# UNIT – II

# (DESIGN OF MECHANICAL SPRINGS & DESIGN OF POWER SCREWS)

**DEFINITION FOR SPRING:**

Springs are elastic bodies (generally metal) that can be twisted, pulled, or stretched by some force. They can return to their original shape when the force is released. In other words it is also termed as a resilient member.

**CLASSIFICATION OF SPRINGS:**

Based on the shape behavior obtained by some applied force, springs are classified into the following ways:

**I. HELICAL SPRINGS:**

**DEFINITON:**

It is made of wire coiled in the form of helix.

**CROSS-SECTION:**

Circular, square or rectangular

**CLASSIFICATION:**

1) Open coil springs (or) Compression helical springs

2) Closed coil springs (or) Tension helical springs

**1) HELICAL TENSION SPRINGS:**

**CHARACTERISTICS:**

- Figure1 shows a helical tension spring. It has some means of transferring the load from the support to the body by means of some arrangement.
- It stretches apart to create load.
- The gap between the successive coils is small.
- The wire is coiled in a sequence that the turn is at right angles to the axis of the spring.
- The spring is loaded along the axis.
- By applying load the spring elongates in action as it mainly depends upon the end hooks as shown in figure2.

**FIGURE1.TENSION HELICAL SPRING**

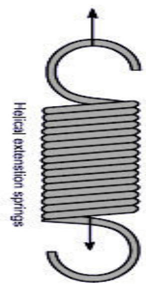
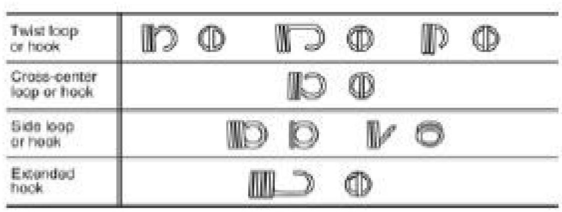



**FIGURE2.TYPES OF END HOOKS OF A HELICAL EXTENSION SPRING**




**APPLICATIONS:**

1) Garage door assemblies
2) Vise-grip pilers
3) carburetors

**2) HELICAL COMPRESSION SPRINGS:**

**CHARACTERISTICS:**

- The gap between the successive coils is larger.
- It is made of round wire and wrapped in cylindrical shape with a constant pitch between the coils.
- By applying the load the spring contracts in action.

- There are mainly four forms of compression springs as shown in figure3.. They are as follows:

  1) Plain end

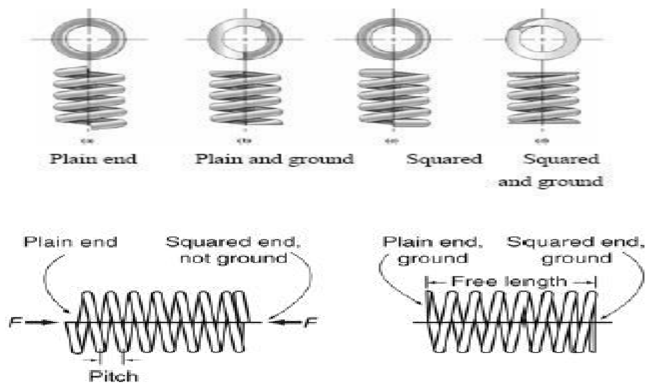
  2) Plain and ground end

  3) Squared end

  4) Squared and ground end

  Among the four types, the plain end type is less expensive to manufacture. It tends to bow sideways when applying a compressive load.

**FIGURE3.COMPRESSION HELICAL SPRING**




**APPLICATIONS:**

1) Ball point pens
2) Pogo sticks
3) Valve assemblies in engines

**3) TORSION SPRINGS:**

**CHARACTERISTICS:**

- It is also a form of helical spring, but it rotates about an axis to create load.
- It releases the load in an arc around the axis as shown in figure4.
- Mainly used for torque transmission
- The ends of the spring are attached to other application objects, so that if the object rotates around the center of the spring, it tends to push the spring to retrieve its normal position.

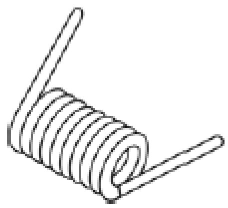


**FIGURE4.TORSION SPRING**

**APPLICATIONS:**

- Mouse tracks
- Rocker switches
- Door hinges
- Clipboards
- Automobile starters

**4) SPIRAL SPRINGS:**

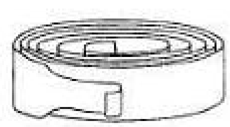
**CHARACTERISTICS:**

- It is made of a band of steel wrapped around itself a number of times to create a geometric shape as shown in figure5.
- Its inner end is attached to an arbor and outer end is attached to a retaining drum.
- It has a few rotations and also contains a thicker band of steel.
- It releases power when it unwinds.

**APPLICATIONS:**

- Alarm timepiece
- Watch
- Automotive seat recliners

**FIGURE5. SPIRAL SPRING**

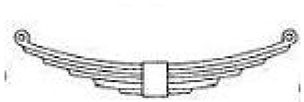


**II. LEAF SPRING:**

**DEFINITION:**

A Leaf spring is a simple form of spring commonly used in the suspension vehicles.

**FIGURE6.LEAF SPRING**



**CHARACTERISTICS:**

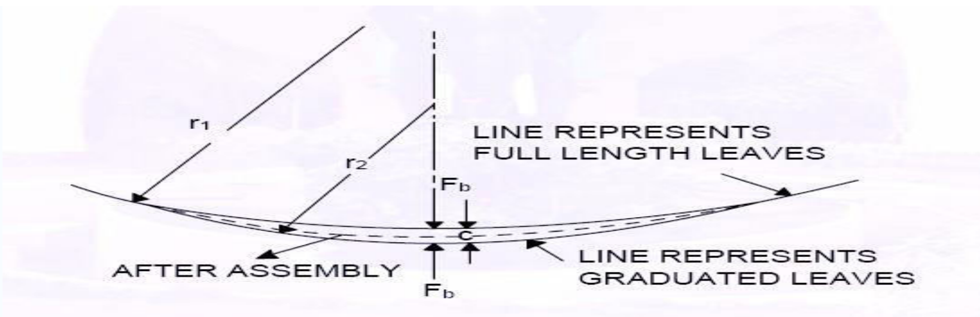
- Figure6 shows a leaf spring.Sometimes it is also called as a semi-elliptical spring, as it takes the form of a slender arc shaped length of spring steel of rectangular cross section.
- The center of the arc provides the location for the axle,while the tie holes are provided at either end for attaching to the vehicle body.
- Heavy vehicles,leaves are stacked one upon the other to ensure rigidity and strenth.
- It provides dampness and springing function.
- It can be attached directly to the frame at the both ends or attached directly to one end,usually at the front,with the other end attched through a shackle,a short swinging arm.
- The shackle takes up the tendency of the leaf spring to elongate when it gets compressed and by which the spring becomes softer.
- Thus depending upon the load bearing capacity of the vehicle the leaf spring is designed with graduated and Ungraduated leaves as shown in figure7.

**FIGURE7.LEAF SPRINGS-FABRICATION STAGES**

- Because of the difference in the leaf length,different stress will be there at each leaf.To compensate the stress level,prestressing is to be done.Prestressing is achieved by bending the leaves to different radius of curvature before they are assembled with the center clip.

- The radius of curvature decreases with shorter leaves.
- The extra intail gap found between the extra full length leaf and graduated length leaf is called as nip.Such prestressing achieved by a difference in the radius of curvature is known as nipping which is shown in figure8.

**FIGURE8.NIPPING IN LEAF SPRINGS**




**APPLICATIONS:**

Mainly in automobiles suspension systems.

**ADVANTAGES:**

- It can carry lateral loads.
- It provides braking torque.
- It takes driving torque and withstand the shocks provided by the vehicles.

**SPRING MATERIALS:**

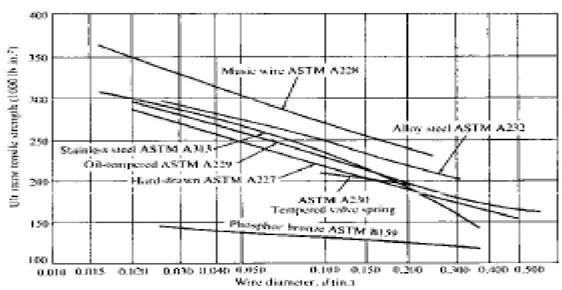
The mainly used material for manufacturing the springs are as follows:

1. Hard drawn high carbon steel.
2. Oil tempered high carbon steel.

3. Stainless steel
4. Copper or nickel based alloys.
5. Phosphor bronze.
6. Iconel.
7. Monel
8. Titanium.
9. Chrome vanadium.
10. Chrome silicon.

Depending upon the strength of the material,The material is Selected for the design of the spring as shown in figure9.

**Min tensile strength of Spring materials**



| NAME OF MATERIAL | SIMILAR SPECIFICATION | DESCRIPTION |
|---|---|---|
| Music wire, | UNS G10850<br>AISI 1085<br>ASTM A228-51 | This is the best, toughest, and most widely used of all spring materials for small springs. It has the highest tensile strength and can withstand higher stresses under repeated loading than any other spring material. Available in diameters 0.12 to 3mm( 0.005 to 0.125 in). Do not use above 120 C (250 F) or at subzero temperature |
| Oil-tempered wire, 0.60-0.70C | UNS G10650<br>AISI 1065<br>ASTM 229-41 | This general-purpose spring steel is used for many types of coil springs where the cost of music wire is prohibitive and in sizes larger thena available in music wire. Not for shock or impact loading. Available in diameters 3 to 12 mm (0.125 to 0.5000 in),but larger and smaller sizes may be obtained. Not for use above 180 C (350 F) or at sub-zero temperatures |
| Hard-drawn wire, 0.60-0.70 | UNS G10660<br>AISI 1066<br>ASTM 227-47 | This is the cheapest general purpose spring steel and should be used only where life, accuracy, and deflection are not too important. Available in diametes 0.8 to 12 mm (0.031 to 0.500 in). Not for use above 120 C (250 F) or at subzero temperatures |
| Chrome Vanadium | UNS G61500<br>AISI 6150<br>ASTM 231-41 | This is the most popular allowy spring steel for conditions involving higher stresses than cn be used with the high- carbon steels and for use wherefatigue resistanceand long endureance are needed. Aslo good for shock and impact loads. Widely used iforaircraft engine valve springs and for temperaturesto 220 C (425 F) Available in annealed or pretemperedsizes 0.r to 12mm(0.031 to 0.500 in ) in diameter |
| Chrome silicon | UNS G92540<br>AISI 9254 | This alloy is an excellent material for highly stressed springs that requrire long lfe and are subjected to shock loading. Rockwell hardnesses of C50 to C53 are quite common, and the material may be used up to 250 C(475 F). Available form 0.8 to 12 mim (0.031 to 0.500 in) in diameter |

**FIGURE9:MATERIAL SELECTION FOR DESIGN OF SPRING**

**NOMENCLATURE OF SPRING:**

Figure10 and Figure11 shows the momenclature of the spring under loading conditions.

**Active Coils**

Those coils which are free to deflect under load.

**Angular relationship of ends**

The relative position of the plane of the hooks or loops of extension spring to each other.

**Buckling**

Bowing or lateral deflection of compression springs when compressed, related to the slenderness ration (L/D).

**Closed ends**

Ends of compression springs where the pitch of the end coils is reduced so that the end coils touch.

**Closed and ground ends**

As with closed ends, except that the end is ground to provide a flat plane.

**Close-wound**

Coiled with adjacent coils touching.

**Deflection**

Motion of the spring ends or arms under the application or removal of an external load.

**Elastic limit**

Maximum stress to which a material may be subjected without permanent set.

**Endurance limit**

Maximum stress at which any given material may operate indefinitely without failure for a given minimum stress.

**Free angle**

Angle between the arms of a torsion spring when the spring is not loaded.

**Free length**

The overall length of a spring in the unloaded position.

**Frequency (natural)**

The lowest inherent rate of free vibration of a spring itself (usually in cycles per second) with ends restrained.

**Hysteresis**

The mechanical energy loss that always occurs under cyclical loading and unloading of a spring, proportional to the are between the loading and unloading load-deflection curves within the elastic range of a spring.

**Initial tension**

The force that tends to keep the coils of an extension spring closed and which must be overcome before the coil starts to open.

**Loops**

Coil-like wire shapes at the ends of extension springs that provide for attachment and force application.

**Mean coil diameter**

Outside wire diameter minus one wire diameter.

**Modulus in shear or torsion**

Coefficient of stiffness for extension and compression springs.

**Modulus in tension or bending**

Coefficient of stiffness used for torsion and flat springs. (Young's modulus).

**Open ends, not ground**

End of a compression spring with a constant pitch for each coil.

**Open ends ground**

"Opens ends, not ground" followed by an end grinding operation.

**Permanent set**

A material that is deflected so far that its elastic properties have been exceeded and it does not return to its original condition upon release of load is said to have taken a "permanent set".

**Pitch**

The distance from center to center of the wire in adjacent active coils.

**Spring Rate (or) Stiffness (or) spring constant**

Changes in load per unit of deflection, generally given in Kilo Newton per meter. (KN/m).

**Remove set**

The process of closing to a solid height a compression spring which has been coiled longer than the desired finished length, so as to increase the elastic limit.

**Set**

Permanent distortion which occurs when a spring is stressed beyond the elastic limit of the material.

**Slenderness ratio**

Ratio of spring length to mean coil diameter.

**Solid height**

Length of a compression spring when under sufficient load to bring all coils into contact with adjacent coils.

**Spring index** Ratio of mean coil diameter to wire diameter.

**Stress range**

The difference in operating stresses at minimum and maximum loads.

**Squareness of ends**

Angular deviation between the axis o a compression spring and a normal to the plane of the other ends.

**Squareness under load** As in *squareness of ends*, except with the spring under load.

**Torque**

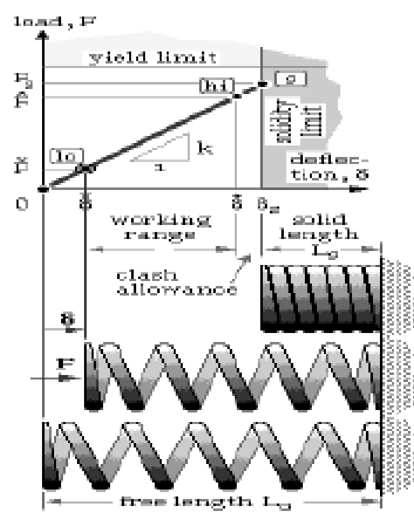
A twisting action in torsion springs which tends to produce rotation, equal to the load multiplied by the distance (or moment arm) from the load to the axis of the spring body. Usually expressed in inch-oz, inch-pounds or in foot-pounds.

**Total number of coils** Number of active coils plus the coils forming the ends.

**Spring index**: The ratio between Mean diameter of coil to the diameter of the wire.

**Solid length**: It is the product of total number of coils and the diameter of the wire when the spring is in the compressed state. It is otherwise called as Solid height also.

Figure10.Nomenclature of Spring



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## 23.5 Terms used in Compression Springs

The following terms used in connection with compression springs are important from the subject point of view.

**1. *Solid length.*** When the compression spring is compressed until the coils come in contact with each other, then the spring is said to be ***solid***. The solid length of a spring is the product of total number of coils and the diameter of the wire. Mathematically,

Solid length of the spring,

$$L_S = n'.d$$

where $n'$ = Total number of coils, and

$d$ = Diameter of the wire.

**2. *Free length.*** The free length of a compression spring, as shown in Fig. 23.6, is the length of the spring in the free or unloaded condition. It is equal to the solid length plus the maximum deflection or compression of the spring and the clearance between the adjacent coils (when fully compressed). Mathematically,

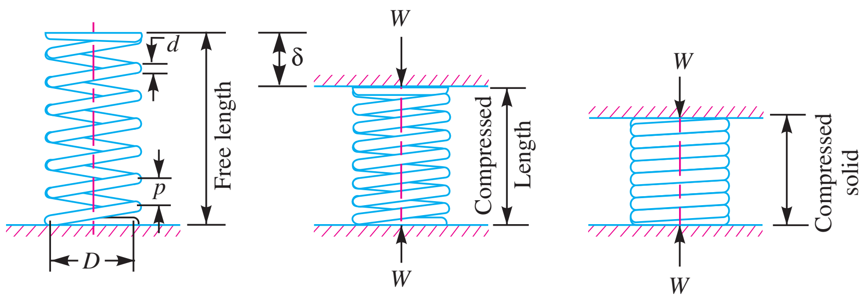



**Fig. 23.6.** Compression spring nomenclature.

Free length of the spring,

$L_F$ = Solid length + Maximum compression + *Clearance between adjacent coils (or clash allowance)

$$= n'.d + \delta_{max} + 0.15\, \delta_{max}$$

The following relation may also be used to find the free length of the spring, *i.e.*

$$L_F = n'.d + \delta_{max} + (n' - 1) \times 1 \text{ mm}$$

In this expression, the clearance between the two adjacent coils is taken as 1 mm.

**3. *Spring index.*** The spring index is defined as the ratio of the mean diameter of the coil to the diameter of the wire. Mathematically,

Spring index, $C = D / d$

where $D$ = Mean diameter of the coil, and

$d$ = Diameter of the wire.

**4. *Spring rate.*** The spring rate (or stiffness or spring constant) is defined as the load required per unit deflection of the spring. Mathematically,

Spring rate, $k = W / \delta$

where $W$ = Load, and

$\delta$ = Deflection of the spring.

* In actual practice, the compression springs are seldom designed to close up under the maximum working load and for this purpose a clearance (or clash allowance) is provided between the adjacent coils to prevent closing of the coils during service. It may be taken as 15 per cent of the maximum deflection.

Top

**5. *Pitch*.** The pitch of the coil is defined as the axial distance between adjacent coils in uncompressed state. Mathematically,

Pitch of the coil, $$p = \frac{\text{Free length}}{n' - 1}$$

The pitch of the coil may also be obtained by using the following relation, *i.e.*

Pitch of the coil, $$p = \frac{L_F - L_S}{n'} + d$$

where $L_F$ = Free length of the spring,

$L_S$ = Solid length of the spring,

$n'$ = Total number of coils, and

$d$ = Diameter of the wire.

In choosing the pitch of the coils, the following points should be noted :

(*a*) The pitch of the coils should be such that if the spring is accidently or carelessly compressed, the stress does not increase the yield point stress in torsion.

(*b*) The spring should not close up before the maximum service load is reached.

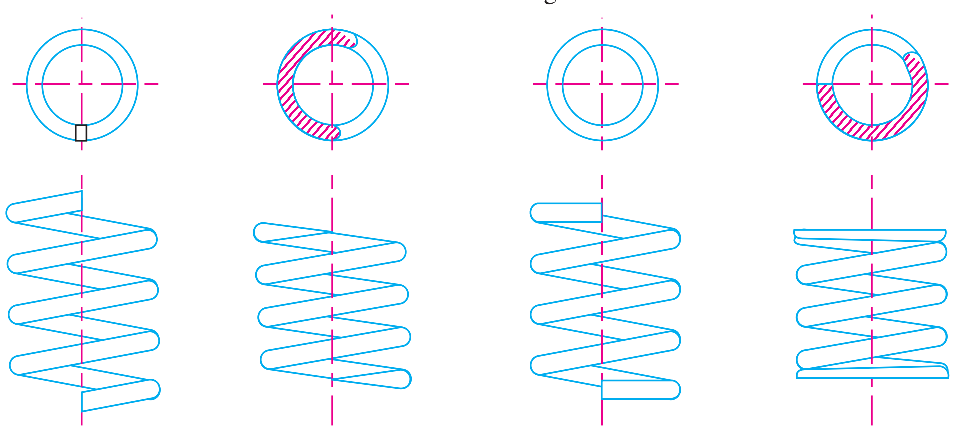
**Note :** In designing a tension spring (See Example 23.8), the minimum gap between two coils when the spring is in the free state is taken as 1 mm. Thus the free length of the spring,

$$L_F = n.d + (n - 1)$$

and pitch of the coil, $$p = \frac{L_F}{n - 1}$$

## 23.6 End Connections for Compression Helical Springs

The end connections for compression helical springs are suitably formed in order to apply the load. Various forms of end connections are shown in Fig. 23.7.



(*a*) Plain ends. (*b*) Ground ends. (*c*) Squared ends. (*d*) Squared and ground ends.

**Fig 23.7.** End connections for compression helical spring.

In all springs, the end coils produce an eccentric application of the load, increasing the stress on one side of the spring. Under certain conditions, especially where the number of coils is small, this effect must be taken into account. The nearest approach to an axial load is secured by squared and ground ends, where the end turns are squared and then ground perpendicular to the helix axis. It may be noted that part of the coil which is in contact with the seat does not contribute to spring action and hence are termed as ***inactive coils***. The turns which impart spring action are known as ***active turns***. As the load increases, the number of inactive coils also increases due to seating of the end coils and the amount of increase varies from 0.5 to 1 turn at the usual working loads. The following table shows the total number of turns, solid length and free length for different types of end connections.

**Table 23.3. Total number of turns, solid length and free length for different types of end connections.**

| *Type of end* | *Total number of turns (n')* | *Solid length* | *Free length* |
|---|---|---|---|
| 1. Plain ends | $n$ | $(n + 1)\ d$ | $p \times n + d$ |
| 2. Ground ends | $n$ | $n \times d$ | $p \times n$ |
| 3. Squared ends | $n + 2$ | $(n + 3)\ d$ | $p \times n + 3d$ |
| 4. Squared and ground ends | $n + 2$ | $(n + 2)\ d$ | $p \times n + 2d$ |

where $n$ = Number of active turns,

$p$ = Pitch of the coils, and

$d$ = Diameter of the spring wire.

*Tension helical spring*

## 23.7 End Connections for Tension Helical Springs

The tensile springs are provided with hooks or loops as shown in Fig. 23.8. These loops may be made by turning whole coil or half of the coil. In a tension spring, large stress concentration is produced at the loop or other attaching device of tension spring.

The main disadvantage of tension spring is the failure of the spring when the wire breaks. A compression spring used for carrying a tensile load is shown in Fig. 23.9.

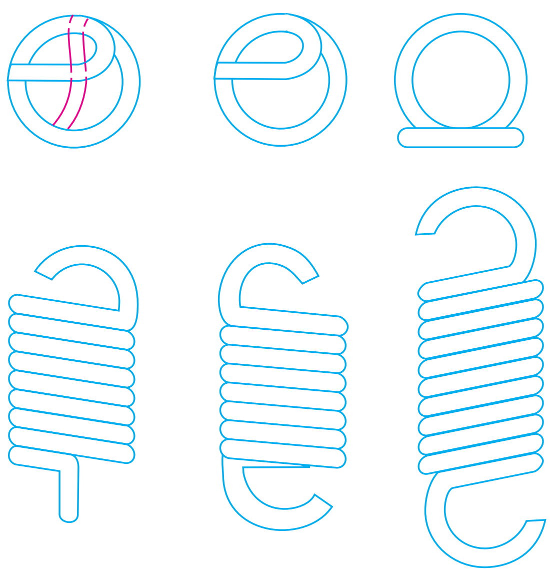


**Fig. 23.8.** End connection for tension helical springs.

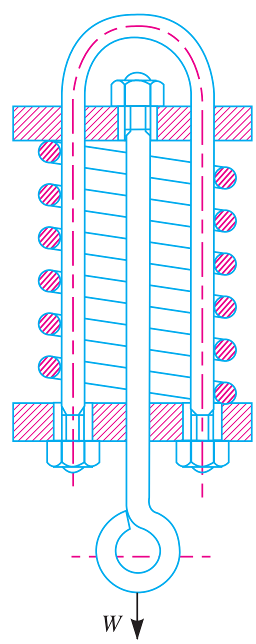



**Fig. 23.9.** Compression spring for carrying tensile load.

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**Note :** The total number of turns of a tension helical spring must be equal to the number of turns ($n$) between the points where the loops start plus the equivalent turns for the loops. It has been found experimentally that half turn should be added for each loop. Thus for a spring having loops on both ends, the total number of active turns,

$$n' = n + 1$$

## 23.8 Stresses in Helical Springs of Circular Wire

Consider a helical compression spring made of circular wire and subjected to an axial load $W$, as shown in Fig. 23.10 ($a$).

Let $D$ = Mean diameter of the spring coil,

$d$ = Diameter of the spring wire,

$n$ = Number of active coils,

$G$ = Modulus of rigidity for the spring material,

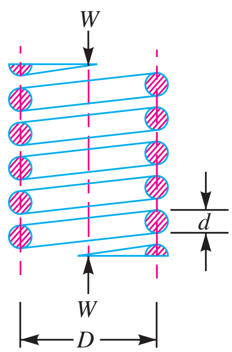
$W$ = Axial load on the spring,

$\tau$ = Maximum shear stress induced in the wire,

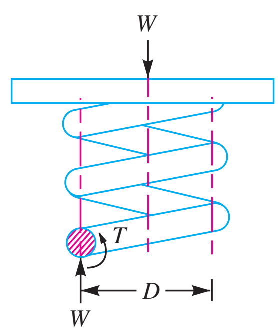
$C$ = Spring index = $D/d$,

$p$ = Pitch of the coils, and

$\delta$ = Deflection of the spring, as a result of an axial load $W$.




($a$) Axially loaded helical spring.




($b$) Free body diagram showing that wire is subjected to torsional shear and a direct shear.

**Fig. 23.10**

Now consider a part of the compression spring as shown in Fig. 23.10 ($b$). The load $W$ tends to rotate the wire due to the twisting moment ( $T$ ) set up in the wire. Thus torsional shear stress is induced in the wire.

A little consideration will show that part of the spring, as shown in Fig. 23.10 ($b$), is in equilibrium under the action of two forces $W$ and the twisting moment $T$. We know that the twisting moment,

$$T = W \times \frac{D}{2} = \frac{\pi}{16} \times \tau_1 \times d^3$$

$$\therefore \quad \tau_1 = \frac{8 W.D}{\pi\, d^3} \qquad \text{...}(i)$$

The torsional shear stress diagram is shown in Fig. 23.11 ($a$).

In addition to the torsional shear stress ($\tau_1$) induced in the wire, the following stresses also act on the wire :

1. Direct shear stress due to the load $W$, and
2. Stress due to curvature of wire.

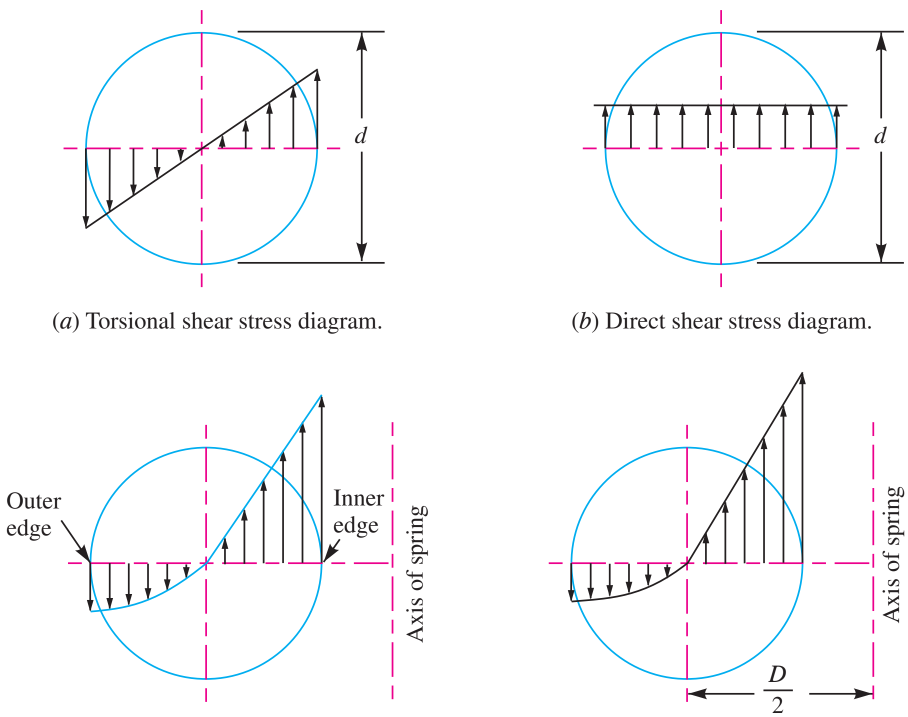
Top

We know that direct shear stress due to the load $W$,

$$\tau_2 = \frac{\text{Load}}{\text{Cross-sectional area of the wire}}$$

$$= \frac{W}{\frac{\pi}{4} \times d^2} = \frac{4W}{\pi\, d^2} \qquad \ldots(ii)$$

The direct shear stress diagram is shown in Fig. 23.11 (*b*) and the resultant diagram of torsional shear stress and direct shear stress is shown in Fig. 23.11 (*c*).




(*a*) Torsional shear stress diagram.

(*b*) Direct shear stress diagram.

(*c*) Resultant torsional shear and direct shear stress diagram.

(*d*) Resultant torsional shear, direct shear and curvature shear stress diagram.

**Fig. 23.11.** Superposition of stresses in a helical spring.

We know that the resultant shear stress induced in the wire,

$$\tau = \tau_1 \pm \tau_2 = \frac{8W.D}{\pi\, d^3} \pm \frac{4W}{\pi\, d^2}$$

The ***positive*** sign is used for the inner edge of the wire and ***negative*** sign is used for the outer edge of the wire. Since the stress is maximum at the inner edge of the wire, therefore

Maximum shear stress induced in the wire,

$$= \text{Torsional shear stress} + \text{Direct shear stress}$$

$$= \frac{8W.D}{\pi\, d^3} + \frac{4W}{\pi\, d^2} = \frac{8W.D}{\pi\, d^3}\left(1 + \frac{d}{2D}\right)$$

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$$= \frac{8\,W.D}{\pi\,d^3}\left(1 + \frac{1}{2C}\right) = K_S \times \frac{8\,W.D}{\pi\,d^3} \qquad ...(iii)$$

... (Substituting $D/d = C$)

where $K_S$ = Shear stress factor $= 1 + \frac{1}{2C}$

From the above equation, it can be observed that the effect of direct shear $\left(\frac{8\,WD}{\pi\,d^3} \times \frac{1}{2C}\right)$ is appreciable for springs of small spring index $C$. Also we have neglected the effect of wire curvature in equation (*iii*). It may be noted that when the springs are subjected to static loads, the effect of wire curvature may be neglected, because yielding of the material will relieve the stresses.

