G. PULLAIAH COLLEGE OF ENGINEERING AND TECHNOLOGY

Accredited by NAAC with 'A' Grade of UGC, Approved by AICTE, New Delhi

Permanently Affiliated to JNTUA, Ananthapuramu

(Recognized by UGC under 2(f) and 12(B) & ISO 9001:2008 Certified Institution)

Nandikotkur Road, Venkayapalli, Kurnool – 518452

Department of Electronics and Communication Engineering

Bridge Course
On
SIGNALS AND SYSTEMS

By

Dr. N.Ramamurthy

Analogy between Vectors and Signals

Consider two vectors V_1 and V_2. If V_1 is to be represented in terms of V_2as

$$V_1 = C_{12}V_2 + V_e \quad .... \ (1)$$

Where V_e is the error vector. This error is minimum when V_1 is projected perpendicularly onto V_2. In this case, C_{12} is computed using dot product between V_1 and V_2.

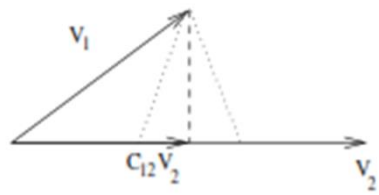

Figure (1): Representation of V_1 in terms of V_2

The error is minimum when V_1 is projected perpendicularly onto V_2. In this case, C_{12} is computed using dot product between V_1 and V_2.

Component of V_1 along V_2 is $\dfrac{V_1.V_2}{\|V_1\|}$ .....(2)

Similarly, the component of V_2 along V_1 is $\dfrac{V_2.V_1}{\|V_2\|}$ .....(3)

Using the above discussion, analogy can be drawn to signal spaces also.

Let f_1 (t) and f_2 (t) be two real signals. Approximation of f_1 (t) by f_2 (t) over a time interval $t_1 < t < t_2$ can be given by

$f_e(t) = f_1(t) - C_{12}f_2(t)$ .....(4) where $f_e(t)$ is the error function.

The goal is to find C_{12} such that $f_e(t)$ is minimum over the interval considered. The energy of the error signal ε given by

$$\varepsilon = \frac{1}{t_2-t_1}\int_{t_1}^{t_2}[\, f_1(t) - C_{12}f_2(t)]^2 dt \quad .....(5)$$

To find C_{12},

$$\frac{\partial \varepsilon}{\partial C_{12}} = 0 \quad .....(6)$$

Solving the above equation we get

$$C_{12} = \frac{\frac{1}{t_2-t_1}\int_{t_1}^{t_2} f_1(t)f_2(t)dt}{\frac{1}{t_2-t_1}\int_{t_1}^{t_2} f_2^2(t)dt} \quad .....(7)$$

Trigonometric, Polar and Exponential Fourier series:

Representation of a function over a certain interval by a linear combination of mutually orthogonal functions is called Fourier series representation.

1. The Trigonometric Fourier series representation of a function f(t) over an interval $(t_0 < t < t_o + \frac{2\pi}{\omega_0})$ is given by

$$f(t) = a_0 + \sum_{n=1}^{\infty}[a_n \operatorname{Cos}(n\omega_0 t) + b_n \sin(n\omega_o t)] \quad ; \quad t_0<t<t_0+\frac{2\pi}{\omega_0}$$

where

$$a_n = \frac{\int_{t_0}^{t_0+T} f(t)\operatorname{Cos}(n\omega_0 t)dt}{\int_{t_0}^{t_0+T} \operatorname{Cos}^2(n\omega_0 t)dt}$$

$$b_n = \frac{\int_{t_0}^{t_0+T} f(t)\operatorname{Sin}(n\omega_0 t)dt}{\int_{t_0}^{t_0+T} \operatorname{Sin}^2(n\omega_0 t)dt}$$

$$a_0 = \frac{1}{T}\int_{t_0}^{t_0+T} f(t)dt$$

2. The Polar series representation of a function f(t) is

$$f(t) = a_0 + \sum_{n=1}^{\infty}[A_n \operatorname{Cos}(n\omega_0 t + \emptyset_n)]$$

Where $A_n = \sqrt{a_n^2 + b_n^2}\emptyset_n = -\operatorname{Tan}^{-1}(\frac{b_n}{a_n})$

3. The Exponential Fourier series representation of a function f(t) is given by

$$f(t) = \sum_{n=-\infty}^{\infty} F_n e^{jn\omega_0 t} \quad ; \quad t_0<t<t_0+\frac{2\pi}{\omega_0}$$

where $T=\frac{2\pi}{\omega_0}$

$$F_n = \frac{1}{T}\int_{t_0}^{t_0+T} f(t)e^{-jn\omega_0 t}dt$$

and $\quad F_0 = \frac{1}{T}\int_{t_0}^{t_0+T} f(t)dt$

4. The relation between Trigonometric and Exponential Fourier series is given by

$$a_0 = F_0, \quad F_n = \frac{1}{2}(a_n - jb_n) \quad \text{and}$$

$$F_{-n} = \frac{1}{2}(a_n + jb_n)$$

Continuous Fourier Transform (CTFT)

The Fourier transform of a continuous time signal x(t) is given by

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}\, dt$$

The inverse Fourier transform of $X(j\omega)$ is

$$x(t) = \int_{-\infty}^{\infty} X(j\omega)e^{j\omega t}\, d\omega$$

The above two equations are known as the Fourier transform pair.

