to expand or contract freely, with the rise or fall of temperature no stress is developed but if free expansion is prevented the stress developed is called temperature stress or strain.

Elastic limit

Some external force is acting on the body, the body tends to deformation. If the force is released from the body its regain to the original position. This is called elastic limit.

Young's modulus

The ratio of stress and strain is constant within the elastic limit.

Bulk-modulus

The ratio of direct stress to volumetric strain.

Lateral strain

When a body is subjected to axial load P. The length of the body is increased. The axial deformation of the length of the body is called lateral strain.

Longitudinal strain

The strain right angle to the direction of the applied load is called lateral strain.

Rigidity modulus

The shear stress is directly proportional to shear strain.

Beam

Structural member which is acted upon a system of external loads at right angles to its axis is known as beam.

Types of beams

1. Cantilever beam
2. Simply supported beam
3. Fixed beam
4. Continuous beam

Types of loads

1. Concentrated load or point load
2. Uniform distributed load
3. Uniform varying load

Shear force and bending moment

SF at any cross section is defined as algebraic sum of all the forces acting either side of beam. BM at any cross section is defined as algebraic sum of the moments of all the forces which are placed either side from that point.

Sagging BM

BM is said to positive if moment on left side of beam is clockwise or right side of the beam is counter clockwise.

Hogging BM

BM is said to negative if moment on left side of beam is counterclockwise or right side of the beam is clockwise.

Define point of contra flexure? In which beam it occurs?

Point at which BM changes to zero is point of contra flexure. It occurs in overhanging beam.

Assumptions in the theory of simple bending

1. The material of the beam is homogeneous and isotropic.

2. The beam material is stressed within the elastic limit and thus obey hooke's law.

3. The transverse section which was plane before bending remains plains after bending also.

4. Each layer of the beam is free to expand or contract independently about the layer, above or below.

5. The value of E is the same in both compression and tension.

Theory of simple bending Equation

$M/I = F/Y = E/R$

M - Maximum bending moment

I - Moment of inertia

F - Maximum stress induced

Y - Distance from the neutral axis

E - Young's modulus

R - Constant.

Shear stress distribution

The variation of shear stress along the depth of the beam is called shear stress distribution.

Section Modulus

It can be defined as the ratio of moment of inertia of a section about the neutral axis to the distance of the outermost layer from the neutral axis. It is denoted by Z.

$Z = I / Y\max$

Since, M/I = σ/y

⇒ $Z = I / Y\max$

$M / I = \sigma\max/Y\max \Rightarrow M = \sigma\max/Z$

Moment Area method

1. Draw Bending moment Diagram.

2. The total area of the BM diagram will give the slope at free end.

3. To find the slope at the other point in the beam. Find the area of the BM diagram from the support to that point, that area would give the slope at that point. A/EI.

4. The total area of the BM diagram multiplied by centroid from free end will give the deflection at the free end. Ax/EI x is centroid from point of deflection to be found.

5. To find the deflection at the other point in the beam. Find the area of the BM diagram from the support to that point multiplied by centroid from that point. That would give the deflection at that point.

Conjugate method for Cantilever beam

It is a modification of Moment Area Method. It is effective where the inertia of section is different along the length of the beam. Conjugate method for cantilever is almost same as moment area method of cantilever.

1. Draw Bending moment Diagram of the given load.

2. The total area of the BM diagram will give the slope at free end.

3. The sum of the area of the BM diagram at varying inertia from a point to the support would give the slope at that point.

4. The sum of moment of the BM diagram at varying section taken from a point to the support would give the deflection at that point

5. To find the deflection at the other point in the beam. Find the area of the BM diagram from the

support to that point multiplied by centroid from that point. That would give the deflection at that point.

Conjugate method simply supported beam

1. Find the reaction of the given load and draw Bending moment Diagram.
2. Find the reaction of the support assuming the bending moment diagram as the load for varying inertia. This beam is known as conjugate beam.
3. The reaction at the supports will give the slope at the supports.
4. The upward load minus downward load of the conjugate beam will give slope at a point.
5. The moment taken at a point from the conjugate beam will give the deflection