In order to consider the effects of both direct shear as well as curvature of the wire, a Wahl's stress factor ($K$) introduced by A.M. Wahl may be used. The resultant diagram of torsional shear, direct shear and curvature shear stress is shown in Fig. 23.11 (*d*).

∴ Maximum shear stress induced in the wire,

$$\tau = K \times \frac{8\,W.D}{\pi\,d^3} = K \times \frac{8\,W.C}{\pi\,d^2} \qquad ...(iv)$$

where $K = \frac{4C - 1}{4C - 4} + \frac{0.615}{C}$

The values of $K$ for a given spring index ($C$) may be obtained from the graph as shown in Fig. 23.12.

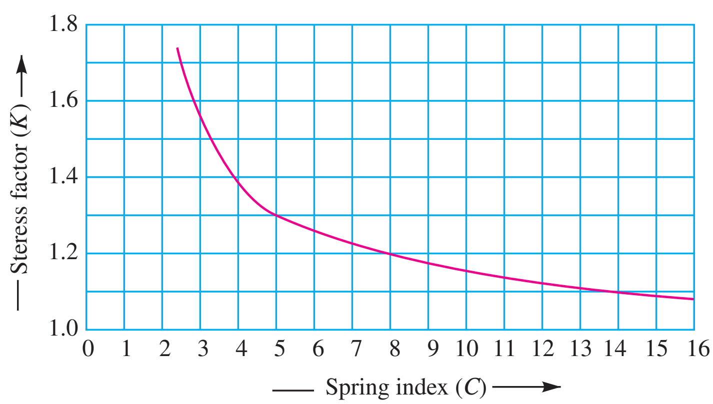


**Fig. 23.12.** Wahl's stress factor for helical springs.

We see from Fig. 23.12 that Wahl's stress factor increases very rapidly as the spring index decreases. The spring mostly used in machinery have spring index above 3.

**Note:** The Wahl's stress factor ($K$) may be considered as composed of two sub-factors, $K_S$ and $K_C$, such that

$$K = K_S \times K_C$$

where $K_S$ = Stress factor due to shear, and

$K_C$ = Stress concentration factor due to curvature.

## 23.9 Deflection of Helical Springs of Circular Wire

In the previous article, we have discussed the maximum shear stress developed in the wire. We know that

Top

Total active length of the wire,

$$l = \text{Length of one coil} \times \text{No. of active coils} = \pi\, D \times n$$

Let $\theta$ = Angular deflection of the wire when acted upon by the torque $T$.

$\therefore$ Axial deflection of the spring,

$$\delta = \theta \times D/2 \qquad \ldots(i)$$

We also know that

$$\frac{T}{J} = \frac{\tau}{D/2} = \frac{G.\theta}{l}$$

$$\therefore \quad \theta = \frac{T.l}{J.G} \qquad \ldots\left(\text{considering } \frac{T}{J} = \frac{G.\theta}{l}\right)$$

where $J$ = Polar moment of inertia of the spring wire

$= \dfrac{\pi}{32} \times d^4$, $d$ being the diameter of spring wire.

and $G$ = Modulus of rigidity for the material of the spring wire.

Now substituting the values of $l$ and $J$ in the above equation, we have

$$\theta = \frac{T.l}{J.G} = \frac{\left(W \times \dfrac{D}{2}\right) \pi\, D.n}{\dfrac{\pi}{32} \times d^4\, G} = \frac{16 W.D^2.n}{G.d^4} \qquad \ldots(ii)$$

Substituting this value of $\theta$ in equation (*i*), we have

$$\delta = \frac{16 W.D^2.n}{G.d^4} \times \frac{D}{2} = \frac{8\, W.D^3.n}{G.d^4} = \frac{8\, W.C^3.n}{G.d} \qquad \ldots (\because\ C = D/d)$$

and the stiffness of the spring or spring rate,

$$\frac{W}{\delta} = \frac{G.d^4}{8\, D^3.n} = \frac{G.d}{8\, C^3.n} = \text{constant}$$

## 23.10 Eccentric Loading of Springs

Sometimes, the load on the springs does not coincide with the axis of the spring, *i.e.* the spring is subjected to an eccentric load. In such cases, not only the safe load for the spring reduces, the stiffness of the spring is also affected. The eccentric load on the spring increases the stress on one side of the spring and decreases on the other side. When the load is offset by a distance $e$ from the spring axis, then the safe load on the spring may be obtained by multiplying the axial load by the factor $\dfrac{D}{2\, e + D}$, where $D$ is the mean diameter of the spring.

## 23.11 Buckling of Compression Springs

It has been found experimentally that when the free length of the spring ($L_F$) is more than four times the mean or pitch diameter ($D$), then the spring behaves like a column and may fail by buckling at a comparatively low load as shown in Fig. 23.13. The critical axial load ($W_{cr}$) that causes buckling may be calculated by using the following relation, *i.e.*

$$W_{cr} = k \times K_B \times L_F$$

where $k$ = Spring rate or stiffness of the spring = $W/\delta$,

$L_F$ = Free length of the spring, and

$K_B$ = Buckling factor depending upon the ratio $L_F / D$.

The buckling factor ($K_B$) for the hinged end and built-in end springs may be taken from the following table.

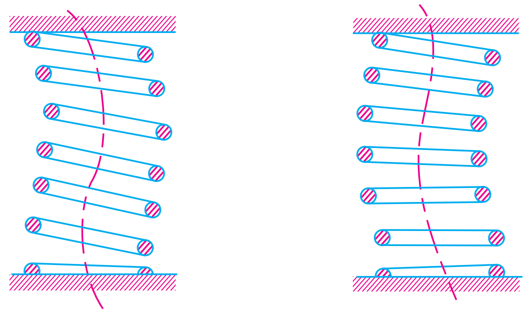


**Fig. 23.13.** Buckling of compression springs.

**Table 23.4. Values of buckling factor ($K_B$).**

| $L_F/D$ | *Hinged end spring* | *Built-in end spring* | $L_F / D$ | *Hinged end spring* | *Built-in end spring* |
|---|---|---|---|---|---|
| 1 | 0.72 | 0.72 | 5 | 0.11 | 0.53 |
| 2 | 0.63 | 0.71 | 6 | 0.07 | 0.38 |
| 3 | 0.38 | 0.68 | 7 | 0.05 | 0.26 |
| 4 | 0.20 | 0.63 | 8 | 0.04 | 0.19 |

It may be noted that a ***hinged end spring*** is one which is supported on pivots at both ends as in case of springs having plain ends where as a ***built-in end spring*** is one in which a squared and ground end spring is compressed between two rigid and parallel flat plates.

It order to avoid the buckling of spring, it is either mounted on a central rod or located on a tube. When the spring is located on a tube, the clearance between the tube walls and the spring should be kept as small as possible, but it must be sufficient to allow for increase in spring diameter during compression.

*In railway coaches strongs springs are used for suspension.*

## 23.12 Surge in Springs

When one end of a helical spring is resting on a rigid support and the other end is loaded suddenly, then all the coils of the spring will not suddenly deflect equally, because some time is required for the propagation of stress along the spring wire. A little consideration will show that in the beginning, the end coils of the spring in contact with the applied load takes up whole of the deflection and then it transmits a large part of its deflection to the adjacent coils. In this way, a wave of compression propagates through the coils to the supported end from where it is reflected back to the deflected end. This wave of compression travels along the spring indefinitely. If the applied load is of fluctuating type as in the case of valve spring in internal combustion engines and if the time interval between the load applications is equal to the time required for the wave to travel from one end to the other end, then resonance will occur. This results in very large deflections of the coils and correspondingly very high stresses. Under these conditions, it is just possible that the spring may fail. This phenomenon is called ***surge.***

It has been found that the natural frequency of spring should be atleast twenty times the frequency of application of a periodic load in order to avoid resonance with all harmonic frequencies upto twentieth order. The natural frequency for springs clamped between two plates is given by

$$f_n = \frac{d}{2\pi D^2 . n}\sqrt{\frac{6\,G.g}{\rho}} \text{ cycles/s}$$

where $d$ = Diameter of the wire,

$D$ = Mean diameter of the spring,

$n$ = Number of active turns,

$G$ = Modulus of rigidity,

$g$ = Acceleration due to gravity, and

$\rho$ = Density of the material of the spring.

The surge in springs may be eliminated by using the following methods :

**1.** By using friction dampers on the centre coils so that the wave propagation dies out.

**2.** By using springs of high natural frequency.

**3.** By using springs having pitch of the coils near the ends different than at the centre to have different natural frequencies.

**Example 23.1.** *A compression coil spring made of an alloy steel is having the following specifications :*

*Mean diameter of coil = 50 mm ; Wire diameter = 5 mm ; Number of active coils = 20.*

*If this spring is subjected to an axial load of 500 N ; calculate the maximum shear stress (neglect the curvature effect) to which the spring material is subjected.*

**Solution.** Given : $D$ = 50 mm ; $d$ = 5 mm ; *$n$ = 20 ; $W$ = 500 N

We know that the spring index,

$$C = \frac{D}{d} = \frac{50}{5} = 10$$

∴ Shear stress factor,

$$K_S = 1 + \frac{1}{2C} = 1 + \frac{1}{2 \times 10} = 1.05$$

and maximum shear stress (neglecting the effect of wire curvature),

$$\tau = K_S \times \frac{8W.D}{\pi\, d^3} = 1.05 \times \frac{8 \times 500 \times 50}{\pi \times 5^3} = 534.7 \text{ N/mm}^2$$

$= 534.7$ MPa **Ans.**

* Superfluous data.

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**Example 23.2.** *A helical spring is made from a wire of 6 mm diameter and has outside diameter of 75 mm. If the permissible shear stress is 350 MPa and modulus of rigidity 84 kN/mm², find the axial load which the spring can carry and the deflection per active turn.*

**Solution.** Given : $d$ = 6 mm ; $D_o$ = 75 mm ; $\tau$ = 350 MPa = 350 N/mm² ; $G$ = 84 kN/mm² = 84 × 10³ N/mm²

We know that mean diameter of the spring,

$$D = D_o - d = 75 - 6 = 69 \text{ mm}$$

∴ Spring index, $$C = \frac{D}{d} = \frac{69}{6} = 11.5$$

Let $W$ = Axial load, and

$\delta / n$ = Deflection per active turn.

**1.** ***Neglecting the effect of curvature***

We know that the shear stress factor,

$$K_S = 1 + \frac{1}{2C} = 1 + \frac{1}{2 \times 11.5} = 1.043$$

and maximum shear stress induced in the wire ($\tau$),

$$350 = K_S \times \frac{8\, W.D}{\pi\, d^3} = 1.043 \times \frac{8\, W \times 69}{\pi \times 6^3} = 0.848\, W$$

∴ $W = 350 / 0.848 = 412.7$ N **Ans.**

We know that deflection of the spring,

$$\delta = \frac{8\, W.D^3.n}{G.d^4}$$

∴ Deflection per active turn,

$$\frac{\delta}{n} = \frac{8\, W.D^3}{G.d^4} = \frac{8 \times 412.7\, (69)^3}{84 \times 10^3 \times 6^4} = 9.96 \text{ mm}$$ **Ans.**

**2.** ***Considering the effect of curvature***

We know that Wahl's stress factor,

$$K = \frac{4C - 1}{4C - 4} + \frac{0.615}{C} = \frac{4 \times 11.5 - 1}{4 \times 11.5 - 4} + \frac{0.615}{11.5} = 1.123$$

We also know that the maximum shear stress induced in the wire ($\tau$),

$$350 = K \times \frac{8W.C}{\pi\, d^2} = 1.123 \times \frac{8 \times W \times 11.5}{\pi \times 6^2} = 0.913\, W$$

∴ $W = 350 / 0.913 = 383.4$ N **Ans.**

and deflection of the spring,

$$\delta = \frac{8\, W.D^3.n}{G.d^4}$$

∴ Deflection per active turn,

$$\frac{\delta}{n} = \frac{8\, W.D^3}{G.d^4} = \frac{8 \times 383.4\, (69)^3}{84 \times 10^3 \times 6^4} = 9.26 \text{ mm}$$ **Ans.**

**Example 23.3.** *Design a spring for a balance to measure 0 to 1000 N over a scale of length 80 mm. The spring is to be enclosed in a casing of 25 mm diameter. The approximate number of turns is 30. The modulus of rigidity is 85 kN/mm². Also calculate the maximum shear stress induced.*

**Solution.** Given : $W$ = 1000 N ; $\delta$ = 80 mm ; $n$ = 30 ; $G$ = 85 kN/mm² = 85 × 10³ N/mm²

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*Design of spring*

Let $D$ = Mean diameter of the spring coil,

$d$ = Diameter of the spring wire, and

$C$ = Spring index = $D/d$.

Since the spring is to be enclosed in a casing of 25 mm diameter, therefore the outer diameter of the spring coil ($D_o = D + d$ ) should be less than 25 mm.

We know that deflection of the spring ($\delta$),

$$80 = \frac{8\,W.C^3.n}{G.d} = \frac{8 \times 1000 \times C^3 \times 30}{85 \times 10^3 \times d} = \frac{240\,C^3}{85\,d}$$

$$\therefore \quad \frac{C^3}{d} = \frac{80 \times 85}{240} = 28.3$$

Let us assume that $d$ = 4 mm. Therefore

$$C^3 = 28.3\,d = 28.3 \times 4 = 113.2 \quad \text{or} \quad C = 4.84$$

and $D = C.d = 4.84 \times 4 = 19.36$ mm **Ans.**

We know that outer diameter of the spring coil,

$$D_o = D + d = 19.36 + 4 = 23.36 \text{ mm} \quad \textbf{Ans.}$$

Since the value of $D_o$ = 23.36 mm is less than the casing diameter of 25 mm, therefore the assumed dimension, $d$ = 4 mm is correct.

*Maximum shear stress induced*

We know that Wahl's stress factor,

$$K = \frac{4C-1}{4C-4} + \frac{0.615}{C} = \frac{4 \times 4.84 - 1}{4 \times 4.84 - 4} + \frac{0.615}{4.84} = 1.322$$

$\therefore$ Maximum shear stress induced,

$$\tau = K \times \frac{8\,W.C}{\pi\,d^2} = 1.322 \times \frac{8 \times 1000 \times 4.84}{\pi \times 4^2}$$

$$= 1018.2 \text{ N/mm}^2 = 1018.2 \text{ MPa} \quad \textbf{Ans.}$$

**Example 23.4.** *A mechanism used in printing machinery consists of a tension spring assembled with a preload of 30 N. The wire diameter of spring is 2 mm with a spring index of 6. The spring has 18 active coils. The spring wire is hard drawn and oil tempered having following material properties:*

*Design shear stress = 680 MPa*

*Modulus of rigidity = 80 kN/mm²*

*Determine : 1. the initial torsional shear stress in the wire; 2. spring rate; and 3. the force to cause the body of the spring to its yield strength.*

Tension springs are widely used in printing machines.

**Solution.** Given : $W_i$ = 30 N ; $d$ = 2 mm ; $C = D/d = 6$ ; $n$ = 18 ; $\tau$ = 680 MPa = 680 N/mm$^2$ ; $G$ = 80 kN/mm$^2$ = $80 \times 10^3$ N/mm$^2$

**1.** ***Initial torsional shear stress in the wire***

We know that Wahl's stress factor,

$$K = \frac{4C-1}{4C-4} + \frac{0.615}{C} = \frac{4\times6-1}{4\times6-4} + \frac{0.615}{6} = 1.2525$$

∴ Initial torsional shear stress in the wire,

$$\tau_i = K \times \frac{8\,W_i \times C}{\pi\,d^2} = 1.2525 \times \frac{8\times30\times6}{\pi\times2^2} = 143.5 \text{ N/mm}^2$$

$= 143.5$ MPa **Ans.**

**2.** ***Spring rate***

We know that spring rate (or stiffness of the spring),

$$= \frac{G.d}{8\,C^3.n} = \frac{80\times10^3\times2}{8\times6^3\times18} = 5.144 \text{ N/mm} \quad \textbf{Ans.}$$

**3.** ***Force to cause the body of the spring to its yield strength***

Let $W$ = Force to cause the body of the spring to its yield strength.

We know that design or maximum shear stress ($\tau$),

$$680 = K \times \frac{8\,W.C}{\pi\,d^2} = 1.2525 \times \frac{8W\times6}{\pi\times2^2} = 4.78\,W$$

∴ $W = 680 / 4.78 = 142.25$ N **Ans.**

**Example 23.5.** *Design a helical compression spring for a maximum load of 1000 N for a deflection of 25 mm using the value of spring index as 5.*

*The maximum permissible shear stress for spring wire is 420 MPa and modulus of rigidity is 84 kN/mm².*

*Take Wahl's factor,* $K = \dfrac{4C-1}{4C-4} + \dfrac{0.615}{C}$, *where C = Spring index.*

**Solution.** Given : $W = 1000$ N ; $\delta = 25$ mm ; $C = D/d = 5$ ; $\tau = 420$ MPa $= 420$ N/mm² ; $G = 84$ kN/mm² $= 84 \times 10^3$ N/mm²

**1.** ***Mean diameter of the spring coil***

Let $D$ = Mean diameter of the spring coil, and

$d$ = Diameter of the spring wire.

We know that Wahl's stress factor,

$$K = \frac{4C-1}{4C-4} + \frac{0.615}{C} = \frac{4\times5-1}{4\times5-4} + \frac{0.615}{5} = 1.31$$

and maximum shear stress ($\tau$),

$$420 = K \times \frac{8\,W.C}{\pi\,d^2} = 1.31 \times \frac{8\times1000\times5}{\pi\,d^2} = \frac{16\,677}{d^2}$$

∴ $d^2 = 16\,677 / 420 = 39.7$ or $d = 6.3$ mm

From Table 23.2, we shall take a standard wire of size *SWG* 3 having diameter ($d$) = 6.401 mm.

∴ Mean diameter of the spring coil,

$D = C.d = 5\,d = 5 \times 6.401 = 32.005$ mm **Ans.** ... ($\because C = D/d = 5$)

and outer diameter of the spring coil,

$D_o = D + d = 32.005 + 6.401 = 38.406$ mm **Ans.**

**2.** ***Number of turns of the coils***

Let $n$ = Number of active turns of the coils.

We know that compression of the spring ($\delta$),

$$25 = \frac{8W.C^3.n}{G.d} = \frac{8 \times 1000\,(5)^3 n}{84 \times 10^3 \times 6.401} = 1.86\,n$$

$\therefore$ $n = 25 / 1.86 = 13.44$ say 14 **Ans.**

For squared and ground ends, the total number of turns,

$$n' = n + 2 = 14 + 2 = 16 \text{ **Ans.**}$$

**3.** ***Free length of the spring***

We know that free length of the spring

$$= n'.d + \delta + 0.15\,\delta = 16 \times 6.401 + 25 + 0.15 \times 25$$

$$= 131.2 \text{ mm}$$ **Ans.**

**4.** ***Pitch of the coil***

We know that pitch of the coil

$$= \frac{\text{Free length}}{n' - 1} = \frac{131.2}{16 - 1} = 8.75 \text{ mm}$$ **Ans.**

**Example 23.6.** *Design a close coiled helical compression spring for a service load ranging from 2250 N to 2750 N. The axial deflection of the spring for the load range is 6 mm. Assume a spring index of 5. The permissible shear stress intensity is 420 MPa and modulus of rigidity, $G = 84$ kN/mm$^2$.*

*Neglect the effect of stress concentration. Draw a fully dimensioned sketch of the spring, showing details of the finish of the end coils.*

**Solution.** Given : $W_1 = 2250$ N ; $W_2 = 2750$ N ; $\delta = 6$ mm ; $C = D/d = 5$ ; $\tau = 420$ MPa $= 420$ N/mm$^2$ ; $G = 84$ kN/mm$^2$ $= 84 \times 10^3$ N/mm$^2$

**1.** ***Mean diameter of the spring coil***

Let $D$ = Mean diameter of the spring coil for a maximum load of $W_2 = 2750$ N, and

$d$ = Diameter of the spring wire.

We know that twisting moment on the spring,

$$T = W_2 \times \frac{D}{2} = 2750 \times \frac{5d}{2} = 6875\,d \qquad \ldots\left(\because C = \frac{D}{d} = 5\right)$$

We also know that twisting moment ($T$),

$$6875\,d = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 420 \times d^3 = 82.48\,d^3$$

$\therefore$ $d^2 = 6875 / 82.48 = 83.35$ or $d = 9.13$ mm

From Table 23.2, we shall take a standard wire of size *SWG* 3/0 having diameter ($d$) = 9.49 mm.

$\therefore$ Mean diameter of the spring coil,

$$D = 5d = 5 \times 9.49 = 47.45 \text{ mm}$$ **Ans.**

We know that outer diameter of the spring coil,

$$D_o = D + d = 47.45 + 9.49 = 56.94 \text{ mm}$$ **Ans.**

and inner diameter of the spring coil,

$$D_i = D - d = 47.45 - 9.49 = 37.96 \text{ mm}$$ **Ans.**

**2.** ***Number of turns of the spring coil***

Let $n$ = Number of active turns.

It is given that the axial deflection ($\delta$) for the load range from 2250 N to 2750 N (*i.e.* for $W = 500$ N) is 6 mm.

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We know that the deflection of the spring (δ),

$$6 = \frac{8\,W.C^3.n}{G.d} = \frac{8 \times 500\,(5)^3\,n}{84 \times 10^3 \times 9.49} = 0.63\,n$$

∴ $n = 6 / 0.63 = 9.5$ say 10 **Ans.**

For squared and ground ends, the total number of turns,

$$n' = 10 + 2 = 12 \textbf{ Ans.}$$

***3. Free length of the spring***

Since the compression produced under 500 N is 6 mm, therefore maximum compression produced under the maximum load of 2750 N is

$$\delta_{max} = \frac{6}{500} \times 2750 = 33 \text{ mm}$$

We know that free length of the spring,

$$L_F = n'.d + \delta_{max} + 0.15\,\delta_{max}$$
$$= 12 \times 9.49 + 33 + 0.15 \times 33$$
$$= 151.83 \text{ say } 152 \text{ mm } \textbf{Ans.}$$

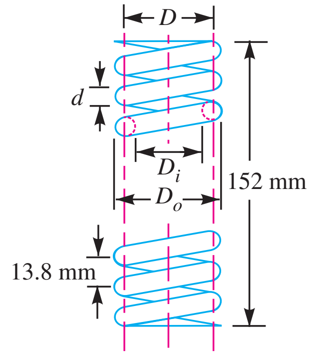


**Fig. 23.14**

***4. Pitch of the coil***

We know that pitch of the coil

$$= \frac{\text{Free length}}{n' - 1} = \frac{152}{12 - 1} = 13.73 \text{ say } 13.8 \text{ mm } \textbf{Ans.}$$

The spring is shown in Fig. 23.14.

**Example 23.7.** *Design and draw a valve spring of a petrol engine for the following operating conditions :*

*Spring load when the valve is open = 400 N*
*Spring load when the valve is closed = 250 N*
*Maximum inside diameter of spring = 25 mm*
*Length of the spring when the valve is open = 40 mm*
*Length of the spring when the valve is closed = 50 mm*
*Maximum permissible shear stress = 400 MPa*

**Solution.** Given : $W_1$ = 400 N ; $W_2$ = 250 N ; $D_i$ = 25 mm ; $l_1$ = 40 mm ; $l_2$ = 50 mm ; τ = 400 MPa = 400 N/mm²

***1. Mean diameter of the spring coil***

Let $d$ = Diameter of the spring wire in mm, and

$D$ = Mean diameter of the spring coil
= Inside dia. of spring + Dia. of spring wire = (25 + $d$) mm

Since the diameter of the spring wire is obtained for the maximum spring load ($W_1$), therefore maximum twisting moment on the spring,

*Petrol engine.*

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$$T = W_1 \times \frac{D}{2} = 400\left(\frac{25 + d}{2}\right) = (5000 + 200\ d)\ \text{N-mm}$$

We know that maximum twisting moment ($T$),

$$(5000 + 200\ d) = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 400 \times d^3 = 78.55\ d^3$$

Solving this equation by hit and trial method, we find that $d = 4.2$ mm.

From Table 23.2, we find that standard size of wire is *SWG* 7 having $d = 4.47$ mm.

Now let us find the diameter of the spring wire by taking Wahl's stress factor ($K$) into consideration.

We know that spring index,

$$C = \frac{D}{d} = \frac{25 + 4.47}{4.47} = 6.6 \qquad \text{... } (\because D = 25 + d)$$

$\therefore$ Wahl's stress factor,

$$K = \frac{4C - 1}{4C - 4} + \frac{0.615}{C} = \frac{4 \times 6.6 - 1}{4 \times 6.6 - 4} + \frac{0.615}{6.6} = 1.227$$

We know that the maximum shear stress ($\tau$),

$$400 = K \times \frac{8\ W_1 . C}{\pi\ d^2} = 1.227 \times \frac{8 \times 400 \times 6.6}{\pi\ d^2} = \frac{8248}{d^2}$$

$$\therefore \qquad d^2 = 8248 / 400 = 20.62 \quad \text{or} \quad d = 4.54 \text{ mm}$$

Taking larger of the two values, we have

$$d = 4.54 \text{ mm}$$

From Table 23.2, we shall take a standard wire of size *SWG* 6 having diameter ($d$) = 4.877 mm.

$\therefore$ Mean diameter of the spring coil

$$D = 25 + d = 25 + 4.877 = 29.877 \text{ mm}$$ **Ans.**

and outer diameter of the spring coil,

$$D_o = D + d = 29.877 + 4.877 = 34.754 \text{ mm}$$ **Ans.**

***2. Number of turns of the coil***

Let $n$ = Number of active turns of the coil.

We are given that the compression of the spring caused by a load of $(W_1 - W_2)$, *i.e.* 400 – 250 = 150 N is $l_2 - l_1$, *i.e.* 50 – 40 = 10 mm. In other words, the deflection ($\delta$) of the spring is 10 mm for a load ($W$) of 150 N

We know that the deflection of the spring ($\delta$),

$$10 = \frac{8\ W . D^3 . n}{G . d^4} = \frac{8 \times 150\ (29.877)^3\ n}{80 \times 10^3\ (4.877)^4} = 0.707\ n$$

... (Taking $G = 80 \times 10^3$ N/mm²)

$$\therefore \qquad n = 10 / 0.707 = 14.2 \text{ say } 15$$ **Ans.**

Taking the ends of the springs as squared and ground, the total number of turns of the spring,

$$n' = 15 + 2 = 17$$ **Ans.**

***3. Free length of the spring***

Since the deflection for 150 N of load is 10 mm, therefore the maximum deflection for the maximum load of 400 N is

$$\delta_{max} = \frac{10}{150} \times 400 = 26.67 \text{ mm}$$

An automobile suspension and shock-absorber. The two links with green ends are turnbuckles.

∴ Free length of the spring,

$$L_F = n'.d + \delta_{max} + 0.15\ \delta_{max}$$
$$= 17 \times 4.877 + 26.67 + 0.15 \times 26.67 = 113.58 \text{ mm} \quad \textbf{Ans.}$$

**4. *Pitch of the coil***

We know that pitch of the coil

$$= \frac{\text{Free length}}{n' - 1} = \frac{113.58}{17 - 1} = 7.1 \text{ mm} \quad \textbf{Ans.}$$

**Example 23.8.** *Design a helical spring for a spring loaded safety valve (Ramsbottom safety valve) for the following conditions :*

*Diameter of valve seat = 65 mm ; Operating pressure = 0.7 N/mm²; Maximum pressure when the valve blows off freely = 0.75 N/mm²; Maximum lift of the valve when the pressure rises from 0.7 to 0.75 N/mm² = 3.5 mm ; Maximum allowable stress = 550 MPa ; Modulus of rigidity = 84 kN/mm²; Spring index = 6.*

*Draw a neat sketch of the free spring showing the main dimensions.*

**Solution.** Given : $D_1$ = 65 mm ; $p_1$ = 0.7 N/mm² ; $p_2$ = 0.75 N/mm² ; $\delta$ = 3.5 mm ; $\tau$ = 550 MPa = 550 N/mm² ; $G$ = 84 kN/mm² = $84 \times 10^3$ N/mm² ; $C$ = 6

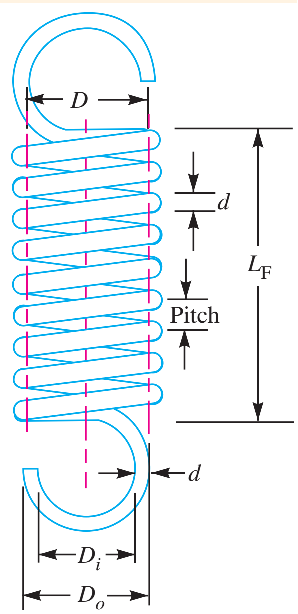


Fig. 23.15

**1. *Mean diameter of the spring coil***

Let $D$ = Mean diameter of the spring coil, and

$d$ = Diameter of the spring wire.

Since the safety valve is a Ramsbottom safety valve, therefore the spring will be under tension. We know that initial tensile force acting on the spring (*i.e.* before the valve lifts),

$$W_1 = \frac{\pi}{4}(D_1)^2\, p_1 = \frac{\pi}{4}(65)^2\, 0.7 = 2323 \text{ N}$$

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and maximum tensile force acting on the spring (*i.e.* when the valve blows off freely),

$$W_2 = \frac{\pi}{4}(D_1)^2\, p_2 = \frac{\pi}{4}(65)^2\, 0.75 = 2489 \text{ N}$$

∴ Force which produces the deflection of 3.5 mm,

$$W = W_2 - W_1 = 2489 - 2323 = 166 \text{ N}$$

Since the diameter of the spring wire is obtained for the maximum spring load ($W_2$), therefore maximum twisting moment on the spring,

$$T = W_2 \times \frac{D}{2} = 2489 \times \frac{6\,d}{2} = 7467\,d \qquad \ldots (\because C = D/d = 6)$$

We know that maximum twisting moment ($T$),

$$7467\,d = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 550 \times d^3 = 108\,d^3$$

$$\therefore \qquad d^2 = 7467 / 108 = 69.14 \quad \text{or} \quad d = 8.3 \text{ mm}$$

From Table 23.2, we shall take a standard wire of size *SWG* 2/0 having diameter ($d$) = 8.839 mm **Ans.**

∴ Mean diameter of the coil,

$$D = 6\,d = 6 \times 8.839 = 53.034 \text{ mm } \textbf{Ans.}$$

Outside diameter of the coil,

$$D_o = D + d = 53.034 + 8.839 = 61.873 \text{ mm } \textbf{Ans.}$$

and inside diameter of the coil,

$$D_i = D - d = 53.034 - 8.839 = 44.195 \text{ mm } \textbf{Ans.}$$

***2. Number of turns of the coil***

Let $n$ = Number of active turns of the coil.