Let us consider a linear system given by

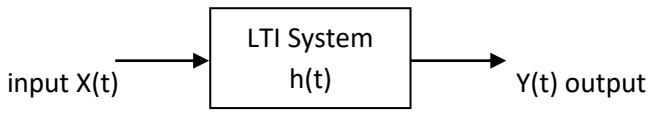

Figure: Linear Continuous –time system

where

output y(t)= h(t)*x(t) =$\int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$

h(t) is the impulse response of the system

Taking Fourier transform on both sides, we get

Y(jω)= H(jω)X(jω) Therefore y(t)= $CTFT^{-1}$[Y(jω)]

Where

Y(jω) is the CTFT of y(t)

H(jω) is the CTFT of h(t)

X(jω) is the CTFT of x(t)

Dirichlet's Conditions:

Any function x(t) can be represented by using the Fourier transform, only when the function satisfies the following Dirichlet's conditions

1. The function x(t) has finite number of minima and maxima
2. There must be finite number of discontinuities in the function in the given integral
3. The function must be absolutely integrable in the given interval of time.

Discrete Time Fourier Transform (DTFT)

The discrete time Fourier transform of a sequence x[n] is given by

$$X(e^{j\omega})=\sum_{n=-\infty}^{\infty} X(e^{j\omega})e^{j\omega n}$$

and its inverse is

$$x(n)=\frac{1}{2\pi}\int_{2\pi} X(e^{j\omega})e^{jwn}d\omega$$

Let us consider a linear discrete-time system given by

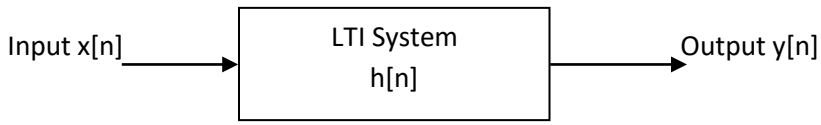

Figure: Linear Discrete-time System

The output y[n] =x[n]*h[n]

$$=\sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

Taking DTFT on both sides

$$Y(e^{j\omega}) = X(e^{j\omega})H(e^{j\omega})$$

Therefore

$$y[n]= DTFT^{-1}[(Y(e^{j\omega})]$$

Where $Y(e^{j\omega})$ is the DTFT of y(n)

$X(e^{j\omega})$ is the DTFT of x(n)

$H(e^{j\omega})$ is the DTFT of h(n)

Laplace Transform and Z-Transform

The Laplace transform of a function f(t) is given by $F(s) = \int_{-\infty}^{\infty} f(t)e^{-st}\,dt$

The Laplace transform of f(t) can be interpreted as the continuous time Fourier transform of $f(t)e^{-\sigma t}$

The Z-transform of a sequence x[n] is given by

$$X(z) = \sum_{n=-\infty}^{\infty} x[n]z^{-n}$$

The Z-transform can be interpreted as the discrete time Fourier transform of $x[n]r^n$.

GPCET
Pioneering Innovative Education

Important Formulas

1. The energy of a continuous time signal x(t) is $E = \int_{-\infty}^{\infty} | x(t) |^2 \, dt$

2. The average power of a continuous time signal x(t) is P $=\lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} | x(t) |^2 \, dt$; where 'T' is the fundamental time period of x(t).

3. The energy of a discrete time signal x[n] is $E = \sum_{-\infty}^{\infty} | x[n] |^2$

4. The average power of a discrete time signal x[n] is $= \lim_{N\to\infty} \frac{1}{2N+1} \sum_{-\infty}^{\infty} | x[n] |^2$. Where 'N' is the fundamental period of x[n].

5. The even and odd parts of a continuous time signal x(t) are

   $X_e(t) = \frac{1}{2}[(x(t) + x(-t)]$ and $X_o(t) = \frac{1}{2}[(x(t) - x(-t)]$

6. The even and odd parts of a discrete time signal x[n] are

   $X_e[n] = \frac{1}{2}[(x[n] + x[-n]]andX_o[n] = \frac{1}{2}[(x[n] - x[-n]]$

7. A continuous time system is said to be time invariant if its output $y(t)$ satisfies the following condition with input x(t)

   $y(t, T) = y(t - T)$; where $y(t, T) = T[x(t - T)]$= $y(t)/_{x(t)=x(t-T)}$ and

   $$y(t - T) = y(t)/_{t=(t-T)}$$

8. A discrete time system is said to be time invariant if its output $y[n]$ satisfies the following condition with input x[n].

   $y[n, k] = y[n - k)$; where$y[n, k] = T[x[(n - k)]$= $y[n]/_{x[n]=x[(n-k)]}$ and

   $y[n - k] = y[n]/_{n=[n-k]}$

9. A continuous time system with impulse response h(t) is said to be BIBO stable, if

   $$\int_{-\infty}^{\infty} | h(t) | \, dt < \infty$$