We know that the deflection of the spring (δ),

$$3.5 = \frac{8\,W.C^3.n}{G.d} = \frac{8 \times 166 \times 6^3 \times n}{84 \times 10^3 \times 8.839} = 0.386\,n$$

$$\therefore \qquad n = 3.5 / 0.386 = 9.06 \text{ say } 10 \textbf{ Ans.}$$

For a spring having loop on both ends, the total number of turns,

$$n' = n + 1 = 10 + 1 = 11 \textbf{ Ans.}$$

***3. Free length of the spring***

Taking the least gap between the adjacent coils as 1 mm when the spring is in free state, the free length of the tension spring,

$$L_F = n.d + (n-1)\,1 = 10 \times 8.839 + (10-1)\,1 = 97.39 \text{ mm } \textbf{Ans.}$$

***4. Pitch of the coil***

We know that pitch of the coil

$$= \frac{\text{Free length}}{n-1} = \frac{97.39}{10-1} = 10.82 \text{ mm } \textbf{Ans.}$$

The tension spring is shown in Fig. 23.15.

**Example 23.9.** *A safety valve of 60 mm diameter is to blow off at a pressure of 1.2 N/mm². It is held on its seat by a close coiled helical spring. The maximum lift of the valve is 10 mm. Design a suitable compression spring of spring index 5 and providing an initial compression of 35 mm. The maximum shear stress in the material of the wire is limited to 500 MPa. The modulus of rigidity for the spring material is 80 kN/mm². Calculate : 1. Diameter of the spring wire, 2. Mean coil diameter, 3. Number of active turns, and 4. Pitch of the coil.*

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*Take Wahl's factor,* $K = \frac{4C-1}{4C-4} + \frac{0.615}{C}$, *where C is the spring index.*

**Solution.** Given : Valve dia. = 60 mm ; Max. pressure = 1.2 N/mm$^2$ ; $\delta_2$ = 10 mm ; $C$ = 5 ; $\delta_1$ = 35 mm ; $\tau$ = 500 MPa = 500 N/mm$^2$ ; $G$ = 80 kN/mm$^2$ = 80 × 10$^3$ N/mm$^2$

**1.** ***Diameter of the spring wire***

Let $d$ = Diameter of the spring wire.

We know that the maximum load acting on the valve when it just begins to blow off,

$$W_1 = \text{Area of the valve} \times \text{Max. pressure}$$

$$= \frac{\pi}{4}(60)^2\ 1.2 = 3394 \text{ N}$$

and maximum compression of the spring,

$$\delta_{max} = \delta_1 + \delta_2 = 35 + 10 = 45 \text{ mm}$$

Since a load of 3394 N keeps the valve on its seat by providing initial compression of 35 mm, therefore the maximum load on the spring when the valve is oepn (*i.e.* for maximum compression of 45 mm),

$$W = \frac{3394}{35} \times 45 = 4364 \text{ N}$$

We know that Wahl's stress factor,

$$K = \frac{4C-1}{4C-4} + \frac{0.615}{C} = \frac{4 \times 5 - 1}{4 \times 5 - 4} + \frac{0.615}{5} = 1.31$$

We also know that the maximum shear stress ($\tau$),

$$500 = K \times \frac{8\,W.C}{\pi\,d^2} = 1.31 \times \frac{8 \times 4364 \times 5}{\pi\,d^2} = \frac{72\,780}{d^2}$$

$$\therefore \quad d^2 = 72\,780 / 500 = 145.6 \quad \text{or} \quad d = 12.06 \text{ mm}$$

From Table 23.2, we shall take a standard wire of size *SWG* 7/0 having diameter ($d$) = 12.7 mm. **Ans.**

**2.** ***Mean coil diameter***

Let $D$ = Mean coil diameter.

We know that the spring index,

$$C = D/d \quad \text{or} \quad D = C.d = 5 \times 12.7 = 63.5 \text{ mm } \textbf{Ans.}$$

**3.** ***Number of active turns***

Let $n$ = Number of active turns.

We know that the maximum compression of the spring ($\delta$),

$$45 = \frac{8\,W.C^3.n}{G.d} = \frac{8 \times 4364 \times 5^3 \times n}{80 \times 10^3 \times 12.7} = 4.3\,n$$

$$\therefore \quad n = 45 / 4.3 = 10.5 \text{ say } 11 \textbf{ Ans.}$$

Taking the ends of the coil as squared and ground, the total number of turns,

$$n' = n + 2 = 11 + 2 = 13 \textbf{ Ans.}$$

**Note :** The valve of $n$ may also be calculated by using

$$\delta_1 = \frac{8\,W_1.C^3.n}{G.d}$$

$$35 = \frac{8 \times 3394 \times 5^3 \times n}{80 \times 10^3 \times 12.7} = 3.34\,n \quad \text{or} \quad n = 35 / 3.34 = 10.5 \text{ say } 11$$

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**4.** ***Pitch of the coil***

We know that free length of the spring,

$$L_F = n'.d + \delta_{max} + 0.15\,\delta_{max} = 13 \times 12.7 + 45 + 0.15 \times 45$$
$$= 216.85 \text{ mm } \textbf{Ans}$$

$$\therefore \qquad \text{Pitch of the coil} = \frac{\text{Free length}}{n' - 1} = \frac{216.85}{13 - 1} = 18.1 \text{ mm } \textbf{Ans.}$$

**Example 23.10.** *In a spring loaded governor as shown in Fig. 23.16, the balls are attached to the vertical arms of the bell crank lever, the horizontal arms of which lift the sleeve against the pressure exerted by a spring. The mass of each ball is 2.97 kg and the lengths of the vertical and horizontal arms of the bell crank lever are 150 mm and 112.5 mm respectively. The extreme radii of rotation of the balls are 100 mm and 150 mm and the governor sleeve begins to lift at 240 r.p.m. and reaches the highest position with a 7.5 percent increase of speed when effects of friction are neglected. Design a suitable close coiled round section spring for the governor.*

*Assume permissible stress in spring steel as 420 MPa, modulus of rigidity 84 kN/mm² and spring index 8. Allowance must be made for stress concentration, factor of which is given by*

$$\frac{4C - 1}{4C - 4} + \frac{0.615}{C}, \text{ where } C \text{ is the spring index.}$$

**Solution.** Given : $m = 2.97$ kg ; $x = 150$ mm $= 0.15$ m ; $y = 112.5$ mm $= 0.1125$ m ; $r_2 = 100$ mm $= 0.1$ m ; $r_1 = 150$ mm $= 0.15$ m ; $N_2 = 240$ r.p.m. ; $\tau = 420$ MPa $= 420$ N/mm² ; $G = 84$ kN/mm² $= 84 \times 10^3$ N/mm² ; $C = 8$

The spring loaded governor, as shown in Fig. 23.16, is a *Hartnell type governor. First of all, let us find the compression of the spring.

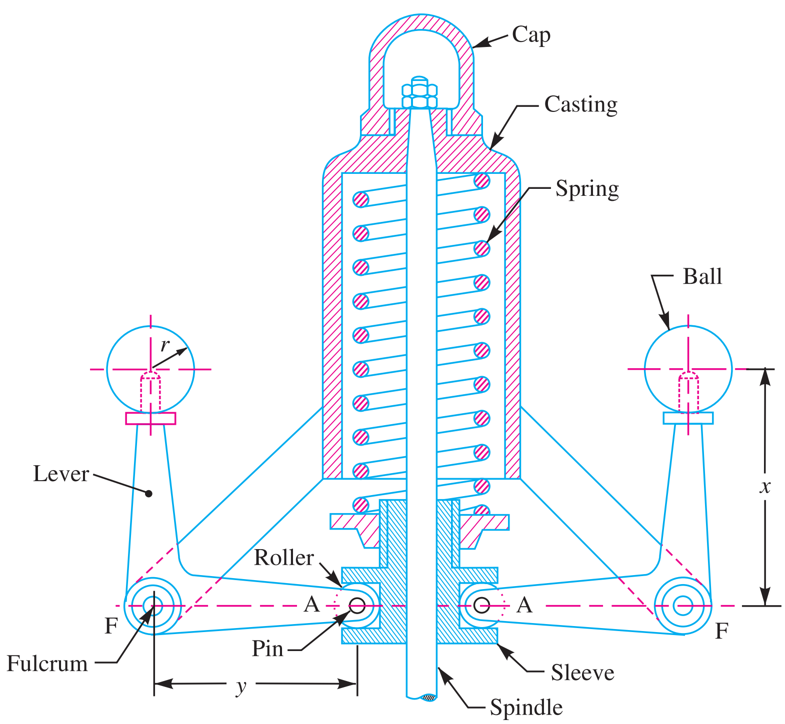


**Fig. 23.16**

* For further details, see authors' popular book on **'Theory of Machines'.**

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We know that minimum angular speed at which the governor sleeve begins to lift,

$$\omega_2 = \frac{2\,\pi\,N_2}{60} = \frac{2\,\pi \times 240}{60} = 25.14 \text{ rad/s}$$

Since the increase in speed is 7.5%, therefore maximum speed,

$$\omega_1 = \omega_2 + \frac{7.5}{100} \times \omega_2 = 25.14 + \frac{7.5}{100} \times 25.14 = 27 \text{ rad/s}$$

The position of the balls and the lever arms at the maximum and minimum speeds is shown in Fig. 23.17 (*a*) and (*b*) respectively.

Let $F_{C1}$ = Centrifugal force at the maximum speed, and

$F_{C2}$ = Centrifugal force at the minimum speed.

We know that the spring force at the maximum speed ($\omega_1$),

$$S_1 = 2\,F_{C1} \times \frac{x}{y} = 2\,m\,(\omega_1)^2\,r_1 \times \frac{x}{y} = 2 \times 2.97\,(27)^2\,0.15 \times \frac{0.15}{0.1125} = 866 \text{ N}$$

Similarly, the spring force at the minimum speed $\omega_2$,

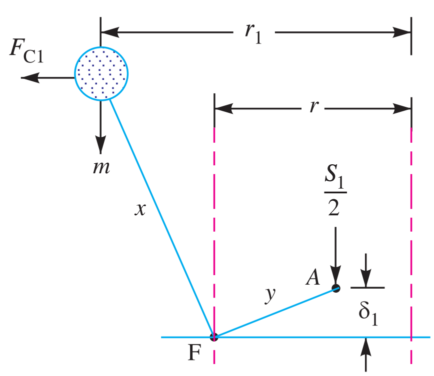
$$S_2 = 2\,F_{C2} \times \frac{x}{y} = 2m\,(\omega_2)^2\,r_2 \times \frac{x}{y} = 2 \times 2.97\,(25.14)^2\,0.1 \times \frac{0.15}{0.1125} = 500 \text{ N}$$

Since the compression of the spring will be equal to the lift of the sleeve, therefore compression of the spring,

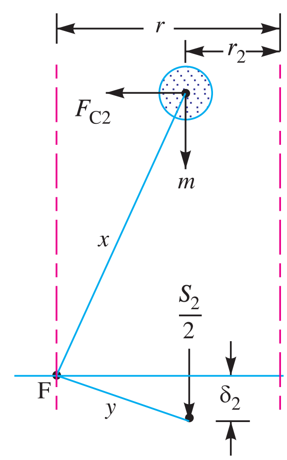
$$\delta = \delta_1 + \delta_2 = (r_1 - r)\,\frac{y}{x} + (r - r_2)\,\frac{y}{x} = (r_1 - r_2)\,\frac{y}{x}$$

$$= (0.15 - 0.1)\,\frac{0.1125}{0.15} = 0.0375 \text{ m} = 37.5 \text{ mm}$$

This compression of the spring is due to the spring force of $(S_1 - S_2)$ *i.e.* (866 – 500) = 366 N.




(*a*) Maximum position.




(*b*) Minimum position.

**Fig. 23.17**

**1.** ***Diameter of the spring wire***

Let $d$ = Diameter of the spring wire in mm.

We know that Wahl's stress factor,

$$K = \frac{4C - 1}{4C - 4} + \frac{0.615}{C} = \frac{4 \times 8 - 1}{4 \times 8 - 4} + \frac{0.615}{8} = 1.184$$

We also know that maximum shear stress ($\tau$),

$$420 = K \times \frac{8\,W.C}{\pi\, d^2} = 1.184 \times \frac{8 \times 866 \times 8}{\pi\, d^2} = \frac{20\,885}{d^2}$$

... (Substituting $W = S_1$, the maximum spring force)

$\therefore$ $d^2 = 20\,885 / 420 = 49.7$ or $d = 7.05$ mm

From Table 23.2, we shall take the standard wire of size *SWG* 1 having diameter ($d$) = 7.62 mm **Ans.**

***2. Mean diameter of the spring coil***

Let $D$ = Mean diameter of the spring coil.

We know that the spring index,

$$C = D/d \quad \text{or} \quad D = C.d = 8 \times 7.62 = 60.96 \text{ mm } \textbf{Ans.}$$

***3. Number of turns of the coil***

Let $n$ = Number of active turns of the coil.

We know that compression of the spring ($\delta$),

$$37.5 = \frac{8\,W.C^3.n}{G.d} = \frac{8 \times 366 \times 8^3 \times n}{84 \times 10^3 \times 7.62} = 2.34\,n$$

... (Substituting $W = S_1 - S_2$)

$\therefore$ $n = 37.5 / 2.34 = 16$ **Ans.**

and total number of turns using squared and ground ends,

$$n' = n + 2 = 16 + 2 = 18$$

***4. Free length of the coil***

Since the compression produced under a force of 366 N is 37.5 mm, therefore maximum compression produced under the maximum load of 866 N is,

$$\delta_{max} = \frac{37.5}{366} \times 866 = 88.73 \text{ mm}$$

We know that free length of the coil,

$$L_F = n'.d + \delta_{max} + 0.15\,\delta_{max}$$
$$= 18 \times 7.62 + 88.73 + 0.15 \times 88.73 = 239.2 \text{ mm } \textbf{Ans.}$$

***5. Pitch of the coil***

We know that pitch of the coil

$$= \frac{\text{Free length}}{n' - 1} = \frac{239.2}{18 - 1} = 14.07 \text{ mm } \textbf{Ans.}$$

**Example 23.11.** *A single plate clutch is to be designed for a vehicle. Both sides of the plate are to be effective. The clutch transmits 30 kW at a speed of 3000 r.p.m. and should cater for an over load of 20%. The intensity of pressure on the friction surface should not exceed 0.085 N/mm² and the surface speed at the mean radius should be limited to 2300 m / min. The outside diameter of the surfaces may be assumed as 1.3 times the inside diameter and the coefficient of friction for the surfaces may be taken as 0.3. If the axial thrust is to be provided by six springs of about 25 mm mean coil diameter, design the springs selecting wire from the following gauges :*

| *SWG* | *4* | *5* | *6* | *7* | *8* | *9* | *10* | *11* | *12* |
|---|---|---|---|---|---|---|---|---|---|
| *Dia. (mm)* | *5.893* | *5.385* | *4.877* | *4.470* | *4.064* | *3.658* | *3.251* | *2.946* | *2.642* |

*Safe shear stress is limited to 420 MPa and modulus of rigidity is 84 kN/mm².*

**Solution.** Given : $P$ = 30 kW = $30 \times 10^3$ W ; $N$ = 3000 r.p.m. ; $p$ = 0.085 N/mm² ; $v$ = 2300 m/min ; $d_1 = 1.3\,d_2$ or $r_1 = 1.3\,r_2$ ; $\mu = 0.3$ ; No. of springs = 6 ; $D$ = 25 mm ; $\tau$ = 420 MPa = 420 N/mm² ; $G$ = 84 kN/mm² = $84 \times 10^3$ N/mm²

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First of all, let us find the maximum load on each spring. We know that the mean torque transmitted by the clutch,

$$T_{mean} = \frac{P \times 60}{2\,\pi\,N} = \frac{30 \times 10^3 \times 60}{2\,\pi \times 3000} = 95.5 \text{ N-m}$$

Since an overload of 20% is allowed, therefore maximum torque to which the clutch should be designed is given by

$$T_{max} = 1.2\,T_{mean} = 1.2 \times 95.5 = 114.6 \text{ N-m} = 114\,600 \text{ N-mm} \qquad \ldots(i)$$

Let $r_1$ and $r_2$ be the outside and inside radii of the friction surfaces. Since maximum intensity of pressure is at the inner radius, therefore for uniform wear,

$$^{*}p \times r_2 = C \text{ (a constant) or } C = 0.085\,r_2$$

We know that the axial thrust transmitted,

$$W = C \times 2\pi\,(r_1 - r_2) \qquad \ldots(ii)$$

Since both sides of the plate are effective, therefore maximum torque transmitted,

$$T_{max} = \tfrac{1}{2}\,\mu \times W\,(r_1 + r_2)\,2 = 2\pi\,\mu.C\,[(r_1)^2 - (r_2)^2] \qquad \ldots\text{[From equation } (ii)]$$

$$114\,600 = 2\pi \times 0.3 \times 0.085\,r_2\,[(1.3\,r_2)^2 - (r_2)^2] = 0.11\,(r_2)^3$$

$$\therefore \quad (r_2)^3 = 114\,600 / 0.11 = 1.04 \times 10^6 \quad \text{or} \quad r_2 = 101.4 \text{ say } 102 \text{ mm}$$

and

$$r_1 = 1.3\,r_2 = 1.3 \times 102 = 132.6 \text{ say } 133 \text{ mm}$$

∴ Mean radius,

$$r = \frac{r_1 + r_2}{2} = \frac{133 + 102}{2} = 117.5 \text{ mm} = 0.1175 \text{ m}$$

We know that surface speed at the mean radius,

$$v = 2\,\pi\,r\,N = 2\,\pi \times 0.1175 \times 3000 = 2215 \text{ m/min}$$

Since the surface speed as obtained above is less than the permissible value of 2300 m/min, therefore the radii of the friction surface are safe.

We know that axial thrust,

$$W = C \times 2\pi\,(r_1 - r_2) = 0.085\,r_2 \times 2\pi\,(r_1 - r_2) \qquad \ldots(\because C = 0.085\,r_2)$$

$$= 0.085 \times 102 \times 2\pi\,(133 - 102) = 1689 \text{ N}$$

Since this axial thrust is to be provided by six springs, therefore maximum load on each spring,

$$W_1 = \frac{1689}{6} = 281.5 \text{ N}$$

**1.** ***Diameter of the spring wire***

Let $d$ = Diameter of the spring wire.

We know that the maximum torque transmitted,

$$T = W_1 \times \frac{D}{2} = 281.5 \times \frac{25}{2} = 3518.75 \text{ N-mm}$$

We also know that the maximum torque transmitted ($T$),

$$3518.75 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 420 \times d^3 = 82.48\,d^3$$

$$\therefore \quad d^3 = 3518.75 / 82.48 = 42.66 \quad \text{or} \quad d = 3.494 \text{ mm}$$

Let us now find out the diameter of the spring wire by taking the stress factor ($K$) into consideration.

We know that the spring index,

$$C = \frac{D}{d} = \frac{25}{3.494} = 7.155$$

* Please refer Chapter 24 on Clutches.

and Wahl's stress factor,

$$K = \frac{4C - 1}{4C - 4} + \frac{0.615}{C} = \frac{4 \times 7.155 - 1}{4 \times 7.155 - 4} + \frac{0.615}{7.155} = 1.21$$

We know that the maximum shear stress ($\tau$),

$$420 = K \times \frac{8\, W_1 . D}{\pi\, d^3} = 1.21 \times \frac{8 \times 281.5 \times 25}{\pi\, d^3} = \frac{21\,681}{d^3}$$

$\therefore$ $d^3 = 21\,681 / 420 = 51.6$ or $d = 3.72$ mm

From Table 23.2, we shall take a standard wire of size *SWG* 8 having diameter ($d$) = 4.064 mm. **Ans.**

Outer diameter of the spring,

$$D_o = D + d = 25 + 4.064 = 29.064 \text{ mm } \textbf{Ans.}$$

and inner diameter of the spring,

$$D_i = D - d = 25 - 4.064 = 20.936 \text{ mm } \textbf{Ans.}$$

***2. Free length of the spring***

Let us assume the active number of coils ($n$) = 8. Therefore compression produced by an axial thrust of 281.5 N per spring,

$$\delta = \frac{8\, W_1 . D^3 . n}{G . d^4} = \frac{8 \times 281.5\, (25)^3\, 8}{84 \times 10^3\, (4.064)^4} = 12.285 \text{ mm}$$

For square and ground ends, the total number of turns of the coil,

$$n' = n + 2 = 8 + 2 = 10$$

We know that free length of the spring,

$$L_F = n'.d + \delta + 0.15\,\delta = 10 \times 4.064 + 12.285 + 0.15 \times 12.285 \text{ mm}$$
$$= 54.77 \text{ mm } \textbf{Ans.}$$

***3. Pitch of the coil***

We know that pitch of the coil

$$= \frac{\text{Free length}}{n' - 1} = \frac{54.77}{10 - 1} = 6.08 \text{ mm } \textbf{Ans.}$$

## 23.13 Energy Stored in Helical Springs of Circular Wire

We know that the springs are used for storing energy which is equal to the work done on it by some external load.

Let $W$ = Load applied on the spring, and

$\delta$ = Deflection produced in the spring due to the load $W$.

Assuming that the load is applied gradually, the energy stored in a spring is,

$$U = \frac{1}{2} W . \delta \qquad \text{...}(i)$$

We have already discussed that the maximum shear stress induced in the spring wire,

$$\tau = K \times \frac{8\, W . D}{\pi\, d^3} \text{ or } W = \frac{\pi\, d^3 . \tau}{8\, K . D}$$

We know that deflection of the spring,

$$\delta = \frac{8\, W . D^3 . n}{G . d^4} = \frac{8 \times \pi\, d^3 . \tau}{8\, K . D} \times \frac{D^3 . n}{G . d^4} = \frac{\pi\, \tau . D^2 . n}{K . d . G}$$

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Substituting the values of $W$ and $\delta$ in equation *(i)*, we have

$$U = \frac{1}{2} \times \frac{\pi\, d^3.\tau}{8\,K.D} \times \frac{\pi\,\tau.D^2.n}{K.d.G}$$

$$= \frac{\tau^2}{4\,K^2.G}(\pi\,D.n)\left(\frac{\pi}{4} \times d^2\right) = \frac{\tau^2}{4\,K^2.G} \times V$$

where $V$ = Volume of the spring wire

= Length of spring wire × Cross-sectional area of spring wire

$$= (\pi\,D.n)\left(\frac{\pi}{4} \times d^2\right)$$

**Note :** When a load (say $P$) falls on a spring through a height $h$, then the energy absorbed in a spring is given by

$$U = P\,(h + \delta) = \tfrac{1}{2}\,W.\delta$$

where $W$ = Equivalent static load *i.e.* the gradually applied load which shall produce the same effect as by the falling load $P$, and

$\delta$ = Deflection produced in the spring.

*Another view of an automobile shock-absorber*

**Example 23.12.** *Find the maximum shear stress and deflection induced in a helical spring of the following specifications, if it has to absorb 1000 N-m of energy.*

*Mean diameter of spring = 100 mm ; Diameter of steel wire, used for making the spring = 20 mm; Number of coils = 30 ; Modulus of rigidity of steel = 85 kN/mm².*

**Solution.** Given : $U$ = 1000 N-m ; $D$ = 100 mm = 0.1 m ; $d$ = 20 mm = 0.02 m ; $n$ = 30 ; $G = 85\ \text{kN/mm}^2 = 85 \times 10^9\ \text{N/m}^2$

***Maximum shear stress induced***

Let $\tau$ = Maximum shear stress induced.

We know that spring index,

$$C = \frac{D}{d} = \frac{0.1}{0.02} = 5$$

∴ Wahl's stress factor,

$$K = \frac{4C - 1}{4C - 4} + \frac{0.615}{C} = \frac{4 \times 5 - 1}{4 \times 5 - 4} + \frac{0.615}{5} = 1.31$$

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Volume of spring wire,

$$V = (\pi\, D.n)\left(\frac{\pi}{4} \times d^2\right) = (\pi \times 0.1 \times 30)\left[\frac{\pi}{4}\,(0.02)^2\right] \text{m}^3$$
$$= 0.002\,96 \text{ m}^3$$

We know that energy absorbed in the spring ($U$),

$$1000 = \frac{\tau^2}{4K^2.G} \times V = \frac{\tau^2}{4\,(1.31)^2\;85 \times 10^9} \times 0.00296 = \frac{5\,\tau^2}{10^{15}}$$

$\therefore$ $\tau^2 = 1000 \times 10^{15} / 5 = 200 \times 10^{15}$

or $\tau = 447.2 \times 10^6 \text{ N/m}^2 = 447.2 \text{ MPa}$ **Ans.**

***Deflection produced in the spring***

We know that deflection produced in the spring,

$$\delta = \frac{\pi\tau.D^2\,n}{K.d.G} = \frac{\pi \times 447.2 \times 10^6\;(0.1)^2\;30}{1.31 \times 0.02 \times 85 \times 10^9} = 0.1893 \text{ m}$$
$$= 189.3 \text{ mm } \textbf{Ans.}$$

**Example 23.13.** *A closely coiled helical spring is made of 10 mm diameter steel wire, the coil consisting of 10 complete turns with a mean diameter of 120 mm. The spring carries an axial pull of 200 N. Determine the shear stress induced in the spring neglecting the effect of stress concentration. Determine also the deflection in the spring, its stiffness and strain energy stored by it if the modulus of rigidity of the material is 80 kN/mm$^2$.*

**Solution.** Given : $d = 10$ mm ; $n = 10$ ; $D = 120$ mm ; $W = 200$ N ; $G = 80 \text{ kN/mm}^2 = 80 \times 10^3 \text{ N/mm}^2$

***Shear stress induced in the spring neglecting the effect of stress concentration***

We know that shear stress induced in the spring neglecting the effect of stress concentration is,

$$\tau = \frac{8\,W.D}{\pi\,d^3}\left(1 + \frac{d}{2D}\right) = \frac{8 \times 200 \times 120}{\pi\,(10)^3}\left[1 + \frac{10}{2 \times 120}\right] \text{N/mm}^2$$
$$= 61.1 \times 1.04 = 63.54 \text{ N/mm}^2 = 63.54 \text{ MPa } \textbf{Ans.}$$

***Deflection in the spring***

We know that deflection in the spring,

$$\delta = \frac{8\,W.D^3 n}{G.d^4} = \frac{8 \times 200\,(120)^3\;10}{80 \times 10^3\;(10)^4} = 34.56 \text{ mm } \textbf{Ans.}$$

***Stiffness of the spring***

We know that stiffness of the spring

$$= \frac{W}{\delta} = \frac{200}{34.56} = 5.8 \text{ N/mm}$$

***Strain energy stored in the spring***

We know that strain energy stored in the spring,

$$U = \frac{1}{2}\,W.\delta = \frac{1}{2} \times 200 \times 34.56 = 3456 \text{ N-mm} = 3.456 \text{ N-m } \textbf{Ans.}$$

**Example 23.14.** *At the bottom of a mine shaft, a group of 10 identical close coiled helical springs are set in parallel to absorb the shock caused by the falling of the cage in case of a failure. The loaded cage weighs 75 kN, while the counter weight has a weight of 15 kN. If the loaded cage falls through a height of 50 metres from rest, find the maximum stress induced in each spring if it is made of 50 mm diameter steel rod. The spring index is 6 and the number of active turns in each spring is 20. Modulus of rigidity, G = 80 kN/mm$^2$.*

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**Solution.** Given : No. of springs = 10 ; $W_1$ = 75 kN = 75 000 N ; $W_2$ = 15 kN = 15 000 N ; $h$ = 50 m = 50 000 mm ; $d$ = 50 mm ; $C$ = 6 ; $n$ = 20 ; $G$ = 80 kN/mm² = $80 \times 10^3$ N/mm²

We know that net weight of the falling load,

$$P = W_1 - W_2 = 75\,000 - 15\,000 = 60\,000 \text{ N}$$

Let $W$ = The equivalent static (or gradually applied) load on each spring which can produce the same effect as by the falling load $P$.

We know that compression produced in each spring,

$$\delta = \frac{8\,W.C^3.n}{G.d} = \frac{8W \times 6^3 \times 20}{80 \times 10^3 \times 50} = 0.008\,64\,W \text{ mm}$$

Since the work done by the falling load is equal to the energy stored in the helical springs which are 10 in number, therefore,

$$P\,(h + \delta) = \frac{1}{2}\,W \times \delta \times 10$$

$$60\,000\,(50\,000 + 0.008\,64\,W) = \frac{1}{2}\,W \times 0.008\,64W \times 10$$

$$3 \times 10^9 + 518.4\,W = 0.0432\,W^2$$

or $\quad W^2 - 12\,000\,W - 69.4 \times 10^9 = 0$

$$\therefore \quad W = \frac{12\,000 \pm \sqrt{(12\,000)^2 + 4 \times 1 \times 69.4 \times 10^9}}{2} = \frac{12\,000 \pm 527\,000}{2}$$

$$= 269\,500 \text{ N} \qquad \text{... (Taking +ve sign)}$$

We know that Wahl's stress factor,

$$K = \frac{4C - 1}{4C - 4} + \frac{0.615}{C} = \frac{4 \times 6 - 1}{4 \times 6 - 4} + \frac{0.615}{6} = 1.25$$

and maximum stress induced in each spring,

$$\tau = K \times \frac{8W.C}{\pi\,d^2} = 1.25 \times \frac{8 \times 269\,500 \times 6}{\pi\,(50)^2} = 2058.6 \text{ N/mm}^2$$

$= 2058.6$ MPa **Ans.**

**Example 23.15.** *A rail wagon of mass 20 tonnes is moving with a velocity of 2 m/s. It is brought to rest by two buffers with springs of 300 mm diameter. The maximum deflection of springs is 250 mm. The allowable shear stress in the spring material is 600 MPa. Design the spring for the buffers.*

*Buffers have springs inside to absorb shock.*

**Solution.** Given : $m$ = 20 $t$ = 20 000 kg ; $v$ = 2 m/s ; $D$ = 300 mm ; $\delta$ = 250 mm ; $\tau$ = 600 MPa = 600 N/mm²

**1.** ***Diameter of the spring wire***

Let $d$ = Diameter of the spring wire.

We know that kinetic energy of the wagon

$$= \frac{1}{2}\,m\,v^2 = \frac{1}{2} \times 20\,000\,(2)^2 = 40\,000 \text{ N-m} = 40 \times 10^6 \text{ N-mm} \qquad \text{...}(i)$$

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Let $W$ be the equivalent load which when applied gradually on each spring causes a deflection of 250 mm. Since there are two springs, therefore

Energy stored in the springs

$$= \frac{1}{2} \times W.\delta \times 2 = W.\delta = W \times 250 = 250\ W \text{ N-mm} \quad \text{...}(ii)$$

From equations *(i)* and *(ii)*, we have

$$\therefore \quad W = 40 \times 10^6 / 250 = 160 \times 10^3 \text{ N}$$

We know that torque transmitted by the spring,

$$T = W \times \frac{D}{2} = 160 \times 10^3 \times \frac{300}{2} = 24 \times 10^6 \text{ N-mm}$$

We also know that torque transmitted by the spring ($T$),

$$24 \times 10^6 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 600 \times d^3 = 117.8\ d^3$$

$$\therefore \quad d^3 = 24 \times 10^6 / 117.8 = 203.7 \times 10^3 \quad \text{or} \quad d = 58.8 \text{ say } 60 \text{ mm}$$ **Ans.**

***2. Number of turns of the spring coil***

Let $n$ = Number of active turns of the spring coil.

We know that the deflection of the spring ($\delta$),

$$250 = \frac{8\ W.D^3.n}{G.d^4} = \frac{8 \times 160 \times 10^3\ (300)^3\ n}{84 \times 10^3\ (60)^4} = 31.7\ n$$

... (Taking $G$ = 84 MPa = $84 \times 10^3$ N/mm$^2$)

$$\therefore \quad n = 250 / 31.7 = 7.88 \text{ say } 8$$ **Ans.**

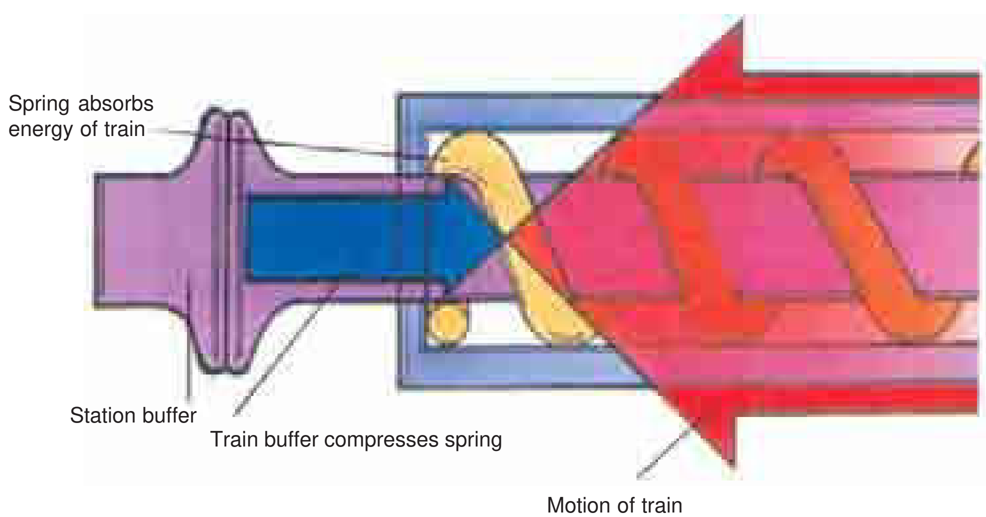
Assuming square and ground ends, total number of turns,

$$n' = n + 2 = 8 + 2 = 10$$ **Ans.**

***3. Free length of the spring***

We know that free length of the spring,

$$L_F = n'.d + \delta + 0.15\ \delta = 10 \times 60 + 250 + 0.15 \times 250 = 887.5 \text{ mm}$$ **Ans.**

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**4. *Pitch of the coil***

We know that pitch of the coil

$$= \frac{\text{Free length}}{n' - 1} = \frac{887.5}{10 - 1} = 98.6 \text{ mm} \quad \textbf{Ans.}$$

## 23.14 Stress and Deflection in Helical Springs of Non-circular Wire

The helical springs may be made of non-circular wire such as rectangular or square wire, in order to provide greater resilience in a given space. However these springs have the following main disadvantages :

1. The quality of material used for springs is not so good.
2. The shape of the wire does not remain square or rectangular while forming helix, resulting in trapezoidal cross-sections. It reduces the energy absorbing capacity of the spring.
3. The stress distribution is not as favourable as for circular wires. But this effect is negligible where loading is of static nature.

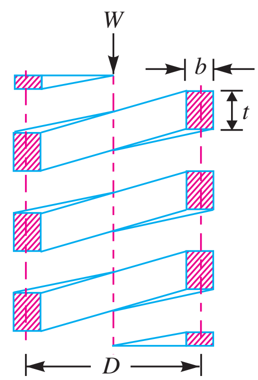



**Fig. 23.18.** Spring of rectangular wire.

For springs made of rectangular wire, as shown in Fig. 23.18, the maximum shear stress is given by

$$\tau = K \times \frac{W.D\,(1.5\,t + 0.9\,b)}{b^2.t^2}$$

This expression is applicable when the longer side (*i.e.* $t > b$) is parallel to the axis of the spring. But when the shorter side (*i.e.* $t < b$) is parallel to the axis of the spring, then maximum shear stress,

$$\tau = K \times \frac{W.D\,(1.5\,b + 0.9\,t)}{b^2.t^2}$$

and deflection of the spring,

$$\delta = \frac{2.45\,W.D^3.n}{G.b^3\,(t - 0.56\,b)}$$

For springs made of square wire, the dimensions $b$ and $t$ are equal. Therefore, the maximum shear stress is given by

$$\tau = K \times \frac{2.4\,W.D}{b^3}$$

and deflection of the spring,

$$\delta = \frac{5.568\,W.D^3.n}{G.b^4} = \frac{5.568\,W.C^3.n}{G.b} \quad \ldots\left(\because C = \frac{D}{b}\right)$$

where $b$ = Side of the square.

**Note :** In the above expressions,

$$K = \frac{4C - 1}{4C - 4} + \frac{0.615}{C}, \text{ and } C = \frac{D}{b}$$

**Example 23.16.** *A loaded narrow-gauge car of mass 1800 kg and moving at a velocity 72 m/min., is brought to rest by a bumper consisting of two helical steel springs of square section. The mean diameter of the coil is six times the side of the square section. In bringing the car to rest, the springs are to be compressed 200 mm. Assuming the allowable shear stress as 365 MPa and spring index of 6, find :*

*1. Maximum load on each spring, 2. Side of the square section of the wire, 3. Mean diameter of coils, and 4. Number of active coils.*

*Take modulus of rigidity as 80 kN/mm*$^2$.

**Solution.** Given : $m$ = 1800 kg ; $v$ = 72 m/min = 1.2 m/s ; $\delta$ = 200 mm ; $\tau$ = 365 MPa = 365 N/mm$^2$ ; $C$ = 6 ; $G$ = 80 kN/mm$^2$ = 80 × 10$^3$ N/mm$^2$

***1. Maximum load on each spring,***

Let $W$ = Maximum load on each spring.

We know that kinetic energy of the car

$$= \frac{1}{2} m.v^2 = \frac{1}{2} \times 1800\ (1.2)^2 = 1296 \text{ N-m} = 1296 \times 10^3 \text{ N-mm}$$

This energy is absorbed in the two springs when compressed to 200 mm. If the springs are loaded gradually from 0 to $W$, then

$$\left(\frac{0 + W}{2}\right) 2 \times 200 = 1296 \times 10^3$$

$\therefore$ $W = 1296 \times 10^3 / 200 = 6480$ N **Ans.**

***2. Side of the square section of the wire***

Let $b$ = Side of the square section of the wire, and

$D$ = Mean diameter of the coil = 6 $b$ ... ($\because$ $C = D/b = 6$)

We know that Wahl's stress factor,

$$K = \frac{4C - 1}{4C - 4} + \frac{0.615}{C} = \frac{4 \times 6 - 1}{4 \times 6 - 4} + \frac{0.615}{6} = 1.2525$$

and maximum shear stres ($\tau$),

$$365 = K \times \frac{2.4\ W.D}{b^3} = 1.2525 \times \frac{2.4 \times 6480 \times 6\ b}{b^3} = \frac{116\ 870}{b^2}$$

$\therefore$ $b^2 = 116\ 870 / 365 = 320$ or $b = 17.89$ say 18 mm **Ans.**

***3. Mean diameter of the coil***

We know that mean diameter of the coil,

$D = 6\ b = 6 \times 18 = 108$ mm **Ans.**

***4. Number of active coils***

Let $n$ = Number of active coils.

We know that the deflection of the spring ($\delta$),

$$200 = \frac{5.568\ W.C^3.n}{G.b} = \frac{5.568 \times 6480 \times 6^3 \times n}{80 \times 10^3 \times 18} = 5.4\ n$$

$\therefore$ $n = 200 / 5.4 = 37$ **Ans.**

## 23.15 Helical Springs Subjected to Fatigue Loading

The helical springs subjected to fatigue loading are designed by using the *Soderberg line method. The spring materials are usually tested for torsional endurance strength under a repeated stress that varies from zero to a maximum. Since the springs are ordinarily loaded in one direction only (the load in springs is never reversed in nature), therefore a modified Soderberg diagram is used for springs, as shown in Fig. 23.19.

The endurance limit for reversed loading is shown at point $A$ where the mean shear stress is equal to $\tau_e / 2$ and the variable shear stress is also equal to $\tau_e / 2$. A line drawn from $A$ to $B$ (the yield point in shear, $\tau_y$) gives the Soderberg's failure stress line. If a suitable factor of safety ($F.S.$) is applied to the yield strength ($\tau_y$), a safe stress line $CD$ may be drawn parallel to the line $AB$, as shown in Fig. 23.19. Consider a design point $P$ on the line $CD$. Now the value of factor of safety may be obtained as discussed below :

* We have discussed the Soderberg method for completely reversed stresses in Chapter 6.

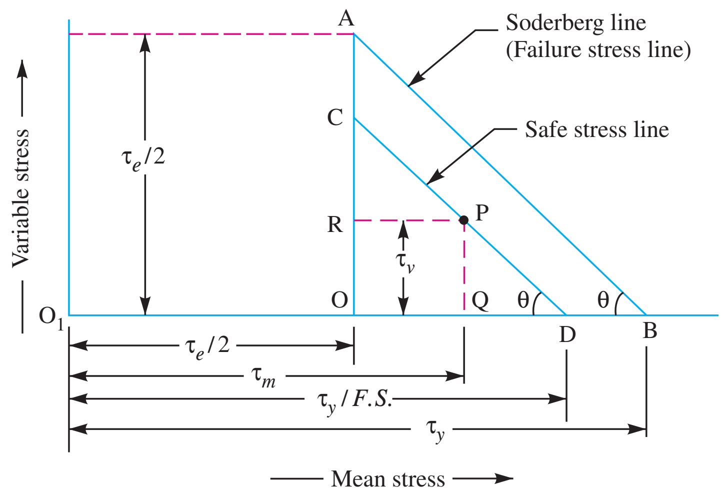



**Fig. 23.19.** Modified Soderberg method for helical springs.

From similar triangles *PQD* and *AOB*, we have

$$\frac{PQ}{QD} = \frac{OA}{OB} \quad \text{or} \quad \frac{PQ}{O_1D - O_1Q} = \frac{OA}{O_1B - O_1O}$$

$$\frac{\tau_v}{\dfrac{\tau_y}{F.S.} - \tau_m} = \frac{\tau_e/2}{\tau_y - \dfrac{\tau_e}{2}} = \frac{\tau_e}{2\tau_y - \tau_e}$$

or
$$2\,\tau_v \,.\, \tau_y - \tau_v .\, \tau_e = \frac{\tau_e \,.\, \tau_y}{F.S.} - \tau_m . \tau_e$$

$\therefore$
$$\frac{\tau_e \,.\, \tau_y}{F.S.} = 2\,\tau_v . \tau_y - \tau_v . \tau_e + \tau_m . \tau_e$$

Dividing both sides by $\tau_e . \tau_y$ and rearranging, we have

$$\frac{1}{F.S.} = \frac{\tau_m - \tau_v}{\tau_y} + \frac{2\tau_v}{\tau_e} \qquad ...(i)$$

**Notes : 1.** From equation (*i*), the expression for the factor of safety (*F.S.*) may be written as

$$F.S. = \frac{\tau_y}{\tau_m - \tau_v + \dfrac{2\,\tau_v . \tau_y}{\tau_e}}$$

**2.** The value of mean shear stress ($\tau_m$) is calculated by using the shear stress factor ($K_S$), while the variable shear stress is calculated by using the full value of the Wahl's factor (*K*). Thus

Mean shear stress,

$$\tau_m = K_s \times \frac{8\,W_m \times D}{\pi d^3}$$

where
$$K_S = 1 + \frac{1}{2C}; \text{ and } W_m = \frac{W_{max} + W_{min}}{2}$$

and variable shear stress,
$$\tau_v = K \times \frac{8\,W_v \times D}{\pi\, d^3}$$

where
$$K = \frac{4C - 1}{4C - 4} + \frac{0.615}{C}; \text{ and } W_v = \frac{W_{max} - W_{min}}{2}$$

**Example 23.17.** *A helical compression spring made of oil tempered carbon steel, is subjected to a load which varies from 400 N to 1000 N. The spring index is 6 and the design factor of safety is 1.25. If the yield stress in shear is 770 MPa and endurance stress in shear is 350 MPa, find : 1. Size of the spring wire, 2. Diameters of the spring, 3. Number of turns of the spring, and 4. Free length of the spring.*

*The compression of the spring at the maximum load is 30 mm. The modulus of rigidity for the spring material may be taken as 80 kN/mm².*

**Solution.** Given : $W_{min}$ = 400 N ; $W_{max}$ = 1000 N ; $C$ = 6 ; $F.S.$ = 1.25 ; $\tau_y$ = 770 MPa = 770 N/mm² ; $\tau_e$ = 350 MPa = 350 N/mm² ; $\delta$ = 30 mm ; $G$ = 80 kN/mm² = 80 × 10³ N/mm²

***1. Size of the spring wire***

Let $d$ = Diameter of the spring wire, and

$D$ = Mean diameter of the spring = $C.d = 6\,d$ ... ($\because D/d = C = 6$)

We know that the mean load,

$$W_m = \frac{W_{max} + W_{min}}{2} = \frac{1000 + 400}{2} = 700 \text{ N}$$

and variable load,

$$W_v = \frac{W_{max} - W_{min}}{2} = \frac{1000 - 400}{2} = 300 \text{ N}$$

Shear stress factor,

$$K_S = 1 + \frac{1}{2C} = 1 + \frac{1}{2 \times 6} = 1.083$$

Wahl's stress factor,

$$K = \frac{4C - 1}{4C - 4} + \frac{0.615}{C} = \frac{4 \times 6 - 1}{4 \times 6 - 4} + \frac{0.615}{6} = 1.2525$$

We know that mean shear stress,

$$\tau_m = K_S \times \frac{8\,W_m \times D}{\pi\,d^3} = 1.083 \times \frac{8 \times 700 \times 6\,d}{\pi\,d^3} = \frac{11\,582}{d^2} \text{ N/mm}^2$$

and variable shear stress,

$$\tau_v = K \times \frac{8\,W_v \times D}{\pi\,d^3} = 1.2525 \times \frac{8 \times 300 \times 6\,d}{\pi\,d^3} = \frac{5740}{d^2} \text{ N/mm}^2$$

We know that

$$\frac{1}{F.S.} = \frac{\tau_m - \tau_v}{\tau_y} + \frac{2\,\tau_v}{\tau_e}$$

$$\frac{1}{1.25} = \frac{\dfrac{11\,582}{d^2} - \dfrac{5740}{d^2}}{770} + \frac{2 \times \dfrac{5740}{d^2}}{350} = \frac{7.6}{d^2} + \frac{32.8}{d^2} = \frac{40.4}{d^2}$$

$\therefore$ $d^2 = 1.25 \times 40.4 = 50.5$ or $d = 7.1$ mm **Ans.**

***2. Diameters of the spring***

We know that mean diameter of the spring,

$D = C.d = 6 \times 7.1 = 42.6$ mm **Ans.**

Outer diameter of the spring,

$D_o = D + d = 42.6 + 7.1 = 49.7$ mm **Ans.**

and inner diameter of the spring,

$D_i = D - d = 42.6 - 7.1 = 35.5$ mm **Ans.**

***3. Number of turns of the spring***

Let $n$ = Number of active turns of the spring.

We know that deflection of the spring (δ),

$$30 = \frac{8\,W.D^3.n}{G.d^4} = \frac{8 \times 1000\,(42.6)^3\;n}{80 \times 10^3\;(7.1)^4} = 3.04\,n$$

$\therefore \quad n = 30 / 3.04 = 9.87$ say 10 **Ans.**

Assuming the ends of the spring to be squared and ground, the total number of turns of the spring,

$$n' = n + 2 = 10 + 2 = 12 \text{ **Ans.**}$$

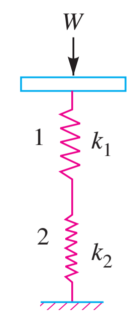
**4. *Free length of the spring***

We know that free length of the spring,

$$L_F = n'.d + \delta + 0.15\,\delta = 12 \times 7.1 + 30 + 0.15 \times 30 \text{ mm}$$
$$= 119.7 \text{ say } 120 \text{ mm}$$ **Ans.**

## 23.16 Springs in Series

Consider two springs connected in series as shown in Fig. 23.20.




Springs in series.
**Fig. 23.20**

Let $W$ = Load carried by the springs,
$\delta_1$ = Deflection of spring 1,
$\delta_2$ = Deflection of spring 2,
$k_1$ = Stiffness of spring 1 = $W / \delta_1$, and
$k_2$ = Stiffness of spring 2 = $W / \delta_2$

A little consideration will show that when the springs are connected in series, then the total deflection produced by the springs is equal to the sum of the deflections of the individual springs.

∴ Total deflection of the springs,

$$\delta = \delta_1 + \delta_2$$

or

$$\frac{W}{k} = \frac{W}{k_1} + \frac{W}{k_2}$$

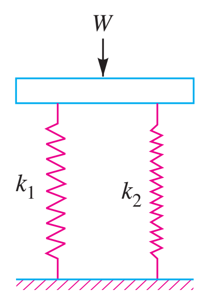
∴

$$\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2}$$

where $k$ = Combined stiffness of the springs.

## 23.17 Springs in Parallel

Consider two springs connected in parallel as shown in Fig 23.21.




Springs in parallel.
**Fig. 23.21**

Let $W$ = Load carried by the springs,
$W_1$ = Load shared by spring 1,
$W_2$ = Load shared by spring 2,
$k_1$ = Stiffness of spring 1, and
$k_2$ = Stiffness of spring 2.

A little consideration will show that when the springs are connected in parallel, then the total deflection produced by the springs is same as the deflection of the individual springs.

We know that $W = W_1 + W_2$

or $\delta.k = \delta.k_1 + \delta.k_2$

$\therefore \quad k = k_1 + k_2$

where $k$ = Combined stiffness of the springs, and
$\delta$ = Deflection produced.

**Example 23.18.** *A close coiled helical compression spring of 12 active coils has a spring stiffness of k. It is cut into two springs having 5 and 7 turns. Determine the spring stiffnesses of resulting springs.*

**Solution.** Given : $n = 12$ ; $n_1 = 5$; $n_2 = 7$

We know that the deflection of the spring,

$$\delta = \frac{8\,W.D^3.n}{G.d^4} \quad \text{or} \quad \frac{W}{\delta} = \frac{G.d^4}{8\,D^3.n}$$

Since $G$, $D$ and $d$ are constant, therefore substituting

$$\frac{G.d^4}{8\,D^3} = X\text{, a constant, we have } \frac{W}{\delta} = k = \frac{X}{n}$$

or $$X = k.n = 12\,k$$

The spring is cut into two springs with $n_1 = 5$ and $n_2 = 7$.

Let $k_1$ = Stiffness of spring having 5 turns, and

$k_2$ = Stiffness of spring having 7 turns.

$$\therefore \quad k_1 = \frac{X}{n_1} = \frac{12\,k}{5} = 2.4\,k \quad \textbf{Ans.}$$

and $$k_2 = \frac{X}{n_2} = \frac{12\,k}{7} = 1.7\,k \quad \textbf{Ans.}$$

## 23.18 Concentric or Composite Springs

A concentric or composite spring is used for one of the following purposes :

**1.** To obtain greater spring force within a given space.

**2.** To insure the operation of a mechanism in the event of failure of one of the springs.

The concentric springs for the above two purposes may have two or more springs and have the same free lengths as shown in Fig. 23.22 (*a*) and are compressed equally. Such springs are used in automobile clutches, valve springs in aircraft, heavy duty diesel engines and rail-road car suspension systems.

Sometimes concentric springs are used to obtain a spring force which does not increase in a direct relation to the deflection but increases faster. Such springs are made of different lengths as shown in Fig. 23.22 (*b*). The shorter spring begins to act only after the longer spring is compressed to a certain amount. These springs are used in governors of variable speed engines to take care of the variable centrifugal force.

*A car shock absorber.*

The adjacent coils of the concentric spring are wound in opposite directions to eliminate any tendency to bind.

If the same material is used, the concentric springs are designed for the same stress. In order to get the same stress factor ($K$), it is desirable to have the same spring index ($C$).

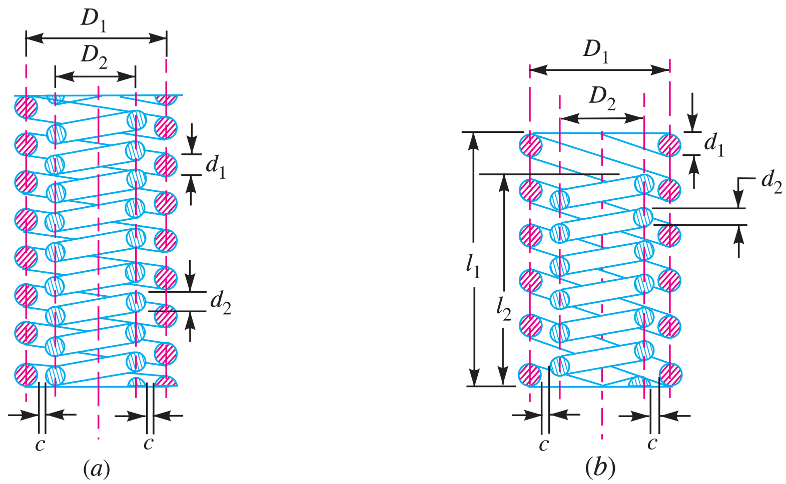



**Fig. 23.22.** Concentric springs.

Consider a concentric spring as shown in Fig. 23.22 (*a*).

Let $W$ = Axial load,

$W_1$ = Load shared by outer spring,

$W_2$ = Load shared by inner spring,

$d_1$ = Diameter of spring wire of outer spring,

$d_2$ = Diameter of spring wire of inner spring,

$D_1$ = Mean diameter of outer spring,

$D_2$ = Mean diameter of inner spring,

$\delta_1$ = Deflection of outer spring,

$\delta_2$ = Deflection of inner spring,

$n_1$ = Number of active turns of outer spring, and

$n_2$ = Number of active turns of inner spring.

Assuming that both the springs are made of same material, then the maximum shear stress induced in both the springs is approximately same, *i.e.*

$$\tau_1 = \tau_2$$

$$\frac{8\,W_1 . D_1 . K_1}{\pi\,(d_1)^3} = \frac{8\,W_2 . D_2 . K_2}{\pi\,(d_2)^3}$$

When stress factor, $K_1 = K_2$, then

$$\frac{W_1 . D_1}{(d_1)^3} = \frac{W_2 . D_2}{(d_2)^3} \qquad ...(i)$$

If both the springs are effective throughout their working range, then their free length and deflection are equal, *i.e.*

$$\delta_1 = \delta_2$$

or $$\frac{8W_1\,(D_1)^3\,n_1}{(d_1)^4\,G} = \frac{8W_2\,(D_2)^3\,n_2}{(d_2)^4\,G} \quad \text{or} \quad \frac{W_1\,(D_1)^3\,n_1}{(d_1)^4} = \frac{W_2\,(D_2)^3\,n_2}{(d_2)^4} \qquad ...(ii)$$

When both the springs are compressed until the adjacent coils meet, then the solid length of both the springs is equal, *i.e.*

$$n_1.d_1 = n_2.d_2$$

∴ The equation **(*ii*)** may be written as

$$\frac{W_1\,(D_1)^3}{(d_1)^5} = \frac{W_2\,(D_2)^3}{(d_2)^5} \qquad ...(iii)$$

Now dividing equation **(*iii*)** by equation **(*i*)**, we have

$$\frac{(D_1)^2}{(d_1)^2} = \frac{(D_2)^2}{(d_2)^2} \quad \text{or} \quad \frac{D_1}{d_1} = \frac{D_2}{d_2} = C, \text{ the spring index} \qquad ...(iv)$$

*i.e.* the springs should be designed in such a way that the spring index for both the springs is same.

From equations **(*i*)** and **(*iv*)**, we have

$$\frac{W_1}{(d_1)^2} = \frac{W_2}{(d_2)^2} \quad \text{or} \quad \frac{W_1}{W_2} = \frac{(d_1)^2}{(d_2)^2} \qquad ..(v)$$

From Fig. 23.22 (*a*), we find that the radial clearance between the two springs,

$$^*c = \left(\frac{D_1}{2} - \frac{D_2}{2}\right) - \left(\frac{d_1}{2} + \frac{d_2}{2}\right)$$

Usually, the radial clearance between the two springs is taken as $\frac{d_1 - d_2}{2}$.

$$\therefore \left(\frac{D_1}{2} - \frac{D_2}{2}\right) - \left(\frac{d_1}{2} + \frac{d_2}{2}\right) = \frac{d_1 - d_2}{2}$$

or
$$\frac{D_1 - D_2}{2} = d_1 \qquad ...(vi)$$

From equation **(*iv*)**, we find that

$$D_1 = C.d_1, \text{ and } D_2 = C.d_2$$

Substituting the values of $D_1$ and $D_2$ in equation **(*vi*)**, we have

$$\frac{C.d_1 - C.d_2}{2} = d_1 \quad \text{or} \quad C.d_1 - 2\,d_1 = C.d_2$$

$$\therefore \quad d_1\,(C-2) = C.d_2 \quad \text{or} \quad \frac{d_1}{d_2} = \frac{C}{C-2} \qquad ...(vii)$$

**Example 23.19.** *A concentric spring for an aircraft engine valve is to exert a maximum force of 5000 N under an axial deflection of 40 mm. Both the springs have same free length, same solid length and are subjected to equal maximum shear stress of 850 MPa. If the spring index for both the springs is 6, find (a) the load shared by each spring, (b) the main dimensions of both the springs, and (c) the number of active coils in each spring.*

*Assume G = 80 kN/mm² and diametral clearance to be equal to the difference between the wire diameters.*

**Solution.** Given : $W$ = 5000 N ; $\delta$ = 40 mm ; $\tau_1 = \tau_2$ = 850 MPa = 850 N/mm² ; $C$ = 6 ; $G$ = 80 kN/mm² = 80 × 10³ N/mm²

The concentric spring is shown in Fig. 23.22 (*a*).

***(a) Load shared by each spring***

Let $W_1$ and $W_2$ = Load shared by outer and inner spring respectively,

$d_1$ and $d_2$ = Diameter of spring wires for outer and inner springs respectively, and

$D_1$ and $D_2$ = Mean diameter of the outer and inner springs respectively.

* The net clearance between the two springs is given by

$$2\,c = (D_1 - D_2) - (d_1 + d_2)$$

Since the diametral clearance is equal to the difference between the wire diameters, therefore

$$(D_1 - D_2) - (d_1 + d_2) = d_1 - d_2$$

or $$D_1 - D_2 = 2\, d_1$$

We know that $D_1 = C.d_1$, and $D_2 = C.d_2$

$\therefore$ $$C.d_1 - C.d_2 = 2\, d_1$$

or $$\frac{d_1}{d_2} = \frac{C}{C-2} = \frac{6}{6-2} = 1.5 \quad ...(i)$$

We also know that $$\frac{W_1}{W_2} = \left(\frac{d_1}{d_2}\right)^2 = (1.5)^2 = 2.25 \quad ...(ii)$$

and $$W_1 + W_2 = W = 5000 \text{ N} \quad ...(iii)$$

From equations *(ii)* and *(iii)*, we find that

$$W_1 = 3462 \text{ N, and } W_2 = 1538 \text{ N}$$ **Ans.**

***(b) Main dimensions of both the springs***

We know that Wahl's stress factor for both the springs,

$$K_1 = K_2 = \frac{4C-1}{4C-4} + \frac{0.615}{C} = \frac{4 \times 6 - 1}{4 \times 6 - 4} + \frac{0.615}{6} = 1.2525$$

and maximum shear stress induced in the outer spring ($\tau_1$),

$$850 = K_1 \times \frac{8\, W_1.C}{\pi (d_1)^2} = 1.2525 \times \frac{8 \times 3462 \times 6}{\pi (d_1)^2} = \frac{66\ 243}{(d_1)^2}$$

$\therefore$ $(d_1)^2 = 66\ 243 / 850 = 78$ or $d_1 = 8.83$ say 10 mm **Ans.**

and $D_1 = C.d_1 = 6\, d_1 = 6 \times 10 = 60$ mm **Ans.**

Similarly, maximum shear stress induced in the inner spring ($\tau_2$),

$$850 = K_2 \times \frac{8 W_2.C}{\pi (d_2)^2} = 1.2525 \times \frac{8 \times 1538 \times 6}{\pi (d_2)^2} = \frac{29\ 428}{(d_2)^2}$$

$\therefore$ $(d_2)^2 = 29\ 428 / 850 = 34.6$ or *$d_2 = 5.88$ say 6 mm **Ans.**

and $D_2 = C.d_2 = 6 \times 6 = 36$ mm **Ans.**

***(c) Number of active coils in each spring***

Let $n_1$ and $n_2$ = Number of active coils of the outer and inner spring respectively.

We know that the axial deflection for the outer spring ($\delta$),

$$40 = \frac{8\, W_1.C^3.n_1}{G.d_1} = \frac{8 \times 3462 \times 6^3 \times n_1}{80 \times 10^3 \times 10} = 7.48\, n_1$$

$\therefore$ $n_1 = 40 / 7.48 = 5.35$ say 6 **Ans.**

Assuming square and ground ends for the spring, the total number of turns of the outer spring,

$$n_1' = 6 + 2 = 8$$

$\therefore$ Solid length of the outer spring,

$$L_{S1} = n_1' \,.\, d_1 = 8 \times 10 = 80 \text{ mm}$$

Let $n_2'$ be the total number of turns of the inner spring. Since both the springs have the same solid length, therefore,

$$n_2'.d_2 = n_1'.d_1$$

* The value of $d_2$ may also be obtained from equation *(i)*, *i.e.*

$$\frac{d_1}{d_2} = 1.5 \quad \text{or} \quad d_2 = \frac{d_1}{1.5} = \frac{8.83}{1.5} = 5.887 \text{ say 6 mm}$$

or $$n_2' = \frac{n_1'.d_1}{d_2} = \frac{8 \times 10}{6} = 13.3 \text{ say } 14$$

and $$n_2 = 14 - 2 = 12 \textbf{ Ans.} \qquad \text{... } (\because n_2' = n_2 + 2)$$

Since both the springs have the same free length, therefore

Free length of outer spring

= Free length of inner spring

$= L_{S1} + \delta + 0.15\,\delta = 80 + 40 + 0.15 \times 40 = 126$ mm **Ans.**

Other dimensions of the springs are as follows:

Outer diameter of the outer spring

$= D_1 + d_1 = 60 + 10 = 70$ mm **Ans.**

Inner diameter of the outer spring

$= D_1 - d_1 = 60 - 10 = 50$ mm **Ans.**

Outer diameter of the inner spring

$= D_2 + d_2 = 36 + 6 = 42$ mm **Ans.**

Inner diameter of the inner spring

$= D_2 - d_2 = 36 - 6 = 30$ mm **Ans.**

*Shock absorbers*

**Example 23.20.** *A composite spring has two closed coil helical springs as shown in Fig. 23.22 (b). The outer spring is 15 mm larger than the inner spring. The outer spring has 10 coils of mean diameter 40 mm and wire diameter 5mm. The inner spring has 8 coils of mean diameter 30 mm and wire diameter 4 mm. When the spring is subjected to an axial load of 400 N, find 1. compression of each spring, 2. load shared by each spring, and 3. shear stress induced in each spring. The modulus of rigidity may be taken as 84 kN/mm².*

**Solution.** Given : $\delta_1 = l_1 - l_2 = 15$ mm ; $n_1 = 10$ ; $D_1 = 40$ mm ; $d_1 = 5$ mm ; $n_2 = 8$ ; $D_2 = 30$ mm ; $d_2 = 4$ mm ; $W = 400$ N ; $G = 84$ kN/mm² $= 84 \times 10^3$ N/mm²

**1. *Compression of each spring***

Since the outer spring is 15 mm larger than the inner spring, therefore the inner spring will not take any load till the outer spring is compressed by 15 mm. After this, both the springs are compressed together. Let $P_1$ be the load on the outer spring to compress it by 15 mm.

We know that compression of the spring ($\delta$),

$$15 = \frac{8\,P_1\,(D_1)^3\,n_1}{G\,(d_1)^4} = \frac{8\,P_1\,(40)^3\,10}{84 \times 10^3 \times 5^4} = 0.0975\,P_1$$

$\therefore$ $$P_1 = 15 / 0.0975 = 154 \text{ N}$$

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Now the remaining load *i.e.* $W - P_1 = 400 - 154 = 246$ N is taken together by both the springs.

Let $\delta_2$ = Further compression of the outer spring or the total compression of the inner spring.

Since for compressing the outer spring by 15 mm, the load required is 154 N, therefore the additional load required by the outer spring to compress it by $\delta_2$ mm is given by

$$P_2 = \frac{P_1}{\delta_1} \times \delta_2 = \frac{154}{15} \times \delta_2 = 10.27\,\delta_2$$

Let $W_2$ = Load taken by the inner spring to compress it by $\delta_2$ mm.

We know that
$$\delta_2 = \frac{8\,W_2\,(D_2)^3\,n_2}{G\,(d_2)^4} = \frac{8\,W_2\,(30)^3\,8}{84 \times 10^3 \times 4^4} = 0.08\,W_2$$

$\therefore$
$$W_2 = \delta_2 / 0.08 = 12.5\,\delta_2$$

and
$$P_2 + W_2 = W - P_1 = 400 - 154 = 246 \text{ N}$$

or
$$10.27\,\delta_2 + 12.5\,\delta_2 = 246 \quad \text{or} \quad \delta_2 = 246 / 22.77 = 10.8 \text{ mm}$$ **Ans.**

$\therefore$ Total compression of the outer spring

$$= \delta_1 + \delta_2 = 15 + 10.8 = 25.8 \text{ mm}$$ **Ans.**

***2. Load shared by each spring***

We know that the load shared by the outer spring,

$$W_1 = P_1 + P_2 = 154 + 10.27\,\delta_2 = 154 + 10.27 \times 10.8 = 265 \text{ N}$$ **Ans.**

and load shared by the inner spring,

$$W_2 = 12.5\,\delta_2 = 12.5 \times 10.8 = 135 \text{ N}$$ **Ans.**

**Note :** The load shared by the inner spring is also given by

$$W_2 = W - W_1 = 400 - 265 = 135 \text{ N}$$ **Ans.**

***3. Shear stress induced in each spring***

We know that the spring index of the outer spring,

$$C_1 = \frac{D_1}{d_1} = \frac{40}{5} = 8$$

and spring index of the inner spring,

$$C_2 = \frac{D_2}{d_2} = \frac{30}{4} = 7.5$$

$\therefore$ Wahl's stress factor for the outer spring,

$$K_1 = \frac{4C_1 - 1}{4C_1 - 4} + \frac{0.615}{C_1} = \frac{4 \times 8 - 1}{4 \times 8 - 4} + \frac{0.615}{8} = 1.184$$

and Wahl's stress factor for the inner spring,

$$K_2 = \frac{4C_2 - 1}{4C_2 - 4} + \frac{0.615}{C_2} = \frac{4 \times 7.5 - 1}{4 \times 7.5 - 4} + \frac{0.615}{7.5} = 1.197$$

We know that shear stress induced in the outer spring,

$$\tau_1 = K_1 \times \frac{8\,W_1.D_1}{\pi\,(d_1)^3} = 1.184 \times \frac{8 \times 265 \times 40}{\pi \times 5^3} = 255.6 \text{ N/mm}^2$$

$$= 255.6 \text{ MPa}$$ **Ans.**

and shear stress induced in the inner spring,

$$\tau_2 = K_2 \times \frac{8\,W_2.D_2}{\pi\,(d_2)^3} = 1.197 \times \frac{8 \times 135 \times 30}{\pi \times 4^3} = 192.86 \text{ N/mm}^2$$

$$= 192.86 \text{ MPa}$$ **Ans.**

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## 23.19 Helical Torsion Springs

The helical torsion springs as shown in Fig. 23.23, may be made from round, rectangular or square wire. These are wound in a similar manner as helical compression or tension springs but the ends are shaped to transmit torque. The primary stress in helical torsion springs is bending stress whereas in compression or tension springs, the stresses are torsional shear stresses. The helical torsion springs are widely used for transmitting small torques as in door hinges, brush holders in electric motors, automobile starters etc.

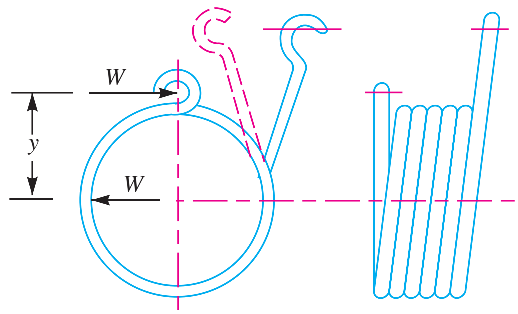


**Fig. 23.23.** Helical torsion spring.

A little consideration will show that the radius of curvature of the coils changes when the twisting moment is applied to the spring. Thus, the wire is under pure bending. According to A.M. Wahl, the bending stress in a helical torsion spring made of round wire is

$$\sigma_b = K \times \frac{32\,M}{\pi\,d^3} = K \times \frac{32\,W.y}{\pi\,d^3}$$

where $K$ = Wahl's stress factor $= \dfrac{4C^2 - C - 1}{4C^2 - 4C}$,

$C$ = Spring index,

$M$ = Bending moment = $W \times y$,

$W$ = Load acting on the spring,

$y$ = Distance of load from the spring axis, and

$d$ = Diameter of spring wire.

and total angle of twist or angular deflection,

$$^*\theta = \frac{M.l}{E.I} = \frac{M \times \pi D.n}{E \times \pi\,d^4/64} = \frac{64\,M.D.n}{E.d^4}$$

where $l$ = Length of the wire = $\pi.D.n$,

$E$ = Young's modulus,

$I$ = Moment of inertia $= \dfrac{\pi}{64} \times d^4$,

$D$ = Diameter of the spring, and

$n$ = Number of turns.

and deflection, $$\delta = \theta \times y = \frac{64\,M.D.n}{E.d^4} \times y$$

When the spring is made of rectangular wire having width $b$ and thickness $t$, then

$$\sigma_b = K \times \frac{6\,M}{t.b^2} = K \times \frac{6\,W \times y}{t.b^2}$$

where $$K = \frac{3C^2 - C - 0.8}{3C^2 - 3C}$$

* We know that $M / I = E / R$, where $R$ is the radius of curvature.

$\therefore$ $R = \dfrac{E.I}{M}$ or $\dfrac{l}{\theta} = \dfrac{E.I}{M}$ or $\theta = \dfrac{M.l}{E.I}$ ...$\left(\because R = \dfrac{l}{\theta}\right)$

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Angular deflection, $\theta = \dfrac{12\,\pi M.D.n}{E.t.b^3}$; and $\delta = \theta.y = \dfrac{12\,\pi M.D.n}{E.t.b^3} \times y$

In case the spring is made of square wire with each side equal to $b$, then substituting $t = b$, in the above relation, we have

$$\sigma_b = K \times \frac{6\,M}{b^3} = K \times \frac{6W \times y}{b^3}$$

$$\theta = \frac{12\,\pi M.D.n}{E.b^4}; \quad \text{and} \quad \delta = \frac{12\,\pi M.D.n}{E.b^4} \times y$$

**Note :** Since the diameter of the spring $D$ reduces as the coils wind up under the applied load, therefore a clearance must be provided when the spring wire is to be wound round a mandrel. A small clearance must also be provided between the adjacent coils in order to prevent sliding friction.

**Example 23.21.** *A helical torsion spring of mean diameter 60 mm is made of a round wire of 6 mm diameter. If a torque of 6 N-m is applied on the spring, find the bending stress induced and the angular deflection of the spring in degrees. The spring index is 10 and modulus of elasticity for the spring material is 200 kN/mm². The number of effective turns may be taken as 5.5.*

**Solution.** Given : $D = 60$ mm ; $d = 6$ mm ; $M = 6$ N-m $= 6000$ N-mm ; $C = 10$ ; $E = 200$ kN/mm$^2$ $= 200 \times 10^3$ N/mm$^2$ ; $n = 5.5$

***Bending stress induced***

We know that Wahl's stress factor for a spring made of round wire,

$$K = \frac{4C^2 - C - 1}{4C^2 - 4C} = \frac{4 \times 10^2 - 10 - 1}{4 \times 10^2 - 4 \times 10} = 1.08$$

∴ Bending stress induced,

$$\sigma_b = K \times \frac{32\,M}{\pi\, d^3} = 1.08 \times \frac{32 \times 6000}{\pi \times 6^3} = 305.5 \text{ N/mm}^2 \text{ or MPa} \quad \textbf{Ans.}$$

***Angular deflection of the spring***

We know that the angular deflection of the spring (in radians),

$$\theta = \frac{64\,M.D.n}{E.d^4} = \frac{64 \times 6000 \times 60 \times 5.5}{200 \times 10^3 \times 6^4} = 0.49 \text{ rad}$$

$$= 0.49 \times \frac{180}{\pi} = 28° \quad \textbf{Ans.}$$

## 23.20 Flat Spiral Spring

A flat spring is a long thin strip of elastic material wound like a spiral as shown in Fig. 23.24. These springs are frequently used in watches and gramophones etc.

When the outer or inner end of this type of spring is wound up in such a way that there is a tendency in the increase of number of spirals of the spring, the strain energy is stored into its spirals. This energy is utilised in any useful way while the spirals open out slowly. Usually the inner end of spring is clamped to an arbor while the outer end may be pinned or clamped. Since the radius of curvature of every spiral decreases when the spring is wound up, therefore the material of the spring is in a state of pure bending.

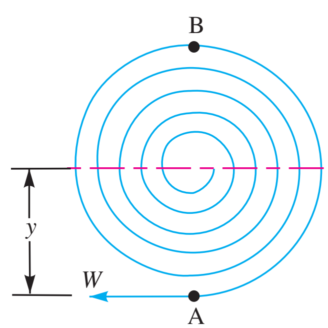


**Fig. 23.24.** Flat spiral spring.

Let $W$ = Force applied at the outer end $A$ of the spring,

$y$ = Distance of centre of gravity of the spring from $A$,

$l$ = Length of strip forming the spring,

$b$ = Width of strip,

$t$ = Thickness of strip,

$I$ = Moment of inertia of the spring section = $b.t^3/12$, and

$Z$ = Section modulus of the spring section = $b.t^2/6$

$$\ldots\left(\because Z = \frac{I}{y} = \frac{b.t^3}{12 \times t/2} = \frac{b.t^2}{6}\right)$$

When the end $A$ of the spring is pulled up by a force $W$, then the bending moment on the spring, at a distance $y$ from the line of action of $W$ is given by

$$M = W \times y$$


*Flat spiral spring of a mechanical clock.*

The greatest bending moment occurs in the spring at $B$ which is at a maximum distance from the application of $W$.

∴ Bending moment at $B$,

$$M_B = M_{max} = W \times 2y = 2W.y = 2M$$

∴ Maximum bending stress induced in the spring material,

$$\sigma_b = \frac{M_{max}}{Z} = \frac{2W \times y}{b.t^2/6} = \frac{12W.y}{b.t^2} = \frac{12M}{b.t^2}$$

Assuming that both ends of the spring are clamped, the angular deflection (in radians) of the spring is given by

$$\theta = \frac{M.l}{E.I} = \frac{12\ M.l}{E.b.t^3} \qquad \ldots\left(\because I = \frac{b.t^3}{12}\right)$$

and the deflection,

$$\delta = \theta \times y = \frac{M.l.y}{E.I}$$

$$= \frac{12\ M.l.y}{E.b.t^3} = \frac{12W.y^2.l}{E.b.t^3} = \frac{\sigma_b.y.l}{E.t} \qquad \ldots\left(\because \sigma_b = \frac{12W.y}{b.t^2}\right)$$

The strain energy stored in the spring

$$= \frac{1}{2}M.\theta = \frac{1}{2}M \times \frac{M.l}{E.I} = \frac{1}{2} \times \frac{M^2.l}{E.I}$$

$$= \frac{1}{2} \times \frac{W^2.y^2.l}{E \times bt^3/12} = \frac{6\ W^2.y^2.l}{E.b.t^3}$$

$$= \frac{6\ W^2.y^2.l}{E.b.t^3} \times \frac{24bt}{24bt} = \frac{144\ W^2 y^2}{Eb^2t^4} \times \frac{b\,t\,l}{24}$$

... (Multiplying the numerator and denominator by 24 $bt$)

$$= \frac{(\sigma_b)^2}{24\ E} \times b\,t\,l = \frac{(\sigma_b)^2}{24\ E} \times \text{Volume of the spring}$$

**Example 23.22.** *A spiral spring is made of a flat strip 6 mm wide and 0.25 mm thick. The length of the strip is 2.5 metres. Assuming the maximum stress of 800 MPa to occur at the point of greatest bending moment, calculate the bending moment, the number of turns to wind up the spring and the strain energy stored in the spring. Take E = 200 kN/mm².*

**Solution.** Given : $b = 6$ mm ; $t = 0.25$ mm ; $l = 2.5$ m = 2500 mm ; $\tau = 800$ MPa = 800 N/mm$^2$ ; $E = 200$ kN/mm$^2$ = $200 \times 10^3$ N/mm$^2$

***Bending moment in the spring***

Let $M$ = Bending moment in the spring.

We know that the maximum bending stress in the spring material ($\sigma_b$),

$$800 = \frac{12\,M}{b.t^2} = \frac{12\,M}{6\,(0.25)^2} = 32\,M$$

$\therefore$ $M = 800 / 32 = 25$ N-mm **Ans.**

***Number of turns to wind up the spring***

We know that the angular deflection of the spring,

$$\theta = \frac{12\,M.l}{E.b.t^3} = \frac{12 \times 25 \times 2500}{200 \times 10^3 \times 6\,(0.25)^3} = 40 \text{ rad}$$

Since one turn of the spring is equal to $2\pi$ radians, therefore number of turns to wind up the spring

$$= 40 / 2\pi = 6.36 \text{ turns } \textbf{Ans.}$$

***Strain energy stored in the spring***

We know that strain energy stored in the spring

$$= \frac{1}{2}\,M.\theta = \frac{1}{2} \times 24 \times 40 = 480 \text{ N-mm } \textbf{Ans.}$$

## 23.21 Leaf Springs

Leaf springs (also known as **flat springs**) are made out of flat plates. The advantage of leaf spring over helical spring is that the ends of the spring may be guided along a definite path as it deflects to act as a structural member in addition to energy absorbing device. Thus the leaf springs may carry lateral loads, brake torque, driving torque etc., in addition to shocks.

Consider a single plate fixed at one end and loaded at the other end as shown in Fig. 23.25. This plate may be used as a flat spring.

Let $t$ = Thickness of plate,

$b$ = Width of plate, and

$L$ = Length of plate or distance of the load $W$ from the cantilever end.

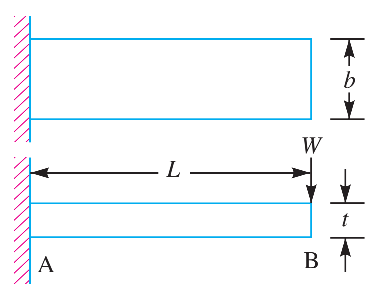


**Fig. 23.25.** Flat spring (cantilever type).

We know that the maximum bending moment at the cantilever end $A$,

$$M = W.L$$

and section modulus,

$$Z = \frac{I}{y} = \frac{b\,t^3/12}{t/2} = \frac{1}{6} \times b.t^2$$

$\therefore$ Bending stress in such a spring,

$$\sigma = \frac{M}{Z} = \frac{W.L}{\frac{1}{6} \times b.t^2} = \frac{6\,W.L}{b.t^2} \quad \text{...}(i)$$

We know that the maximum deflection for a cantilever with concentrated load at the free end is given by

$$\delta = \frac{W.L^3}{3E.I} = \frac{W.L^3}{3E \times b.t^3/12} = \frac{4\,W.L^3}{E.b.t^3} \quad \text{...}(ii)$$

$$= \frac{2\,\sigma.L^2}{3\,E.t} \quad \text{...}\left(\because \sigma = \frac{6W.L}{b.t^2}\right)$$

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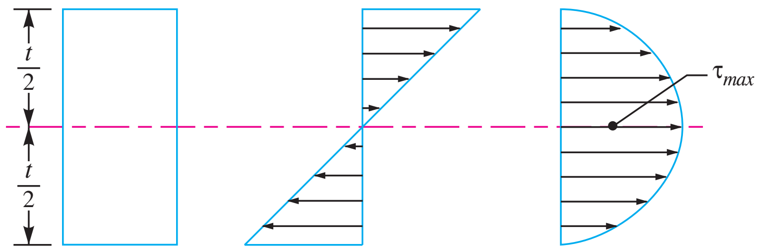
It may be noted that due to bending moment, top fibres will be in tension and the bottom fibres are in compression, but the shear stress is zero at the extreme fibres and maximum at the centre, as shown in Fig. 23.26. Hence for analysis, both stresses need not to be taken into account simultaneously. We shall consider the bending stress only.




(*a*) Cross-section of plate. (*b*) Bending stress diagram. (*c*) Shear stress diagram.

**Fig. 23.26**

If the spring is not of cantilever type but it is like a simply supported beam, with length $2L$ and load $2W$ in the centre, as shown in Fig. 23.27, then

Maximum bending moment in the centre,

$$M = W.L$$

Section modulus, $Z = b.t^2 / 6$

$\therefore$ Bending stress,

$$\sigma = \frac{M}{Z} = \frac{W.L}{b.t^2/6} = \frac{6\,W.L}{b.t^2}$$

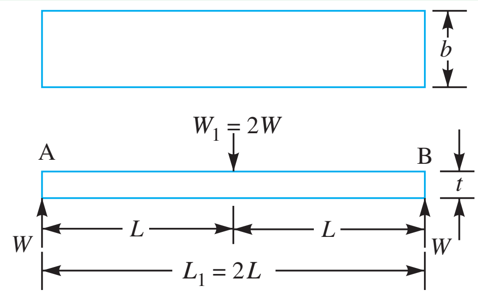



**Fig. 23.27.** Flat spring (simply supported beam type).

We know that maximum deflection of a simply supported beam loaded in the centre is given by

*Leaf spring*

$$\delta = \frac{W_1\,(L_1)^3}{48\,E.I} = \frac{(2W)\,(2L)^3}{48\,E.I} = \frac{W.L^3}{3\,E.I}$$

...($\because$ In this case, $W_1 = 2W$, and $L_1 = 2L$)

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From above we see that a spring such as automobile spring (semi-elliptical spring) with length $2L$ and loaded in the centre by a load $2W$, may be treated as a double cantilever.

If the plate of cantilever is cut into a series of $n$ strips of width $b$ and these are placed as shown in Fig. 23.28, then equations (*i*) and (*ii*) may be written as

$$\sigma = \frac{6\,W.L}{n.b.t^2} \qquad \ldots(iii)$$

and
$$\delta = \frac{4\,W.L^3}{n.E.b.t^3} = \frac{2\,\sigma.L^2}{3\,E.t} \qquad \ldots(iv)$$

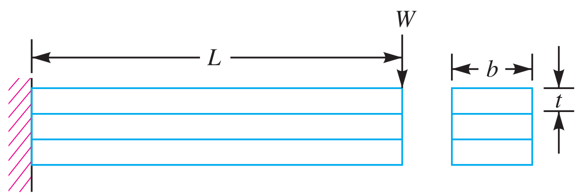


**Fig. 23.28**

The above relations give the stress and deflection of a leaf spring of uniform cross-section. The stress at such a spring is maximum at the support.

If a triangular plate is used as shown in Fig. 23.29 (*a*), the stress will be uniform throughout. If this triangular plate is cut into strips of uniform width and placed one below the other, as shown in Fig. 23.29 (*b*) to form a graduated or laminated leaf spring, then

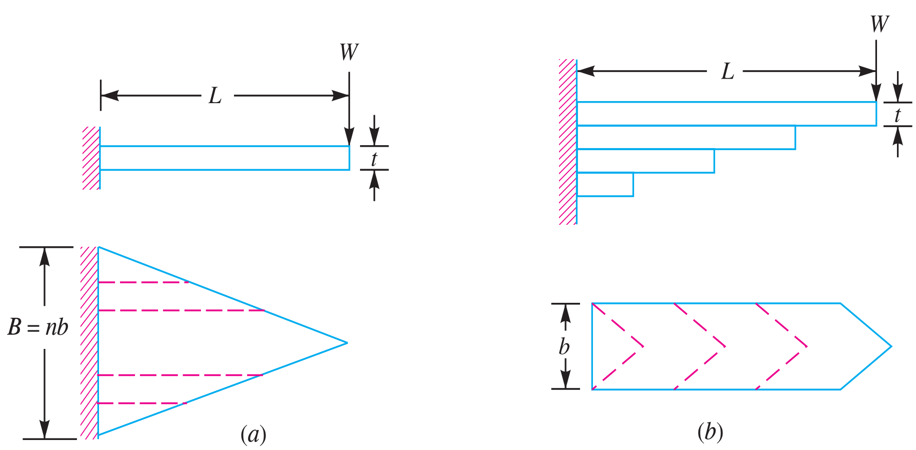


**Fig. 23.29.** Laminated leaf spring.

$$\sigma = \frac{6\,W.L}{n.b.t^2} \qquad \ldots(v)$$

and
$$\delta = \frac{6\,W.L^3}{n.E.b.t^3} = \frac{\sigma.L^2}{E.t} \qquad \ldots(vi)$$

where $n$ = Number of graduated leaves.

A little consideration will show that by the above arrangement, the spring becomes compact so that the space occupied by the spring is considerably reduced.

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When bending stress alone is considered, the graduated leaves may have zero width at the loaded end. But sufficient metal must be provided to support the shear. Therefore, it becomes necessary to have one or more leaves of uniform cross-section extending clear to the end. We see from equations (*iv*) and (*vi*) that for the same deflection, the stress in the uniform cross-section leaves (*i.e.* full length leaves) is 50% greater than in the graduated leaves, assuming that each spring element deflects according to its own elastic curve. If the suffixes $_F$ and $_G$ are used to indicate the full length (or uniform cross-section) and graduated leaves, then

$$\sigma_F = \frac{3}{2}\sigma_G$$

$$\frac{6W_F.L}{n_F.b.t^2} = \frac{3}{2}\left[\frac{6\,W_G.L}{n_G.b.t^2}\right] \quad \text{or} \quad \frac{W_F}{n_F} = \frac{3}{2}\times\frac{W_G}{n_G}$$

$$\therefore \quad \frac{W_F}{W_G} = \frac{3\,n_F}{2\,n_G} \qquad \text{...(vii)}$$

Adding 1 to both sides, we have

$$\frac{W_F}{W_G} + 1 = \frac{3\,n_F}{2\,n_G} + 1 \quad \text{or} \quad \frac{W_F + W_G}{W_G} = \frac{3\,n_F + 2\,n_G}{2\,n_G}$$

$$\therefore \quad W_G = \left(\frac{2\,n_G}{3\,n_F + 2\,n_G}\right)(W_F + W_G) = \left(\frac{2\,n_G}{3\,n_F + 2\,n_G}\right)W \qquad \text{...(viii)}$$

where $W$ = Total load on the spring = $W_G + W_F$

$W_G$ = Load taken up by graduated leaves, and

$W_F$ = Load taken up by full length leaves.

From equation (*vii*), we may write

$$\frac{W_G}{W_F} = \frac{2\,n_G}{3\,n_F}$$

or $$\frac{W_G}{W_F} + 1 = \frac{2\,n_G}{3\,n_F} + 1 \qquad \text{... (Adding 1 to both sides)}$$

$$\frac{W_G + W_F}{W_F} = \frac{2\,n_G + 3\,n_F}{3\,n_F}$$

$$\therefore \quad W_F = \left(\frac{3\,n_F}{2\,n_G + 3\,n_F}\right)(W_G + W_F) = \left(\frac{3\,n_F}{2\,n_G + 3\,n_F}\right)W \qquad \text{...(ix)}$$

∴ Bending stress for full length leaves,

$$\sigma_F = \frac{6\,W_F.L}{n_F.b\,t^2} = \frac{6\,L}{n_F.b.t^2}\left(\frac{3\,n_F}{2\,n_G + 3\,n_F}\right)W = \frac{18\,W.L}{b.t^2\,(2\,n_G + 3\,n_F)}$$

Since $\sigma_F = \frac{3}{2}\sigma_G$, therefore

$$\sigma_G = \frac{2}{3}\sigma_F = \frac{2}{3}\times\frac{18\,W.L}{b.t^2\,(2\,n_G + 3\,n_F)} = \frac{12\,W.L}{b.t^2\,(2\,n_G + 3\,n_F)}$$

The deflection in full length and graduated leaves is given by equation (*iv*), *i.e.*

$$\delta = \frac{2\,\sigma_F \times L^2}{3\,E.t} = \frac{2\,L^2}{3\,E.t}\left[\frac{18\,W.L}{b.t^2\,(2\,n_G + 3\,n_F)}\right] = \frac{12\,W.L^3}{E.b.t^3\,(2\,n_G + 3\,n_F)}$$

## 23.22 Construction of Leaf Spring

A leaf spring commonly used in automobiles is of semi-elliptical form as shown in Fig. 23.30.

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It is built up of a number of plates (known as leaves). The leaves are usually given an initial curvature or cambered so that they will tend to straighten under the load. The leaves are held together by means of a band shrunk around them at the centre or by a bolt passing through the centre. Since the band exerts a stiffening and strengthening effect, therefore the effective length of the spring for bending will be overall length of the spring *minus* width of band. In case of a centre bolt, two-third distance between centres of *U*-bolt should be subtracted from the overall length of the spring in order to find effective length. The spring is clamped to the axle housing by means of *U*-bolts.

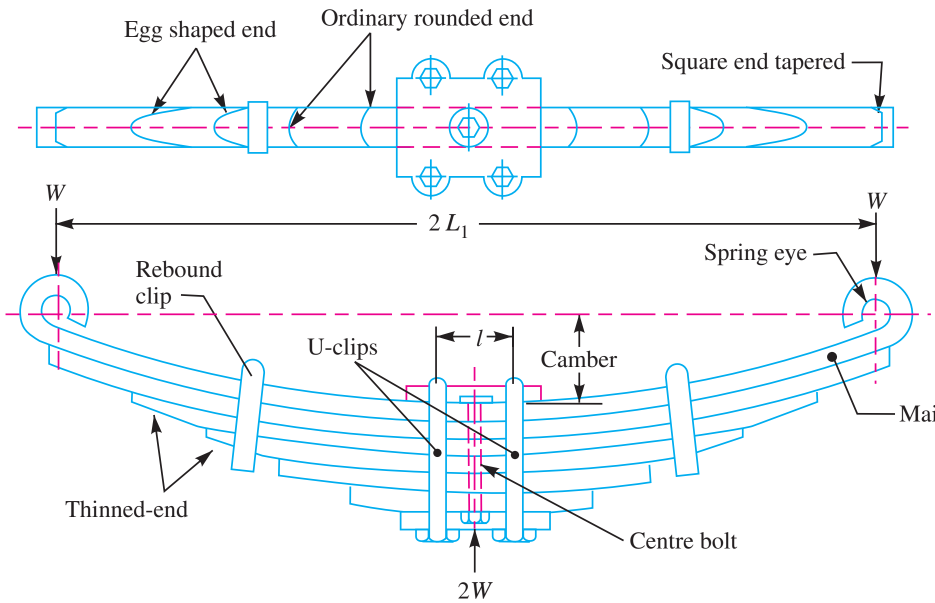


**Fig. 23.30.** Semi-elliptical leaf spring.

*Leaf spring fatigue testing system.*

The longest leaf known as ***main leaf*** or ***master leaf*** has its ends formed in the shape of an eye through which the bolts are passed to secure the spring to its supports. Usually the eyes, through which the spring is attached to the hanger or shackle, are provided with bushings of some antifriction material such as bronze or rubber. The other leaves of the spring are known as ***graduated leaves.*** In order to prevent digging in the adjacent leaves, the ends of the graduated leaves are trimmed in various forms as shown in Fig. 23.30. Since the master leaf has to with stand vertical bending loads as well as loads due to sideways of the vehicle and twisting, therefore due to the presence of stresses caused by these loads, it is usual to provide two full length leaves and the rest graduated leaves as shown in Fig. 23.30.

Rebound clips are located at intermediate positions in the length of the spring, so that the graduated leaves also share the stresses induced in the full length leaves when the spring rebounds.

## 23.23 Equalised Stress in Spring Leaves (Nipping)

We have already discussed that the stress in the full length leaves is 50% greater than the stress in the graduated leaves. In order to utilise the material to the best advantage, all the leaves should be equally stressed. This condition may be obtained in the following two ways :

**1.** By making the full length leaves of smaller thickness than the graduated leaves. In this way, the full length leaves will induce smaller bending stress due to small distance from the neutral axis to the edge of the leaf.

**2.** By giving a greater radius of curvature to the full length leaves than graduated leaves, as shown in Fig. 23.31, before the leaves are assembled to form a spring. By doing so, a gap or clearance will be left between the leaves. This initial gap, as shown by $C$ in Fig. 23.31, is called ***nip.*** When the central bolt, holding the various leaves together, is tightened, the full length leaf will bend back as shown dotted in Fig. 23.31 and have an initial stress in a direction opposite to that of the normal load. The graduated

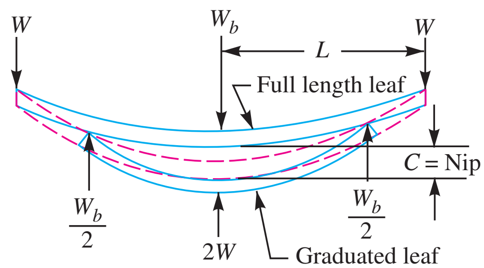


**Fig. 23.31**

leaves will have an initial stress in the same direction as that of the normal load. When the load is gradually applied to the spring, the full length leaf is first relieved of this initial stress and then stressed in opposite direction. Consequently, the full length leaf will be stressed less than the graduated leaf. The initial gap between the leaves may be adjusted so that under maximum load condition the stress in all the leaves is equal, or if desired, the full length leaves may have the lower stress. This is desirable in automobile springs in which full length leaves are designed for lower stress because the full length leaves carry additional loads caused by the swaying of the car, twisting and in some cases due to driving the car through the rear springs. Let us now find the value of initial gap or nip $C$.

Consider that under maximum load conditions, the stress in all the leaves is equal. Then at maximum load, the total deflection of the graduated leaves will exceed the deflection of the full length leaves by an amount equal to the initial gap $C$. In other words,

$$\delta_G = \delta_F + C$$

$$\therefore \quad C = \delta_G - \delta_F = \frac{6\,W_G \cdot L^3}{n_G\, E.b.t^3} - \frac{4\,W_F.L^3}{n_F.E.b.t^3} \qquad ...(i)$$

Since the stresses are equal, therefore

$$\sigma_G = \sigma_F$$

$$\frac{6\,W_G.L}{n_G.b.t^2} = \frac{6\,W_F.L}{n_F.b.t^2} \quad \text{or} \quad \frac{W_G}{n_G} = \frac{W_F}{n_F}$$

$$\therefore \quad W_G = \frac{n_G}{n_F} \times W_F = \frac{n_G}{n} \times W$$

and

$$W_F = \frac{n_F}{n_G} \times W_G = \frac{n_F}{n} \times W$$

Substituting the values of $W_G$ and $W_F$ in equation (*i*), we have

$$C = \frac{6W.L^3}{n.E.b.t^3} - \frac{4W.L^3}{n.E.b.t^3} = \frac{2W.L^3}{n.E.b.t^3} \qquad ...(ii)$$

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The load on the clip bolts ($W_b$) required to close the gap is determined by the fact that the gap is equal to the initial deflections of full length and graduated leaves.

$$\therefore \qquad C = \delta_F + \delta_G$$

$$\frac{2W.L^3}{n.E.b.t^3} = \frac{4L^3}{n_F.E.b.t^3} \times \frac{W_b}{2} + \frac{6\,L^3}{n_G.E.b.t^3} \times \frac{W_b}{2}$$

or
$$\frac{W}{n} = \frac{W_b}{n_F} + \frac{3\,W_b}{2\,n_G} = \frac{2\,n_G.W_b + 3\,n_F.W_b}{2\,n_F.n_G} = \frac{W_b\,(2\,n_G + 3\,n_F)}{2\,n_F.n_G}$$

$$\therefore \qquad W_b = \frac{2\,n_F.n_G.W}{n\,(2\,n_G + 3\,n_F)} \qquad ...(iii)$$

The final stress in spring leaves will be the stress in the full length leaves due to the applied load ***minus*** the initial stress.

$$\therefore \text{ Final stress, } \quad \sigma = \frac{6\,W_F.L}{n_F.b.t^2} - \frac{6\,L}{n_F.b.t^2} \times \frac{W_b}{2} = \frac{6\,L}{n_F.b.t^2}\left(W_F - \frac{W_b}{2}\right)$$

$$= \frac{6\,L}{n_F.b.t^2}\left[\frac{3n_F}{2n_G + 3n_F} \times W - \frac{n_F.n_G.W}{n\,(2n_G + 3n_F)}\right]$$

$$= \frac{6W.L}{b.t^2}\left[\frac{3}{2n_G + 3n_F} - \frac{n_G}{n\,(2n_G + 3n_F)}\right]$$

$$= \frac{6\,W.L}{b.t^2}\left[\frac{3n - n_G}{n\,(2n_G + 3n_F)}\right]$$

$$= \frac{6\,W.L}{b.t^2}\left[\frac{3(n_F + n_G) - n_G}{n\,(2n_G + 3n_F)}\right] = \frac{6\,W.L}{n.b.t^2} \qquad ...(iv)$$

... (Substituting $n = n_F + n_G$)

**Notes : 1.** The final stress in the leaves is also equal to the stress in graduated leaves due to the applied load *plus* the initial stress.

**2.** The deflection in the spring due to the applied load is same as without initial stress.

## 23.24 Length of Leaf Spring Leaves

The length of the leaf spring leaves may be obtained as discussed below :

Let $2L_1$ = Length of span or overall length of the spring,

$l$ = Width of band or distance between centres of $U$-bolts. It is the ineffective length of the spring,

$n_F$ = Number of full length leaves,

$n_G$ = Number of graduated leaves, and

$n$ = Total number of leaves = $n_F + n_G$.

We have already discussed that the effective length of the spring,

$$2L = 2L_1 - l \qquad \text{...(When band is used)}$$

$$= 2L_1 - \frac{2}{3}\,l \qquad \text{... (When } U\text{-bolts are used)}$$

It may be noted that when there is only one full length leaf (*i.e.* master leaf only), then the number of leaves to be cut will be $n$ and when there are two full length leaves (including one master leaf), then the number of leaves to be cut will be $(n - 1)$. If a leaf spring has two full length leaves, then the length of leaves is obtained as follows :

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$$\text{Length of smallest leaf} = \frac{\text{Effective length}}{n-1} + \text{Ineffective length}$$

$$\text{Length of next leaf} = \frac{\text{Effective length}}{n-1} \times 2 + \text{Ineffective length}$$

Similarly, length of $(n-1)$th leaf

$$= \frac{\text{Effective length}}{n-1} \times (n-1) + \text{Ineffective length}$$

The $n$th leaf will be the master leaf and it is of full length. Since the master leaf has eyes on both sides, therefore

$$\text{Length of master leaf} = 2\,L_1 + \pi\,(d + t) \times 2$$

where $d$ = Inside diameter of eye, and

$t$ = Thickness of master leaf.

The approximate relation between the radius of curvature ($R$) and the camber ($y$) of the spring is given by

$$R = \frac{(L_1)^2}{2y}$$

The exact relation is given by

$$y\,(2R + y) = (L_1)^2$$

where $L_1$ = Half span of the spring.

**Note :** The maximum deflection ( $\delta$ ) of the spring is equal to camber ($y$) of the spring.

## 23.25 Standard Sizes of Automobile Suspension Springs

Following are the standard sizes for the automobile suspension springs:

1. Standard nominal widths are : 32, 40*, 45, 50*, 55, 60*, 65, 70*, 75, 80, 90, 100 and 125 mm. (Dimensions marked* are the preferred widths)
2. Standard nominal thicknesses are : 3.2, 4.5, 5, 6, 6.5, 7, 7.5, 8, 9, 10, 11, 12, 14 and 16 mm.
3. At the eye, the following bore diameters are recommended :
   19, 20, 22, 23, 25, 27, 28, 30, 32, 35, 38, 50 and 55 mm.
4. Dimensions for the centre bolts, if employed, shall be as given in the following table.

**Table 23.5. Dimensions for centre bolts.**

| *Width of leaves in mm* | *Dia. of centre bolt in mm* | *Dia. of head in mm* | *Length of bolt head in mm* |
|---|---|---|---|
| Upto and including 65 | 8 or 10 | 12 or 15 | 10 or 11 |
| Above 65 | 12 or 16 | 17 or 20 | 11 |

5. Minimum clip sections and the corresponding sizes of rivets and bolts used with the clips shall be as given in the following table (See Fig. 23.32).

**Table 23.6. Dimensions of clip, rivet and bolts.**

| *Spring width (B) in mm* | *Clip section ($b \times t$) in mm × mm* | *Dia. of rivet ($d_1$) in mm* | *Dia. of bolt ($d_2$) in mm* |
|---|---|---|---|
| Under 50 | 20 × 4 | 6 | 6 |
| 50, 55 and 60 | 25 × 5 | 8 | 8 |
| 65, 70, 75 and 80 | 25 × 6 | 10 | 8 |
| 90, 100 and 125 | 32 × 6 | 10 | 10 |

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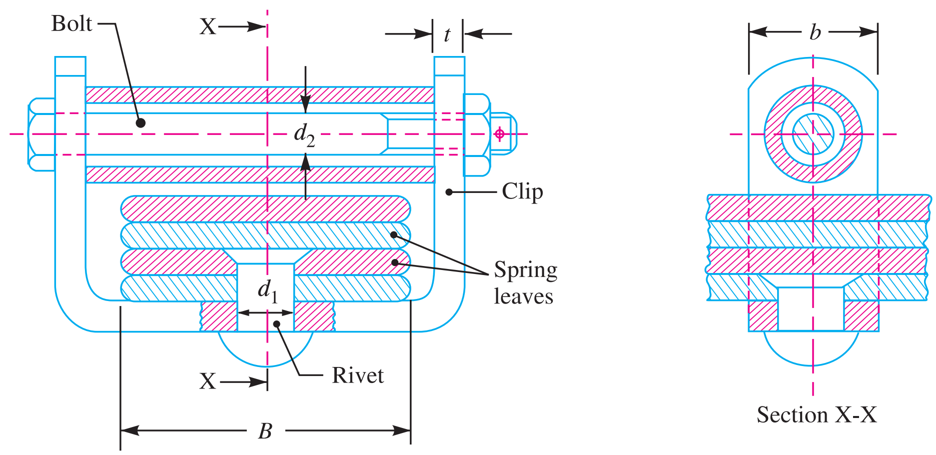


**Fig. 23.32.** Spring clip.

**Notes : 1.** For springs of width below 65 mm, one rivet of 6, 8 or 10 mm may be used. For springs of width above 65 mm, two rivets of 6 or 8 mm or one rivet of 10 mm may be used.

**2.** For further details, the following Indian Standards may be referred :

(*a*) IS : 9484 – 1980 (Reaffirmed 1990) on 'Specification for centre bolts for leaf springs'.

(*b*) IS : 9574 – 1989 (Reaffirmed 1994) on 'Leaf springs assembly-Clips-Specification'.

## 23.26 Materials for Leaf Springs

The material used for leaf springs is usually a plain carbon steel having 0.90 to 1.0% carbon. The leaves are heat treated after the forming process. The heat treatment of spring steel produces greater strength and therefore greater load capacity, greater range of deflection and better fatigue properties.

According to Indian standards, the recommended materials are :

1. For automobiles : 50 Cr 1, 50 Cr 1 V 23, and 55 Si 2 Mn 90 all used in hardened and tempered state.
2. For rail road springs : C 55 (water-hardened), C 75 (oil-hardened), 40 Si 2 Mn 90 (water-hardened) and 55 Si 2 Mn 90 (oil-hardened).
3. The physical properties of some of these materials are given in the following table. All values are for oil quenched condition and for single heat only.

**Table 23.7. Physical properties of materials commonly used for leaf springs.**

| *Material* | *Condition* | *Ultimate tensile strength (MPa)* | *Tensile yield strength (MPa)* | *Brinell hardness number* |
|---|---|---|---|---|
| 50 Cr 1 | Hardened | 1680 – 2200 | 1540 – 1750 | 461 – 601 |
| 50 Cr 1 V 23 | and | 1900 – 2200 | 1680 – 1890 | 534 – 601 |
| 55 Si 2 Mn 90 | tempered | 1820 – 2060 | 1680 – 1920 | 534 – 601 |

**Note :** For further details, Indian Standard [IS : 3431 – 1982 (Reaffirmed 1992)] on 'Specification for steel for the manufacture of volute, helical and laminated springs for automotive suspension' may be referred.

**Example 23.23.** *Design a leaf spring for the following specifications :*

*Total load = 140 kN ; Number of springs supporting the load = 4 ; Maximum number of leaves = 10; Span of the spring = 1000 mm ; Permissible deflection = 80 mm.*

*Take Young's modulus, E = 200 kN/mm*$^2$ *and allowable stress in spring material as 600 MPa.*

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**Solution.** Given : Total load = 140 kN ; No. of springs = 4; $n$ = 10 ; $2L$ = 1000 mm or $L$ = 500 mm ; $\delta$ = 80 mm ; $E$ = 200 kN/mm$^2$ = 200 × 10$^3$ N/mm$^2$ ; $\sigma$ = 600 MPa = 600 N/mm$^2$

We know that load on each spring,

$$2W = \frac{\text{Total load}}{\text{No. of springs}} = \frac{140}{4} = 35 \text{ kN}$$

$\therefore$ $$W = 35 / 2 = 17.5 \text{ kN} = 17\,500 \text{ N}$$

Let $t$ = Thickness of the leaves, and

$b$ = Width of the leaves.

We know that bending stress ($\sigma$),

$$600 = \frac{6\,W.L}{n.b.t^2} = \frac{6 \times 17\,500 \times 500}{n.b.t^2} = \frac{52.5 \times 10^6}{n.b.t^2}$$

$\therefore$ $$n.b.t^2 = 52.5 \times 10^6 / 600 = 87.5 \times 10^3 \quad ...(i)$$

and deflection of the spring ($\delta$),

$$80 = \frac{6\,W.L^3}{n.E.b.t^3} = \frac{6 \times 17\,500\,(500)^3}{n \times 200 \times 10^3 \times b \times t^3} = \frac{65.6 \times 10^6}{n.b.t^3}$$

$\therefore$ $$n.b.t^3 = 65.6 \times 10^6 / 80 = 0.82 \times 10^6 \quad ...(ii)$$

Dividing equation (*ii*) by equation (*i*), we have

$$\frac{n.b.t^3}{n.b.t^2} = \frac{0.82 \times 10^6}{87.5 \times 10^3} \quad \text{or} \quad t = 9.37 \text{ say } 10 \text{ mm} \textbf{ Ans.}$$

Now from equation (*i*), we have

$$b = \frac{87.5 \times 10^3}{n.t^2} = \frac{87.5 \times 10^3}{10\,(10)^2} = 87.5 \text{ mm}$$

and from equation (*ii*), we have

$$b = \frac{0.82 \times 10^6}{n.t^3} = \frac{0.82 \times 10^6}{10\,(10)^3} = 82 \text{ mm}$$

Taking larger of the two values, we have width of leaves,

$$b = 87.5 \text{ say } 90 \text{ mm} \textbf{ Ans.}$$

**Example 23.24.** *A truck spring has 12 number of leaves, two of which are full length leaves. The spring supports are 1.05 m apart and the central band is 85 mm wide. The central load is to be 5.4 kN with a permissible stress of 280 MPa. Determine the thickness and width of the steel spring leaves. The ratio of the total depth to the width of the spring is 3. Also determine the deflection of the spring.*

**Solution.** Given : $n$ = 12 ; $n_F$ = 2 ; $2L_1$ = 1.05 m = 1050 mm ; $l$ = 85 mm ; $2W$ = 5.4 kN = 5400 N or $W$ = 2700 N ; $\sigma_F$ = 280 MPa = 280 N/mm$^2$

***Thickness and width of the spring leaves***

Let $t$ = Thickness of the leaves, and

$b$ = Width of the leaves.

Since it is given that the ratio of the total depth of the spring ($n \times t$) and width of the spring ($b$) is 3, therefore

$$\frac{n \times t}{b} = 3 \quad \text{or} \quad b = n \times t / 3 = 12 \times t / 3 = 4\,t$$

We know that the effective length of the spring,

$$2L = 2L_1 - l = 1050 - 85 = 965 \text{ mm}$$

$\therefore$ $$L = 965 / 2 = 482.5 \text{ mm}$$

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and number of graduated leaves,

$$n_G = n - n_F = 12 - 2 = 10$$

Assuming that the leaves are not initially stressed, therefore maximum stress or bending stress for full length leaves ($\sigma_F$),

$$280 = \frac{18\,W.L}{b.t^2\,(2n_G + 3n_F)} = \frac{18 \times 2700 \times 482.5}{4\,t \times t^2\,(2 \times 10 + 3 \times 2)} = \frac{225\,476}{t^3}$$

∴ $t^3 = 225\,476 / 280 = 805.3$ or $t = 9.3$ say 10 mm **Ans.**

and $b = 4\,t = 4 \times 10 = 40$ mm **Ans.**

*Deflection of the spring*

We know that deflection of the spring,

$$\delta = \frac{12\,W.L^3}{E.b.t^3\,(2n_G + 3n_F)}$$

$$= \frac{12 \times 2700 \times (482.5)^3}{210 \times 10^3 \times 40 \times 10^3\,(2 \times 10 + 3 \times 2)}\text{ mm}$$

$= 16.7$ mm **Ans.** ... (Taking $E = 210 \times 10^3$ N/mm²)

**Example 23.25.** *A locomotive semi-elliptical laminated spring has an overall length of 1 m and sustains a load of 70 kN at its centre. The spring has 3 full length leaves and 15 graduated leaves with a central band of 100 mm width. All the leaves are to be stressed to 400 MPa, when fully loaded. The ratio of the total spring depth to that of width is 2. E = 210 kN/mm². Determine :*

*1. The thickness and width of the leaves.*

*2. The initial gap that should be provided between the full length and graduated leaves before the band load is applied.*

*3. The load exerted on the band after the spring is assembled.*

**Solution.** Given : $2L_1 = 1$ m $= 1000$ mm ; $2W = 70$ kN or $W = 35$ kN $= 35 \times 10^3$ N ; $n_F = 3$ ; $n_G = 15$ ; $l = 100$ mm ; $\sigma = 400$ MPa $= 400$ N/mm² ; $E = 210$ kN/mm² $= 210 \times 10^3$ N/mm²

**1.** *Thickness and width of leaves*

Let $t$ = Thickness of leaves, and

$b$ = Width of leaves.

We know that the total number of leaves,

$$n = n_F + n_G = 3 + 15 = 18$$

Since it is given that ratio of the total spring depth ($n \times t$) and width of leaves is 2, therefore

$$\frac{n \times t}{b} = 2 \quad\text{or}\quad b = n \times t / 2 = 18 \times t / 2 = 9\,t$$

We know that the effective length of the leaves,

$$2L = 2L_1 - l = 1000 - 100 = 900\text{ mm} \quad\text{or}\quad L = 900 / 2 = 450\text{ mm}$$

Since all the leaves are equally stressed, therefore final stress ($\sigma$),

$$400 = \frac{6\,W.L}{n.b.t^2} = \frac{6 \times 35 \times 10^3 \times 450}{18 \times 9\,t \times t^2} = \frac{583 \times 10^3}{t^3}$$

∴ $t^3 = 583 \times 10^3 / 400 = 1458$ or $t = 11.34$ say 12 mm **Ans.**

and $b = 9\,t = 9 \times 12 = 108$ mm **Ans.**

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**2.** ***Initial gap***

We know that the initial gap ($C$) that should be provided between the full length and graduated leaves before the band load is applied, is given by

$$C = \frac{2\,W.L^3}{n.E.b.t^3} = \frac{2 \times 35 \times 10^3\ (450)^3}{18 \times 210 \times 10^3 \times 108\ (12)^3} = 9.04 \text{ mm} \quad \textbf{Ans.}$$

**3.** ***Load exerted on the band after the spring is assembled***

We know that the load exerted on the band after the spring is assembled,

$$W_b = \frac{2\,n_F.n_G.W}{n\,(2n_G + 3n_F)} = \frac{2 \times 3 \times 15 \times 35 \times 10^3}{18\ (2 \times 15 + 3 \times 3)} = 4487 \text{ N} \quad \textbf{Ans.}$$

**Example 23.26.** *A semi-elliptical laminated vehicle spring to carry a load of 6000 N is to consist of seven leaves 65 mm wide, two of the leaves extending the full length of the spring. The spring is to be 1.1 m in length and attached to the axle by two U-bolts 80 mm apart. The bolts hold the central portion of the spring so rigidly that they may be considered equivalent to a band having a width equal to the distance between the bolts. Assume a design stress for spring material as 350 MPa. Determine :*

*1. Thickness of leaves, 2. Deflection of spring, 3. Diameter of eye, 4. Length of leaves, and 5. Radius to which leaves should be initially bent.*

*Sketch the semi-elliptical leaf-spring arrangement.*

*The standard thickness of leaves are : 5, 6, 6.5, 7, 7.5, 8, 9, 10, 11 etc. in mm.*

**Solution.** Given : $2W = 6000$ N or $W = 3000$ N ; $n = 7$ ; $b = 65$ mm ; $n_F = 2$ ; $2L_1 = 1.1$ m $= 1100$ mm or $L_1 = 550$ mm ; $l = 80$ mm ; $\sigma = 350$ MPa $= 350$ N/mm$^2$

**1.** ***Thickness of leaves***

Let $t$ = Thickness of leaves.

We know that the effective length of the spring,

$$2L = 2L_1 - l = 1100 - 80 = 1020 \text{ mm}$$

$$\therefore \quad L = 1020 / 2 = 510 \text{ mm}$$

and number of graduated leaves,

$$n_G = n - n_F = 7 - 2 = 5$$

Assuming that the leaves are not initially stressed, the maximum stress ($\sigma_F$),

$$350 = \frac{18\,W.L}{b.t^2\ (2n_G + 3n_F)} = \frac{18 \times 3000 \times 510}{65 \times t^2\ (2 \times 5 + 3 \times 2)} = \frac{26\,480}{t^2} \quad \text{... } (\sigma_F = \sigma)$$

$$\therefore \quad t^2 = 26\,480 / 350 = 75.66 \quad \text{or} \quad t = 8.7 \text{ say } 9 \text{ mm} \quad \textbf{Ans.}$$

**2.** ***Deflection of spring***

We know that deflection of spring,

$$\delta = \frac{12\,W.L^3}{E.b.t^3\ (2n_G + 3n_F)} = \frac{12 \times 3000\ (510)^3}{210 \times 10^3 \times 65 \times 9^3\ (2 \times 5 + 3 \times 2)}$$

$$= 30 \text{ mm} \quad \textbf{Ans.} \qquad \text{... (Taking } E = 210 \times 10^3 \text{ N/mm}^2\text{)}$$

**3.** ***Diameter of eye***

The inner diameter of eye is obtained by considering the pin in the eye in bearing, because the inner diameter of the eye is equal to the diameter of the pin.

Let $d$ = Inner diameter of the eye or diameter of the pin,

$l_1$ = Length of the pin which is equal to the width of the eye or leaf (*i.e.* $b$) = 65 mm ...(Given)

$p_b$ = Bearing pressure on the pin which may be taken as 8 N/mm$^2$.

We know that the load on pin ($W$),

$$3000 = d \times l_1 \times p_b$$
$$= d \times 65 \times 8 = 520\ d$$
$$\therefore \quad d = 3000 / 520$$
$$= 5.77 \text{ say } 6 \text{ mm}$$

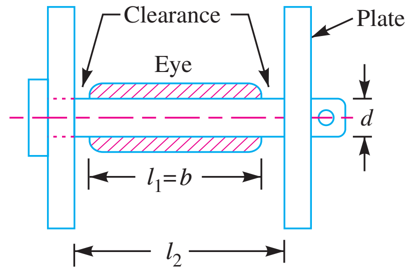


Fig. 23.33

Let us now consider the bending of the pin. Since there is a clearance of about 2 mm between the shackle (or plate) and eye as shown in Fig. 23.33, therefore length of the pin under bending,

$$l_2 = l_1 + 2 \times 2 = 65 + 4 = 69 \text{ mm}$$

*Leaf spring.*

Maximum bending moment on the pin,

$$M = \frac{W \times l_2}{4} = \frac{3000 \times 69}{4} = 51\ 750 \text{ N-mm}$$

and section modulus,
$$Z = \frac{\pi}{32} \times d^3 = 0.0982\ d^3$$

We know that bending stress ($\sigma_b$),

$$80 = \frac{M}{Z} = \frac{51\ 750}{0.0982\ d^3} = \frac{527 \times 10^3}{d^3} \qquad \text{... (Taking } \sigma_b = 80 \text{ N/mm}^2\text{)}$$

$$\therefore \quad d^3 = 527 \times 10^3 / 80 = 6587 \quad \text{or} \quad d = 18.7 \text{ say } 20 \text{ mm}$$ **Ans.**

We shall take the inner diameter of eye or diameter of pin ($d$) as 20 mm **Ans.**

Let us now check the pin for induced shear stress. Since the pin is in double shear, therefore load on the pin ($W$),

$$3000 = 2 \times \frac{\pi}{4} \times d^2 \times \tau = 2 \times \frac{\pi}{4}(20)^2\ \tau = 628.4\ \tau$$

$$\therefore \quad \tau = 3000 / 628.4 = 4.77 \text{ N/mm}^2, \text{ which is safe.}$$

***4. Length of leaves***

We know that ineffective length of the spring

$$= l = 80 \text{ mm} \qquad \text{... } (\because U\text{-bolts are considered equivalent to a band})$$

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$\therefore$ Length of the smallest leaf $= \dfrac{\text{Effective length}}{n-1} + \text{Ineffective length}$

$$= \frac{1020}{7-1} + 80 = 250 \text{ mm} \quad \textbf{Ans.}$$

Length of the 2nd leaf $= \dfrac{1020}{7-1} \times 2 + 80 = 420$ mm **Ans.**

Length of the 3rd leaf $= \dfrac{1020}{7-1} \times 3 + 80 = 590$ mm **Ans.**

Length of the 4th leaf $= \dfrac{1020}{7-1} \times 4 + 80 = 760$ mm **Ans.**

Length of the 5th leaf $= \dfrac{1020}{7-1} \times 5 + 80 = 930$ mm **Ans.**

Length of the 6th leaf $= \dfrac{1020}{7-1} \times 6 + 80 = 1100$ mm **Ans.**

The 6th and 7th leaves are full length leaves and the 7th leaf (*i.e.* the top leaf) will act as a master leaf.

We know that length of the master leaf

$$= 2L_1 + \pi\,(d+t)\,2 = 1100 + \pi\,(20+9)2 = 1282.2 \text{ mm} \quad \textbf{Ans.}$$

**5. *Radius to which the leaves should be initially bent***

Let $R$ = Radius to which the leaves should be initially bent, and

$y$ = Camber of the spring.

We know that

$$y\,(2R - y) = (L_1)^2$$

$$30(2R - 30) = (550)^2 \quad \text{or} \quad 2R - 30 = (550)^2/30 = 10\,083 \qquad \ldots (\because\ y = \delta)$$

$$\therefore \quad R = \frac{10\,083 + 30}{2} = 5056.5 \text{ mm} \quad \textbf{Ans.}$$

## EXERCISES

1. Design a compression helical spring to carry a load of 500 N with a deflection of 25 mm. The spring index may be taken as 8. Assume the following values for the spring material:

   Permissible shear stress = 350 MPa

   Modulus of rigidity = 84 kN/mm$^2$

   Wahl's factor $= \dfrac{4C-1}{4C-4} + \dfrac{0.615}{C}$, where $C$ = spring index.

   **[Ans. $d$ = 5.893 mm ; $D$ = 47.144 mm ; $n$ = 6]**

2. A helical valve spring is to be designed for an operating load range of approximately 90 to 135 N. The deflection of the spring for the load range is 7.5 mm. Assume a spring index of 10. Permissible shear stress for the material of the spring = 480 MPa and its modulus of rigidity = 80 kN/mm$^2$. Design the spring.

   Take Wahl's factor $= \dfrac{4C-1}{4C-4} + \dfrac{0.615}{C}$, $C$ being the spring index.

   **[Ans. $d$ = 2.74 mm ; $D$ = 27.4 mm ; $n$ = 6 ]**

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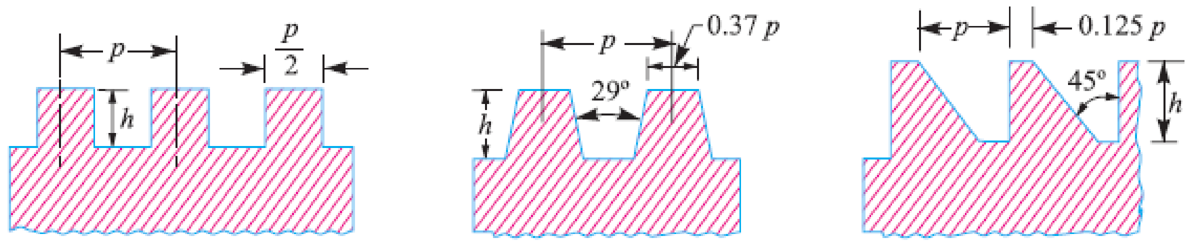
**POWER SCREWS**

## Introduction

The power screws (also known as translation screws) are used to convert rotary motion into translatory motion. For example, in the case of the lead screw of lathe, the rotary motion is available but the tool has to be advanced in the direction of the cut against the cutting resistance of the material. In case of screw jack, a small force applied in the horizontal plane is used to raise or lower a large load. Power screws are also used in vices, testing machines, presses, etc. In most of the power screws, the nut has axial motion against the resisting axial force while the screw rotates in its bearings. In some screws, the screw rotates and moves axially against the resisting force while the nut is stationary and in others the nut rotates while the screw moves axially with no rotation.

## Types of Screw Threads used for Power Screws

Following are the three types of screw threads mostly used for power screws :




$h = 0.5\,p$ (*a*) Square thread. $h = 0.5\,p + 0.25$ mm (*b*) Acme thread. $h = 0.75\,p$ (*c*) Buttress thread.

**1.** ***Square thread.*** A square thread, as shown in Fig. (*a*), is adapted for the transmission of power in either direction. This thread results in maximum efficiency and minimum radial or bursting pressure on the nut. It is difficult to cut with taps and dies. It is usually cut on a lathe with a single point tool and it cannot be easily compensated for wear. The square threads are employed in screw jacks, presses and clamping devices.

**2.** ***Acme or trapezoidal thread.*** An acme or trapezoidal thread, as shown in Fig. (*b*), is a modification of square thread. The slight slope given to its sides lowers the efficiency slightly than square thread and it also introduce some bursting pressure on the nut, but increases its area in shear. It is used where a split nut is required and where provision is made to take up wear as in the lead screw of a lathe. Wear may be taken up by means of an adjustable split nut. An acme thread may be cut by means of dies and hence it is more easily manufactured than square thread.

**3.** ***Buttress thread.*** A buttress thread, as shown in Fig. (*c*), is used when large forces act along the screw axis in one direction only. This thread combines the higher efficiency of square thread and the ease of cutting and the adaptability to a split nut of acme thread. It is stronger than other threads because of greater thickness at the base of the thread. The buttress thread has limited use for power transmission. It is employed as the thread for light jack screws and vices.

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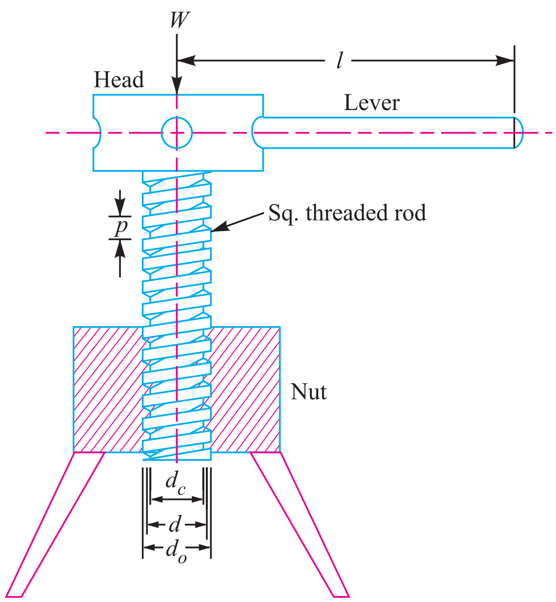
## 17.3 Multiple Threads

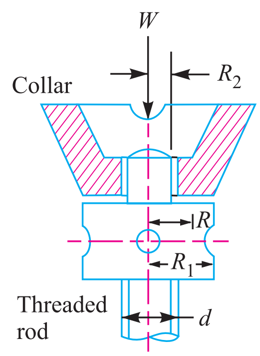
The power screws with multiple threads such as double, triple etc. are employed when it is desired to secure a large lead with fine threads or high efficiency. Such type of threads are usually found in high speed actuators.

## 17.4 Torque Required to Raise Load by Square Threaded Screws

The torque required to raise a load by means of square threaded screw may be determined by considering a screw jack as shown in Fig. 17.2 (*a*). The load to be raised or lowered is placed on the head of the square threaded rod which is rotated by the application of an effort at the end of lever for lifting or lowering the load.



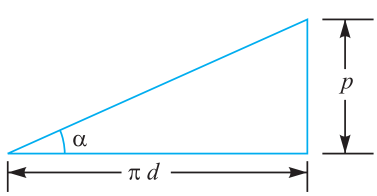

(*a*) Screw jack.



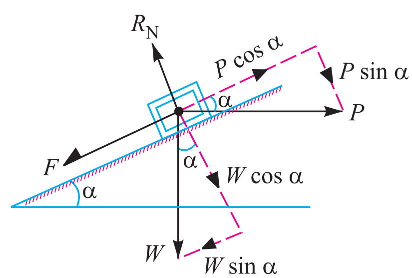

(*b*) Thrust collar.

**Fig. 17.2**

A little consideration will show that if one complete turn of a screw thread be imagined to be unwound, from the body of the screw and developed, it will form an inclined plane as shown in Fig. 17.3 (*a*).




(*a*) Development of a screw.




(*b*) Forces acting on the screw.

**Fig. 17.3**

Let $p$ = Pitch of the screw,

$d$ = Mean diameter of the screw,

$\alpha$ = Helix angle,

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$P$ = Effort applied at the circumference of the screw to lift the load,

$W$ = Load to be lifted, and

= Coefficient of friction, between the screw and nut

= tan $\phi$, where $\phi$ is the friction angle.

From the geometry of the Fig. 17.3 (*a*), we find that

$$\tan \alpha = p / \pi d$$

Since the principle, on which a screw jack works is similar to that of an inclined plane, therefore the force applied on the circumference of a screw jack may be considered to be horizontal as shown in Fig. 17.3 (*b*).

Since the load is being lifted, therefore the force of friction ($F = \mu . R_N$) will act downwards. All the forces acting on the body are shown in Fig. 17.3 (*b*).

Screw jack

Resolving the forces along the plane,

$$P \cos \alpha = W \sin \alpha + F = W \sin \alpha + \mu . R_N \quad \text{...}(i)$$

and resolving the forces perpendicular to the plane,

$$R_N = P \sin \alpha + W \cos \alpha \quad \text{...}(ii)$$

Substituting this value of $R_N$ in equation (*i*), we have

$$P \cos \alpha = W \sin \alpha + \mu (P \sin \alpha + W \cos \alpha)$$
$$= W \sin \alpha + \mu P \sin \alpha + \mu W \cos \alpha$$

or $$P \cos \alpha - \mu P \sin \alpha = W \sin \alpha + \mu W \cos \alpha$$

or $$P (\cos \alpha - \mu \sin \alpha) = W (\sin \alpha + \mu \cos \alpha)$$

$$\therefore \quad P = W \times \frac{(\sin \alpha + \mu \cos \alpha)}{(\cos \alpha - \mu \sin \alpha)}$$

Substituting the value of $\mu = \tan \phi$ in the above equation, we get

or $$P = W \times \frac{\sin \alpha + \tan \phi \cos \alpha}{\cos \alpha - \tan \phi \sin \alpha}$$

Multiplying the numerator and denominator by cos $\phi$, we have

$$P = W \times \frac{\sin \alpha \cos \phi + \sin \phi \cos \alpha}{\cos \alpha \cos \phi - \sin \alpha \sin \phi}$$

$$= W \times \frac{\sin (\alpha + \phi)}{\cos (\alpha + \phi)} = W \tan (\alpha + \phi)$$

$\therefore$ Torque required to overcome friction between the screw and nut,

$$T_1 = P \times \frac{d}{2} = W \tan (\alpha + \phi) \frac{d}{2}$$

When the axial load is taken up by a thrust collar as shown in Fig. 17.2 (*b*), so that the load does not rotate with the screw, then the torque required to overcome friction at the collar,

$$T_2 = \frac{2}{3} \times \mu_1 \times W \left[ \frac{(R_1)^3 - (R_2)^3}{(R_1)^2 - (R_2)^2} \right]$$

... (Assuming uniform pressure conditions)

$$= \mu_1 \times W \left( \frac{R_1 + R_2}{2} \right) = \mu_1 W R$$

....(Assuming uniform wear conditions)

where $R_1$ and $R_2$ = Outside and inside radii of collar,

$R$ = Mean radius of collar $= \frac{R_1 + R_2}{2}$, and

$\mu_1$ = Coefficient of friction for the collar.

∴ Total torque required to overcome friction (*i.e.* to rotate the screw),

$$T = T_1 + T_2$$

If an effort $P_1$ is applied at the end of a lever of arm length $l$, then the total torque required to overcome friction must be equal to the torque applied at the end of lever, *i.e.*

$$T = P \times \frac{d}{2} = P_1 \times l$$

**Notes: 1.** When the *nominal diameter ($d_o$) and the **core diameter ($d_c$) of the screw is given, then

Mean diameter of screw, $$d = \frac{d_o + d_c}{2} = d_o - \frac{p}{2} = d_c + \frac{p}{2}$$

**2.** Since the mechanical advantage is the ratio of the load lifted ($W$) to the effort applied ($P_1$) at the end of the lever, therefore mechanical advantage,

$$\text{M.A.} = \frac{W}{P_1} = \frac{W \times 2\,l}{P \times d} \qquad \ldots\left(\because P \times \frac{d}{2} = P_1 \times l \text{ or } P_1 = \frac{P \times d}{2l}\right)$$

$$= \frac{W \times 2\,l}{W \tan(\alpha + \phi)\, d} = \frac{2\,l}{d \tan(\alpha + \phi)}$$

## 17.5 Torque Required to Lower Load by Square Threaded Screws

A little consideration will show that when the load is being lowered, the force of friction ($F = \mu.R_N$) will act upwards. All the forces acting on the body are shown in Fig. 17.4.

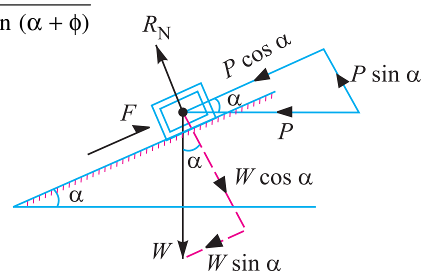


**Fig. 17.4**

Resolving the forces along the plane,

$$P \cos \alpha = F - W \sin \alpha$$

$$= \mu\, R_N - W \sin \alpha \qquad ..(i)$$

and resolving the forces perpendicular to the plane,

$$R_N = W \cos \alpha - P \sin \alpha \qquad ..(ii)$$

Substituting this value of $R_N$ in equation (*i*), we have,

$$P \cos \alpha = \mu\,(W \cos \alpha - P \sin \alpha) - W \sin \alpha$$

$$= \mu\, W \cos \alpha - \mu\, P \sin \alpha - W \sin \alpha$$

or $$P \cos \alpha + \mu\, P \sin \alpha = \mu W \cos \alpha - W \sin \alpha$$

$$P\,(\cos \alpha + \mu \sin \alpha) = W\,(\mu \cos \alpha - \sin \alpha)$$

or $$P = W \times \frac{(\mu \cos \alpha - \sin \alpha)}{(\cos \alpha + \mu \sin \alpha)}$$

Substituting the value of $\mu = \tan \phi$ in the above equation, we have

$$P = W \times \frac{(\tan \phi \cos \alpha - \sin \alpha)}{(\cos \alpha + \tan \phi \sin \alpha)}$$

Multiplying the numerator and denominator by $\cos \phi$, we have

$$P = W \times \frac{(\sin \phi \cos \alpha - \cos \phi \sin \alpha)}{(\cos \phi \cos \alpha + \sin \phi \sin \alpha)}$$

$$= W \times \frac{\sin(\phi - \alpha)}{\cos(\phi - \alpha)} = W \tan(\phi - \alpha)$$

* The nominal diameter of a screw thread is also known as ***outside diameter*** or ***major diameter***.

** The core diameter of a screw thread is also known as ***inner diameter*** or ***root diameter*** or ***minor diameter***.

∴ Torque required to overcome friction between the screw and nut,

$$T_1 = P \times \frac{d}{2} = W \tan(\phi - \alpha)\,\frac{d}{2}$$

**Note :** When $\alpha > \phi$, then $P = W \tan(\alpha - \phi)$.

## 17.6 Efficiency of Square Threaded Screws

The efficiency of square threaded screws may be defined as the ratio between the ideal effort (*i.e.* the effort required to move the load, neglecting friction) to the actual effort (*i.e.* the effort required to move the load taking friction into account).

We have seen in Art. 17.4 that the effort applied at the circumference of the screw to lift the load is

$$P = W \tan(\alpha + \phi) \qquad \text{...}(i)$$

where $W$ = Load to be lifted,

$\alpha$ = Helix angle,

$\phi$ = Angle of friction, and

$\mu$ = Coefficient of friction between the screw and nut = $\tan \phi$.

If there would have been no friction between the screw and the nut, then $\phi$ will be equal to zero. The value of effort $P_0$ necessary to raise the load, will then be given by the equation,

$$P_0 = W \tan \alpha \qquad [\text{Substituting } \phi = 0 \text{ in equation } (i)]$$

∴ Efficiency,

$$\eta = \frac{\text{Ideal effort}}{\text{Actual effort}} = \frac{P_0}{P} = \frac{W \tan\alpha}{W \tan(\alpha + \phi)} = \frac{\tan\alpha}{\tan(\alpha + \phi)}$$

This shows that the efficiency of a screw jack, is independent of the load raised.

In the above expression for efficiency, only the screw friction is considered. However, if the screw friction and collar friction is taken into account, then

$$\eta = \frac{\text{Torque required to move the load, neglecting friction}}{\text{Torque required to move the load, including screw and collar friction}}$$

$$= \frac{T_0}{T} = \frac{P_0 \times d/2}{P \times d/2 + \mu_1 . W . R}$$

**Note:** The efficiency may also be defined as the ratio of mechanical advantage to the velocity ratio.

We know that mechanical advantage,

$$\text{M.A.} = \frac{W}{P_1} = \frac{W \times 2l}{P \times d} = \frac{W \times 2l}{W \tan(\alpha + \phi)\, d} = \frac{2l}{d \tan(\alpha + \phi)} \qquad \text{...(Refer Art .17.4)}$$

and velocity ratio,

$$V.R. = \frac{\text{Distance moved by the effort } (P_1) \text{ in one revolution}}{\text{Distance moved by the load } (W) \text{ in one revolution}}$$

$$= \frac{2\pi l}{p} = \frac{2\pi l}{\tan\alpha \times \pi d} = \frac{2l}{d \tan\alpha} \qquad \text{... } (\because \tan\alpha = p/\pi d)$$

∴ Efficiency,

$$\eta = \frac{M.A.}{V.R.} = \frac{2l}{d \tan(\alpha + \phi)} \times \frac{d \tan\alpha}{2l} = \frac{\tan\alpha}{\tan(\alpha + \phi)}$$

## 17.7 Maximum Efficiency of a Square Threaded Screw

We have seen in Art. 17.6 that the efficiency of a square threaded screw,

$$\eta = \frac{\tan\alpha}{\tan(\alpha + \phi)} = \frac{\sin\alpha / \cos\alpha}{\sin(\alpha + \phi) / \cos(\alpha + \phi)} = \frac{\sin\alpha \times \cos(\alpha + \phi)}{\cos\alpha \times \sin(\alpha + \phi)} \qquad \text{...}(i)$$

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Multiplying the numerator and denominator by 2, we have,

$$\eta = \frac{2\sin\alpha \times \cos(\alpha+\phi)}{2\cos\alpha \times \sin(\alpha+\phi)} = \frac{\sin(2\alpha+\phi) - \sin\phi}{\sin(2\alpha+\phi)+\sin\phi} \quad \text{...(ii)}$$

$$\text{...}\left[\begin{array}{l} \because 2\sin A\cos B = \sin(A+B)+\sin(A-B) \\ 2\cos A\sin B = \sin(A+B)-\sin(A-B)\end{array}\right]$$

The efficiency given by equation *(ii)* will be maximum when $\sin(2\alpha+\phi)$ is maximum, *i.e.* when

$$\sin(2\alpha+\phi) = 1 \quad \text{or} \quad \text{when} \quad 2\alpha+\phi = 90°$$

$$\therefore \quad 2\alpha = 90° - \phi \quad \text{or} \quad \alpha = 45° - \phi/2$$

Substituting the value of $2\alpha$ in equation *(ii)*, we have maximum efficiency,

$$\eta_{max} = \frac{\sin(90°-\phi+\phi)-\sin\phi}{\sin(90°-\phi+\phi)+\sin\phi} = \frac{\sin 90° - \sin\phi}{\sin 90° + \sin\phi} = \frac{1-\sin\phi}{1+\sin\phi}$$

**Example 17.1.** *A vertical screw with single start square threads of 50 mm mean diameter and 12.5 mm pitch is raised against a load of 10 kN by means of a hand wheel, the boss of which is threaded to act as a nut. The axial load is taken up by a thrust collar which supports the wheel boss and has a mean diameter of 60 mm. The coefficient of friction is 0.15 for the screw and 0.18 for the collar. If the tangential force applied by each hand to the wheel is 100 N, find suitable diameter of the hand wheel.*

**Solution.** Given : $d$ = 50 mm ; $p$ = 12.5 mm ; $W$ = 10 kN = 10 × $10^3$N ; $D$ = 60 mm or $R$ = 30 mm ; $\mu = \tan\phi = 0.15$ ; $\mu_1 = 0.18$ ; $P_1$ = 100 N

We know that $\tan\alpha = \dfrac{p}{\pi d} = \dfrac{12.5}{\pi \times 50} = 0.08$

and the tangential force required at the circumference of the screw,

$$P = W\tan(\alpha+\phi) = W\left(\frac{\tan\alpha+\tan\phi}{1-\tan\alpha\tan\phi}\right)$$

$$= 10\times10^3\left[\frac{0.08+0.15}{1-0.08\times0.15}\right] = 2328 \text{ N}$$

We also know that the total torque required to turn the hand wheel,

$$T = P\times\frac{d}{2} + \mu_1 W R = 2328\times\frac{50}{2} + 0.18\times10\times10^3\times30 \text{ N-mm}$$

$$= 58\ 200 + 54\ 000 = 112\ 200 \text{ N-mm} \quad \text{...(i)}$$

Let $D_1$ = Diameter of the hand wheel in mm.

We know that the torque applied to the handwheel,

$$T = 2P_1\times\frac{D_1}{2} = 2\times100\times\frac{D_1}{2} = 100\,D_1 \text{ N-mm} \quad \text{...(ii)}$$

Equating equations *(i)* and *(ii)*,

$D_1$ = 112 200 / 100 = 1122 mm = 1.122 m **Ans.**

**Example 17.2.** *An electric motor driven power screw moves a nut in a horizontal plane against a force of 75 kN at a speed of 300 mm / min. The screw has a single square thread of 6 mm pitch on a major diameter of 40 mm. The coefficient of friction at screw threads is 0.1. Estimate power of the motor.*

**Solution.** Given : $W$ = 75 kN = 75 × $10^3$ N ; $v$ = 300 mm/min ; $p$ = 6 mm ; $d_o$ = 40 mm ; $\mu = \tan\phi = 0.1$

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We know that mean diameter of the screw,

$$d = d_o - p/2 = 40 - 6/2 = 37 \text{ mm}$$

and $$\tan\alpha = \frac{p}{\pi d} = \frac{6}{\pi \times 37} = 0.0516$$

We know that tangential force required at the circumference of the screw,

$$P = W\tan(\alpha + \phi) = W\left[\frac{\tan\alpha + \tan\phi}{1 - \tan\alpha\tan\phi}\right]$$

$$= 75 \times 10^3\left[\frac{0.0516 + 0.1}{1 - 0.0516 \times 0.1}\right] = 11.43 \times 10^3 \text{ N}$$

and torque required to operate the screw,

$$T = P \times \frac{d}{2} = 11.43 \times 10^3 \times \frac{37}{2} = 211.45 \times 10^3 \text{ N-mm} = 211.45 \text{ N-m}$$

Since the screw moves in a nut at a speed of 300 mm / min and the pitch of the screw is 6 mm, therefore speed of the screw in revolutions per minute (r.p.m.),

$$N = \frac{\text{Speed in mm/min.}}{\text{Pitch in mm}} = \frac{300}{6} = 50 \text{ r.p.m.}$$

and angular speed, $$\omega = 2\pi N/60 = 2\pi \times 50/60 = 5.24 \text{ rad/s}$$

∴ Power of the motor $= T.\omega = 211.45 \times 5.24 = 1108 \text{ W} = 1.108 \text{ kW}$ **Ans.**

**Example. 17.3.** *The cutter of a broaching machine is pulled by square threaded screw of 55 mm external diameter and 10 mm pitch. The operating nut takes the axial load of 400 N on a flat surface of 60 mm and 90 mm internal and external diameters respectively. If the coefficient of friction is 0.15 for all contact surfaces on the nut, determine the power required to rotate the operating nut when the cutting speed is 6 m/min. Also find the efficiency of the screw.*

**Solution.** Given : $d_o$ = 55 mm ; $p$ = 10 mm = 0.01 m ; $W$ = 400 N ; $D_2$ = 60 mm or $R_2$ = 30 mm ; $D_1$ = 90 mm or $R_1$ = 45 mm ; $\mu = \tan\phi = \mu_1 = 0.15$ ; Cutting speed = 6 m / min

***Power required to operate the nut***

We know that the mean diameter of the screw,

$$d = d_o - p/2 = 55 - 10/2 = 50 \text{ mm}$$

∴ $$\tan\alpha = \frac{p}{\pi d} = \frac{10}{\pi \times 50} = 0.0637$$

and force required at the circumference of the screw,

$$P = W\tan(\alpha + \phi) = W\left[\frac{\tan\alpha + \tan\phi}{1 - \tan\alpha\tan\phi}\right]$$

$$= 400\left[\frac{0.0637 + 0.15}{1 - 0.0637 \times 0.15}\right] = 86.4 \text{ N}$$

We know that mean radius of the flat surface,

$$R = \frac{R_1 + R_2}{2} = \frac{45 + 30}{2} = 37.5 \text{ mm}$$

∴ Total torque required,

$$T = P \times \frac{d}{2} + \mu_1 W R = 86.4 \times \frac{50}{2} + 0.15 \times 400 \times 37.5 \text{ N-mm}$$

$$= 4410 \text{ N-mm} = 4.41 \text{ N-m}$$

We know that speed of the screw,

$$N = \frac{\text{Cutting speed}}{\text{Pitch}} = \frac{6}{0.01} = 600 \text{ r.p.m}$$

and angular speed, $\omega = 2\pi N / 60 = 2\pi \times 600 / 60 = 62.84$ rad / s

$\therefore$ Power required to operate the nut

$= T.\omega = 4.41 \times 62.84 = 277$ W $= 0.277$ kW **Ans.**

*Efficiency of the screw*

We know that the efficiency of the screw,

$$\eta = \frac{T_0}{T} = \frac{W \tan\alpha \times d/2}{T} = \frac{400 \times 0.0637 \times 50/2}{4410}$$

$= 0.144$ or $14.4\%$ **Ans.**

**Example 17.4.** *A vertical two start square threaded screw of a 100 mm mean diameter and 20 mm pitch supports a vertical load of 18 kN. The axial thrust on the screw is taken by a collar bearing of 250 mm outside diameter and 100 mm inside diameter. Find the force required at the end of a lever which is 400 mm long in order to lift and lower the load. The coefficient of friction for the vertical screw and nut is 0.15 and that for collar bearing is 0.20.*

**Solution.** Given : $d = 100$ mm ; $p = 20$ mm ; $W = 18$ kN $= 18 \times 10^3$N ; $D_1 = 250$ mm or $R_1 = 125$ mm ; $D_2 = 100$ mm or $R_2 = 50$ mm ; $l = 400$ mm ; $\mu = \tan\phi = 0.15$ ; $\mu_1 = 0.20$

*Force required at the end of lever*

Let $P$ = Force required at the end of lever.

Since the screw is a two start square threaded screw, therefore lead of the screw

$= 2p = 2 \times 20 = 40$ mm

We know that $\tan\alpha = \dfrac{\text{Lead}}{\pi d} = \dfrac{40}{\pi \times 100} = 0.127$

**1.** *For raising the load*

We know that tangential force required at the circumference of the screw,

$$P = W\tan(\alpha + \phi) = W\left[\frac{\tan\alpha + \tan\phi}{1 - \tan\alpha\tan\phi}\right]$$

$$= 18 \times 10^3 \left[\frac{0.127 + 0.15}{1 - 0.127 \times 0.15}\right] = 5083 \text{ N}$$

and mean radius of the collar,

$$R = \frac{R_1 + R_2}{2} = \frac{125 + 50}{2} = 87.5 \text{ mm}$$

$\therefore$ Total torque required at the end of lever,

$$T = P \times \frac{d}{2} + \mu_1 WR$$

$$= 5083 \times \frac{100}{2} + 0.20 \times 18 \times 10^3 \times 87.5 = 569\ 150 \text{ N-mm}$$

We know that torque required at the end of lever ($T$),

$569\ 150 = P_1 \times l = P_1 \times 400$ or $P_1 = 569\ 150/400 = 1423$ N **Ans.**

**2.** *For lowering the load*

We know that tangential force required at the circumference of the screw,

$$P = W\tan(\phi - \alpha) = W\left[\frac{\tan\phi - \tan\alpha}{1 + \tan\phi\tan\alpha}\right]$$

$$= 18 \times 10^3 \left[\frac{0.15 - 0.127}{1 + 0.15 \times 0.127}\right] = 406.3 \text{ N}$$

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and the total torque required the end of lever,

$$T = P \times \frac{d}{2} + \mu_1 \, W \, R$$

$$= 406.3 \times \frac{100}{2} + 0.20 \times 18 \times 10^3 \times 87.5 = 335\ 315 \text{ N-mm}$$

We know that torque required at the end of lever ( $T$ ),

$$335\ 315 = P_1 \times l = P_1 \times 400 \quad \text{or} \quad P_1 = 335\ 315 / 400 = 838.3 \text{ N}$$ **Ans.**

**Example 17.5.** The *mean diameter of the square threaded screw having pitch of 10 mm is 50 mm. A load of 20 kN is lifted through a distance of 170 mm. Find the work done in lifting the load and the efficiency of the screw, when*

*1. The load rotates with the screw, and*

*2. The load rests on the loose head which does not rotate with the screw.*

*The external and internal diameter of the bearing surface of the loose head are 60 mm and 10 mm respectively. The coefficient of friction for the screw and the bearing surface may be taken as 0.08.*

**Solution.** Given : $p$ = 10 mm ; $d$ = 50 mm ; $W$ = 20 kN = 20 × $10^3$ N ; $D_1$ = 60 mm or $R_2$ = 30 mm ; $D_2$ = 10 mm or $R_2$ = 5 mm ; $\mu = \tan \phi = \mu_1 = 0.08$

We know that $\tan \alpha = \dfrac{p}{\pi\, d} = \dfrac{10}{\pi \times 50} = 0.0637$

∴ Force required at the circumference of the screw to lift the load,

$$P = W \tan(\alpha + \phi) = W \left[ \frac{\tan \alpha + \tan \phi}{1 - \tan \alpha \tan \phi} \right]$$

$$= 20 \times 10^3 \left[ \frac{0.0637 + 0.08}{1 - 0.0673 \times 0.08} \right] = 2890 \text{ N}$$

and torque required to overcome friction at the screw,

$$T = P \times d / 2 = 2890 \times 50 / 2 = 72\ 250 \text{ N-mm} = 72.25 \text{ N-m}$$

Since the load is lifted through a vertical distance of 170 mm and the distance moved by the screw in one rotation is 10 mm (equal to pitch), therefore number of rotations made by the screw,

$$N = 170 / 10 = 17$$

**1. *When the load rotates with the screw***

We know that workdone in lifting the load

$$= T \times 2\,\pi\,N = 72.25 \times 2\pi \times 17 = 7718 \text{ N-m}$$ **Ans.**

and efficiency of the screw,

$$\eta = \frac{\tan \alpha}{\tan(\alpha + \phi)} = \frac{\tan \alpha\,(1 - \tan \alpha \tan \phi)}{\tan \alpha + \tan \phi}$$

$$= \frac{0.0637\,(1 - 0.0637 \times 0.08)}{0.0637 + 0.08} = 0.441 \text{ or } 44.1\%$$ **Ans.**

**2. *When the load does not rotate with the screw***

We know that mean radius of the bearing surface,

$$R = \frac{R_1 + R_2}{2} = \frac{30 + 5}{2} = 17.5 \text{ mm}$$

and torque required to overcome friction at the screw and the collar,

$$T = P \times \frac{d}{2} + \mu_1 \, W \, R$$

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$$= 2890 \times \frac{50}{2} + 0.08 \times 20 \times 10^3 \times 17.5 = 100\ 250 \text{ N-mm}$$

$$= 100.25 \text{ N-m}$$

∴ Workdone by the torque in lifting the load

$$= T \times 2\pi N = 100.25 \times 2\pi \times 17 = 10\ 710 \text{ N-m} \quad \textbf{Ans.}$$

We know that torque required to lift the load, neglecting friction,

$$T_0 = P_0 \times d/2 = W \tan\alpha \times d/2 \qquad \text{... } (P_o = W \tan\alpha)$$

$$= 20 \times 10^3 \times 0.0637 \times 50/2 = 31\ 850 \text{ N-mm} = 31.85 \text{ N-m}$$

∴ Efficiency of the screw,

$$\eta = \frac{T_0}{T} = \frac{31.85}{100.25} = 0.318 \text{ or } 31.8\% \quad \textbf{Ans.}$$

## 17.8 Efficiency Vs Helix Angle

We have seen in Art. 17.6 that the efficiency of a square threaded screw depends upon the helix angle $\alpha$ and the friction angle $\phi$. The variation of efficiency of a square threaded screw for raising the load with the helix angle $\alpha$ is shown in Fig. 17.5. We see that the efficiency of a square threaded screw increases rapidly upto helix angle of 20°, after which the increase in efficiency is slow. The efficiency is maximum for helix angle between 40 to 45°.

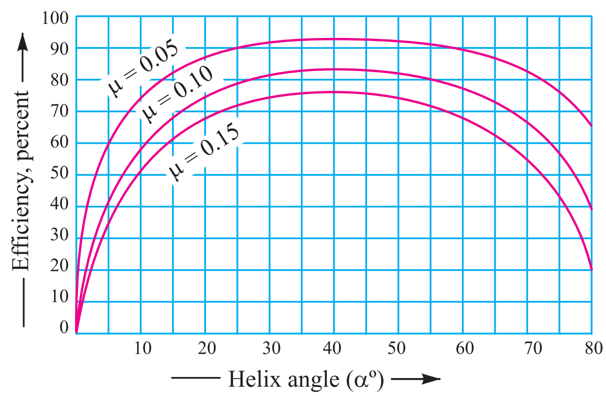


**Fig. 17.5.** Graph between efficiency and helix angle.

When the helix angle further increases say 70°, the efficiency drops. This is due to the fact that the normal thread force becomes large and thus the force of friction and the work of friction becomes large as compared with the useful work. This results in low efficiency.

## 17.9 Over Hauling and Self Locking Screws

We have seen in Art. 17.5 that the effort required at the circumference of the screw to lower the load is

$$P = W \tan(\phi - \alpha)$$

and the torque required to lower the load,

$$T = P \times \frac{d}{2} = W \tan(\phi - \alpha)\frac{d}{2}$$

In the above expression, if $\phi < \alpha$, then torque required to lower the load will be ***negative***. In other words, the load will start moving downward without the application of any torque. Such a condition is known as ***over hauling of screws.*** If however, $\phi > \alpha$, the torque required to lower the load will be ***positive***, indicating that an effort is applied to lower the load. Such a screw is known as

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***self locking screw***. In other words, a screw will be self locking if the friction angle is greater than helix angle or coefficient of friction is greater than tangent of helix angle *i.e.* $\mu$ or $\tan \phi > \tan \alpha$.

## 17.10 Efficiency of Self Locking Screws

We know that the efficiency of screw,

$$\eta = \frac{\tan \phi}{\tan (\alpha + \phi)}$$

and for self locking screws, $\phi \geq \alpha$ or $\alpha \leq \phi$.

$\therefore$ Efficiency for self locking screws,

$$\eta \leq \frac{\tan \phi}{\tan (\phi + \phi)} \leq \frac{\tan \phi}{\tan 2\phi} \leq \frac{\tan \phi (1 - \tan^2 \phi)}{2 \tan \phi} \leq \frac{1}{2} - \frac{\tan^2 \phi}{2}$$

$$\ldots \left( \because \tan 2\phi = \frac{2 \tan \phi}{1 - \tan^2 \phi} \right)$$

From this expression we see that efficiency of self locking screws is less than $\frac{1}{2}$ or 50%. If the efficiency is more than 50%, then the screw is said to be overhauling.

**Note:** It can be proved as follows:

Let $W$ = Load to be lifted, and

$h$ = Distance through which the load is lifted.

$\therefore$ Output = $W.h$

and Input $= \dfrac{\text{Output}}{\eta} = \dfrac{W.h}{\eta}$

$\therefore$ Work lost in overcoming friction

$$= \text{Input} - \text{Output} = \frac{W.h}{\eta} - W.h = W.h \left( \frac{1}{\eta} - 1 \right)$$

For self locking,

$$W.h \left( \frac{1}{\eta} - 1 \right) \leq W.h$$

$$\therefore \quad \frac{1}{\eta} - 1 \leq 1 \quad \text{or} \quad \eta \leq \frac{1}{2} \quad \text{or} \quad 50\%$$

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## 17.11 Coefficient of Friction

The coefficient of friction depends upon various factors like *material of screw and nut, workmanship in cutting screw, quality of lubrication, unit bearing pressure and the rubbing speeds. The value of coefficient of friction does not vary much with different combination of material, load or rubbing speed, except under starting conditions. The coefficient of friction, with good lubrication and average workmanship, may be assumed between 0.10 and 0.15. The various values for coefficient of friction for steel screw and cast iron or bronze nut, under different conditions are shown in the following table.

**Table 17.5. Coefficient of friction under different conditions.**

| *S.No.* | *Condition* | *Average coefficient of friction* | |
|---|---|---|---|
| | | *Starting* | *Running* |
| 1. | High grade materials and workmanship and best running conditions. | 0.14 | 0.10 |
| 2. | Average quality of materials and workmanship and average running conditions. | 0.18 | 0.13 |
| 3. | Poor workmanship or very slow and in frequent motion with indifferent lubrication or newly machined surface. | 0.21 | 0.15 |

If the thrust collars are used, the values of coefficient of friction may be taken as shown in the following table.

**Table 17.6. Coefficient of friction when thrust collars are used.**

| *S.No.* | *Materials* | *Average coefficient of friction* | |
|---|---|---|---|
| | | *Starting* | *Running* |
| 1. | Soft steel on cast iron | 0.17 | 0.12 |
| 2. | Hardened steel on cast iron | 0.15 | 0.09 |
| 3. | Soft steel on bronze | 0.10 | 0.08 |
| 4. | Hardened steel on bronze | 0.08 | 0.06 |

## 17.12 Acme or Trapezoidal Threads

We know that the normal reaction in case of a square threaded screw is

$$R_N = W \cos \alpha,$$

where $\alpha$ is the helix angle.

But in case of Acme or trapezoidal thread, the normal reaction between the screw and nut is increased because the axial component of this normal reaction must be equal to the axial load ($W$).

Consider an Acme or trapezoidal thread as shown in Fig. 17.6.

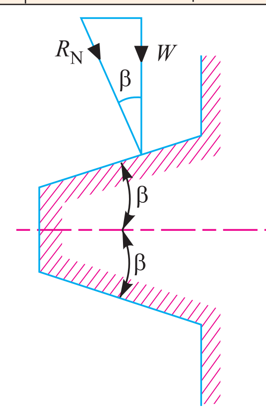


**Fig. 17.6.** Acme or trapeonidal threads.

Let **$2\beta$ = Angle of the Acme thread, and

$\beta$ = Semi-angle of the thread.

* The material of screw is usually steel and the nut is made of cast iron, gun metal, phosphor bronze in order to keep the wear to a mininum.

** For Acme threads, $2\beta = 29°$, and for trapezoidal threads, $2\beta = 30°$.

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$$\therefore \qquad R_N = \frac{W}{\cos \beta}$$

and frictional force, $\quad F = \mu . R_N = \mu \times \frac{W}{\cos \beta} = \mu_1 . W$

where $\quad \mu / \cos \beta = \mu_1$, known as ***virtual coefficient of friction.***

**Notes : 1.** When coefficient of friction, $\mu_1 = \frac{\mu}{\cos \beta}$ is considered, then the Acme thread is equivalent to a square thread.

**2.** All equations of square threaded screw also hold good for Acme threads. In case of Acme threads, $\mu_1$ (*i.e.* tan $\phi_1$) may be substituted in place of $\mu$ (*i.e.* tan $\phi$). Thus for Acme threads,

$$P = W \tan (\alpha + \phi_1)$$

where $\quad \phi_1$ = Virtual friction angle, and $\tan \phi_1 = \mu_1$.

**Example 17.6.** *The lead screw of a lathe has Acme threads of 50 mm outside diameter and 8 mm pitch. The screw must exert an axial pressure of 2500 N in order to drive the tool carriage. The thrust is carried on a collar 110 mm outside diameter and 55 mm inside diameter and the lead screw rotates at 30 r.p.m. Determine* (a) *the power required to drive the screw; and* (b) *the efficiency of the lead screw. Assume a coefficient of friction of 0.15 for the screw and 0.12 for the collar.*

**Solution.** Given : $d_o$ = 50 mm ; $p$ = 8 mm ; $W$ = 2500 N ; $D_1$ = 110 mm or $R_1$ = 55 mm ; $D_2$ = 55 mm or $R_2$ = 27.5 mm ; $N$ = 30 r.p.m. ; $\mu = \tan \phi = 0.15$ ; $\mu_2 = 0.12$

***(a) Power required to drive the screw***

We know that mean diameter of the screw,

$$d = d_o - p / 2 = 50 - 8 / 2 = 46 \text{ mm}$$

$$\therefore \qquad \tan \alpha = \frac{p}{\pi d} = \frac{8}{\pi \times 46} = 0.055$$

Since the angle for Acme threads is $2\beta = 29°$ or $\beta = 14.5°$, therefore virtual coefficient of friction,

$$\mu_1 = \tan \phi_1 = \frac{\mu}{\cos \beta} = \frac{0.15}{\cos 14.5°} = \frac{0.15}{0.9681} = 0.155$$

We know that the force required to overcome friction at the screw,

$$P = W \tan (\alpha + \phi_1) = W \left[ \frac{\tan \alpha + \tan \phi_1}{1 - \tan \alpha \tan \phi_1} \right]$$

$$= 2500 \left[ \frac{0.055 + 0.155}{1 - 0.055 \times 0.155} \right] = 530 \text{ N}$$

and torque required to overcome friction at the screw.

$$T_1 = P \times d / 2 = 530 \times 46 / 2 = 12\ 190 \text{ N-mm}$$

We know that mean radius of collar,

$$R = \frac{R_1 + R_2}{2} = \frac{55 + 27.5}{2} = 41.25 \text{ mm}$$

Assuming uniform wear, the torque required to overcome friction at collars,

$$T_2 = \mu_2 W R = 0.12 \times 2500 \times 41.25 = 12\ 375 \text{ N-mm}$$

$\therefore$ Total torque required to overcome friction,

$$T = T_1 + T_2 = 12\ 190 + 12\ 375 = 24\ 565 \text{ N-mm} = 24.565 \text{ N-m}$$

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We know that power required to drive the screw

$$= T.\omega = \frac{T \times 2\pi N}{60} = \frac{24.565 \times 2\pi \times 30}{60} = 77 \text{ W} = 0.077 \text{ kW} \quad \textbf{Ans.}$$

$$\ldots (\because \omega = 2\pi N / 60)$$

***(b) Efficiency of the lead screw***

We know that the torque required to drive the screw with no friction,

$$T_o = W \tan \alpha \times \frac{d}{2} = 2500 \times 0.055 \times \frac{46}{2} = 3163 \text{ N-mm} = 3.163 \text{ N-m}$$

∴ Efficiency of the lead screw,

$$\eta = \frac{T_o}{T} = \frac{3.163}{24.565} = 0.13 \text{ or } 13\% \quad \textbf{Ans.}$$

## 17.13 Stresses in Power Screws

A power screw must have adequate strength to withstand axial load and the applied torque. Following types of stresses are induced in the screw.

**1. *Direct tensile or compressive stress due to an axial load.*** The direct stress due to the axial load may be determined by dividing the axial load ($W$) by the minimum cross-sectional area of the screw ($A_c$) *i.e.* area corresponding to minor or core diameter ($d_c$).

∴ Direct stress (tensile or compressive)

$$= \frac{W}{A_c}$$

This is only applicable when the axial load is compressive and the unsupported length of the screw between the load and the nut is short. But when the screw is axially loaded in compression and the unsupported length of the screw between the load and the nut is too great, then the design must be based on column theory assuming suitable end conditions. In such cases, the cross-sectional area corresponding to core diameter may be obtained by using Rankine-Gordon formula or J.B. Johnson's formula. According to this,

$$W_{cr} = A_c \times \sigma_y \left[ 1 - \frac{\sigma_y}{4\, C\, \pi^2\, E} \left( \frac{L}{k} \right)^2 \right]$$

$$\therefore \quad \sigma_c = \frac{W}{A_c} \left[ \frac{1}{1 - \dfrac{\sigma_y}{4\, C\, \pi^2\, E} \left( \dfrac{L}{k} \right)^2} \right]$$

where $W_{cr}$ = Critical load,

$\sigma_y$ = Yield stress,

$L$ = Length of screw,

$k$ = Least radius of gyration,

$C$ = End-fixity coefficient,

$E$ = Modulus of elasticity, and

$\sigma_c$ = Stress induced due to load $W$.

**Note :** In actual practice, the core diameter is first obtained by considering the screw under simple compression and then checked for critical load or buckling load for stability of the screw.

**2. *Torsional shear stress.*** Since the screw is subjected to a twisting moment, therefore torsional shear stress is induced. This is obtained by considering the minimum cross-section of the screw. We know that torque transmitted by the screw,

$$T = \frac{\pi}{16} \times \tau\, (d_c)^3$$

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or shear stress induced,

$$\tau = \frac{16\,T}{\pi (d_c)^3}$$

When the screw is subjected to both direct stress and torsional shear stress, then the design must be based on maximum shear stress theory, according to which maximum shear stress on the minor diameter section,

$$\tau_{max} = \frac{1}{2}\sqrt{(\sigma_t \text{ or } \sigma_c)^2 + 4\,\tau^2}$$

It may be noted that when the unsupported length of the screw is short, then failure will take place when the maximum shear stress is equal to the shear yield strength of the material. In this case, shear yield strength,

$$\tau_y = \tau_{max} \times \text{Factor of safety}$$

**3. *Shear stress due to axial load.*** The threads of the screw at the core or root diameter and the threads of the nut at the major diameter may shear due to the axial load. Assuming that the load is uniformly distributed over the threads in contact, we have

Shear stress for screw,

$$\tau_{(screw)} = \frac{W}{\pi\, n\, .d_c\, .\, t}$$

and shear stress for nut,

$$\tau_{(nut)} = \frac{W}{\pi\, n\, .d_o\, .\, t}$$

where $W$ = Axial load on the screw,

$n$ = Number of threads in engagement,

$d_c$ = Core or root diameter of the screw,

$d_o$ = Outside or major diameter of nut or screw, and

$t$ = Thickness or width of thread.

*Friction between the threads of screw and nut plays important role in determining the efficiency and locking properties of a screw*

**4. *Bearing pressure.*** In order to reduce wear of the screw and nut, the bearing pressure on the thread surfaces must be within limits. In the design of power screws, the bearing pressure depends upon the materials of the screw and nut, relative velocity between the nut and screw and the nature of lubrication. Assuming that the load is uniformly distributed over the threads in contact, the bearing pressure on the threads is given by

$$p_b = \frac{W}{\frac{\pi}{4}\left[(d_o)^2 - (d_c)^2\right] n} = \frac{{}^{*}W}{\pi\, d\, .t\, .n}$$

where $d$ = Mean diameter of screw,

$t$ = Thickness or width of screw = $p$ / 2, and

$n$ = Number of threads in contact with the nut

$$= \frac{\text{Height of the nut}}{\text{Pitch of threads}} = \frac{h}{p}$$

Therefore, from the above expression, the height of nut or the length of thread engagement of the screw and nut may be obtained.

The following table shows some limiting values of bearing pressures.

* We know that $\frac{(d_o)^2 - (d_c)^2}{4} = \frac{d_o + d_c}{2} \times \frac{d_o - d_c}{2} = d \times \frac{p}{2} = d.t$

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**Table 17.7. Limiting values of bearing pressures.**

| *Application of screw* | *Material* | | *Safe bearing pressure in N/mm²* | *Rubbing speed at thread pitch diameter* |
|---|---|---|---|---|
| | *Screw* | *Nut* | | |
| 1. Hand press | Steel | Bronze | 17.5 - 24.5 | Low speed, well lubricated |
| 2. Screw jack | Steel | Cast iron | 12.6 – 17.5 | Low speed < 2.4 m / min |
| | Steel | Bronze | 11.2 – 17.5 | Low speed < 3 m / min |
| 3. Hoisting screw | Steel | Cast iron | 4.2 – 7.0 | Medium speed 6 – 12 m / min |
| | Steel | Bronze | 5.6 – 9.8 | Medium speed 6 – 12 m / min |
| 4. Lead screw | Steel | Bronze | 1.05 – 1.7 | High speed > 15 m / min |

**Example 17.7.** *A power screw having double start square threads of 25 mm nominal diameter and 5 mm pitch is acted upon by an axial load of 10 kN. The outer and inner diameters of screw collar are 50 mm and 20 mm respectively. The coefficient of thread friction and collar friction may be assumed as 0.2 and 0.15 respectively. The screw rotates at 12 r.p.m. Assuming uniform wear condition at the collar and allowable thread bearing pressure of 5.8 N/mm², find: 1. the torque required to rotate the screw; 2. the stress in the screw; and 3. the number of threads of nut in engagement with screw.*

**Solution.** Given : $d_o$ = 25 mm ; $p$ = 5 mm ; $W$ = 10 kN = 10 × $10^3$ N ; $D_1$ = 50 mm or $R_1$ = 25 mm ; $D_2$ = 20 mm or $R_2$ = 10 mm ; $\mu = \tan\phi = 0.2$ ; $\mu_1 = 0.15$ ; $N$ = 12 r.p.m. ; $p_b$ = 5.8 N/mm²

**1. *Torque required to rotate the screw***

We know that mean diameter of the screw,

$$d = d_o - p / 2 = 25 - 5 / 2 = 22.5 \text{ mm}$$

Since the screw is a double start square threaded screw, therefore lead of the screw,

$$= 2\,p = 2 \times 5 = 10 \text{ mm}$$

$$\therefore \qquad \tan\alpha = \frac{\text{Lead}}{\pi d} = \frac{10}{\pi \times 22.5} = 0.1414$$

We know that tangential force required at the circumference of the screw,

$$P = W \tan(\alpha + \phi) = W \left[\frac{\tan\alpha + \tan\phi}{1 - \tan\alpha \tan\phi}\right]$$

$$= 10 \times 10^3 \left[\frac{0.1414 + 0.2}{1 - 0.1414 \times 0.2}\right] = 3513 \text{ N}$$

and mean radius of the screw collar,

$$R = \frac{R_1 + R_2}{2} = \frac{25 + 10}{2} = 17.5$$

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∴ Total torque required to rotate the screw,

$$T = P \times \frac{d}{2} + \mu_1 \; W \; R = 3513 \times \frac{22.5}{2} + 0.15 \times 10 \times 10^3 \times 17.5 \text{ N-mm}$$

$$= 65\,771 \text{ N-mm} = 65.771 \text{ N-m}$$ **Ans.**

**2.** ***Stress in the screw***

We know that the inner diameter or core diameter of the screw,

$$d_c = d_o - p = 25 - 5 = 20 \text{ mm}$$

∴ Corresponding cross-sectional area of the screw,

$$A_c = \frac{\pi}{4}(d_c)^2 = \frac{\pi}{4}(20)^2 = 314.2 \text{ mm}^2$$

We know that direct stress,

$$\sigma_c = \frac{W}{A_c} = \frac{10 \times 10^3}{314.2} = 31.83 \text{ N/mm}^2$$

and shear stress, $$\tau = \frac{16\,T}{\pi (d_c)^3} = \frac{16 \times 65771}{\pi (20)^3} = 41.86 \text{ N/mm}^2$$

We know that maximum shear stress in the screw,

$$\tau_{max} = \frac{1}{2}\sqrt{(\sigma_c)^2 + 4\tau^2} = \frac{1}{2}\sqrt{(31.83)^2 + 4(41.86)^2}$$

$$= 44.8 \text{ N/mm}^2 = 44.8 \text{ MPa}$$ **Ans.**

**3.** ***Number of threads of nut in engagement with screw***

Let $n$ = Number of threads of nut in engagement with screw, and

$t$ = Thickness of threads = $p / 2 = 5 / 2 = 2.5$ mm

We know that bearing pressure on the threads ($p_b$),

$$5.8 = \frac{W}{\pi d \times t \times n} = \frac{10 \times 10^3}{\pi \times 22.5 \times 2.5 \times n} = \frac{56.6}{n}$$

∴ $n = 56.6 / 5.8 = 9.76$ say 10 **Ans.**

**Example 17.8.** *The screw of a shaft straightener exerts a load of 30 kN as shown in Fig. 17.7. The screw is square threaded of outside diameter 75 mm and 6 mm pitch. Determine:*

*1. Force required at the rim of a 300 mm diameter hand wheel, assuming the coefficient of friction for the threads as 0.12;*

*2. Maximum compressive stress in the screw, bearing pressure on the threads and maximum shear stress in threads; and*

*3. Efficiency of the straightner.*

**Solution.** Given : $W = 30$ kN $= 30 \times 10^3$ N ; $d_o = 75$ mm ; $p = 6$ mm ; $D = 300$ mm ; $\mu = \tan \phi = 0.12$

**1.** ***Force required at the rim of handwheel***

Let $P_1$ = Force required at the rim of handwheel.

We know that the inner diameter or core diameter of the screw,

$$d_c = d_o - p = 75 - 6 = 69 \text{ mm}$$

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Mean diameter of the screw,

$$*d = \frac{d_o + d_c}{2} = \frac{75 + 69}{2} = 72 \text{ mm}$$

and $$\tan \alpha = \frac{p}{\pi d} = \frac{6}{\pi \times 72} = 0.0265$$

∴ Torque required to overcome friction at the threads,

$$T = P \times \frac{d}{2} = W \tan (\alpha + \phi) \frac{d}{2}$$

$$= W \left( \frac{\tan \alpha + \tan \phi}{1 - \tan \alpha \tan \phi} \right) \frac{d}{2}$$

$$= 30 \times 10^3 \left( \frac{0.0265 + 0.12}{1 - 0.0265 \times 0.12} \right) \frac{72}{2}$$

$$= 158\ 728 \text{ N-mm}$$

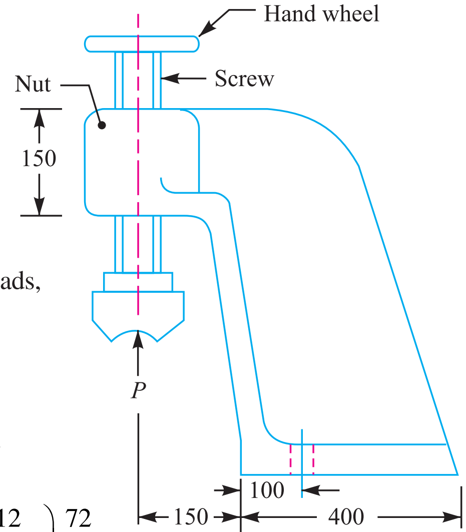


**Fig. 17.7**

We know that the torque required at the rim of handwheel ($T$),

$$158\ 728 = P_1 \times \frac{D}{2} = P_1 \times \frac{300}{2} = 150\ P_1$$

∴ $$P_1 = 158\ 728 / 150 = 1058 \text{ N}$$ **Ans.**

***2. Maximum compressive stress in the screw***

We know that maximum compressive stress in the screw,

$$\sigma_c = \frac{W}{A_c} = \frac{W}{\frac{\pi}{4}(d_c)^2} = \frac{30 \times 10^3}{\frac{\pi}{4}(69)^2} = 8.02 \text{ N/mm}^2 = 8.02 \text{ MPa}$$ **Ans.**

***Bearing pressure on the threads***

We know that number of threads in contact with the nut,

$$n = \frac{\text{Height of nut}}{\text{Pitch of threads}} = \frac{150}{6} = 25 \text{ threads}$$

and thickness of threads, $t = p/2 = 6/2 = 3$ mm

We know that bearing pressure on the threads,

$$p_b = \frac{W}{\pi\, d\,.\,t\,.\,n} = \frac{30 \times 10^3}{\pi \times 72 \times 3 \times 25} = 1.77 \text{ N/mm}^2$$ **Ans.**

***Maximum shear stress in the threads***

We know that shear stress in the threads,

$$\tau = \frac{16\, T}{\pi (d_c)^3} = \frac{16 \times 158\ 728}{\pi\ (69)^3} = 2.46 \text{ N/mm}^2$$ **Ans.**

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* The mean diameter of the screw ($d$) is also given by

$$d = d_o - p/2 = 75 - 6/2 = 72 \text{ mm}$$

∴ Maximum shear stress in the threads,

$$\tau_{max} = \frac{1}{2}\sqrt{(\sigma_c)^2 + 4\,\tau^2} = \frac{1}{2}\sqrt{(8.02)^2 + 4\,(2.46)^2}$$

$$= 4.7 \text{ N/mm}^2 = 4.7 \text{ MPa} \quad \textbf{Ans.}$$

**3. *Efficiency of the straightener***

We know that the torque required with no friction,

$$T_0 = W \tan\alpha \times \frac{d}{2} = 30 \times 10^3 \times 0.0265 \times \frac{72}{2} = 28\,620 \text{ N-mm}$$

∴ Efficiency of the straightener,

$$\eta = \frac{T_0}{T} = \frac{28\,620}{158\,728} = 0.18 \text{ or } 18\% \quad \textbf{Ans.}$$

**Example 17.9.** *A sluice gate weighing 18 kN is raised and lowered by means of square threaded screws, as shown in Fig. 17.8. The frictional resistance induced by water pressure against the gate when it is in its lowest position is 4000 N.*

*The outside diameter of the screw is 60 mm and pitch is 10 mm. The outside and inside diameter of washer is 150 mm and 50 mm respectively. The coefficient of friction between the screw and nut is 0.1 and for the washer and seat is 0.12. Find :*

*1. The maximum force to be exerted at the ends of the lever raising and lowering the gate, 2. Efficiency of the arrangement, and 3. Number of threads and height of nut, for an allowable bearing pressure of 7 N/mm².*

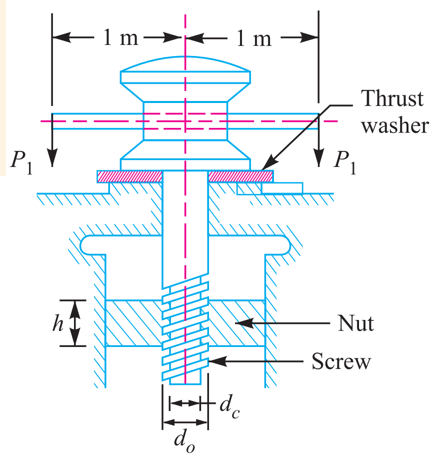



Fig. 17.8

**Solution.** Given : $W_1$ = 18 kN = 18 000 N ; $F$ = 4000 N ; $d_o$ = 60 mm ; $p$ = 10 mm ; $D_1$ = 150 mm or $R_1$ = 75 mm ; $D_2$ = 50 mm or $R_2$ = 25 mm ; $\mu = \tan\phi$ = 0.1 ; $\mu_1$ = 0.12 ; $p_b$ = 7 N/mm²

**1. *Maximum force to be exerted at the ends of lever***

Let $P_1$ = Maximum force exerted at each end of the lever 1 m (1000 mm) long.

We know that inner diameter or core diameter of the screw,

$$d_c = d_o - p = 60 - 10 = 50 \text{ mm}$$

Mean diameter of the screw,

$$d = \frac{d_o + d_c}{2} = \frac{60 + 50}{2} = 55 \text{ mm}$$

and
$$\tan\alpha = \frac{p}{\pi\, d} = \frac{10}{\pi \times 55} = 0.058$$

***(a) For raising the gate***

Since the frictional resistance acts in the opposite direction to the motion of screw, therefore for raising the gate, the frictional resistance ( $F$ ) will act downwards.

∴ Total load acting on the screw,

$$W = W_1 + F = 18\,000 + 4000 = 22\,000 \text{ N}$$

and torque required to overcome friction at the screw,

$$T_1 = P \times \frac{d}{2} = W \tan(\alpha + \phi)\frac{d}{2} = W\left(\frac{\tan\alpha + \tan\phi}{1 - \tan\alpha\tan\phi}\right)\frac{d}{2}$$

$$= 22\ 000 \left( \frac{0.058 + 0.1}{1 - 0.058 \times 0.1} \right) \frac{55}{2} = 96\ 148 \text{ N-mm}$$

Mean radius of washer,

$$R = \frac{R_1 + R_2}{2} = \frac{75 + 25}{2} = 50 \text{ mm}$$

∴ Torque required to overcome friction at the washer,

$$T_2 = \mu_1\, W R = 0.12 \times 22\ 000 \times 50 = 132\ 000 \text{ N-mm}$$

and total torque required to overcome friction,

$$T = T_1 + T_2 = 96\ 148 + 132\ 000 = 228\ 148 \text{ N-mm}$$

We know that the torque required at the end of lever ($T$),

$$228\ 148 = 2P_1 \times \text{Length of lever} = 2P_1 \times 1000 = 2000\ P_1$$

∴ $P_1 = 228\ 148 / 2000 = 141.1$ N **Ans.**

***(b) For lowering the gate***

Since the gate is being lowered, therefore the frictional resistance ($F$) will act upwards,

∴ Total load acting on the screw,

$$W = W_1 - F = 18\ 000 - 4000 = 14\ 000 \text{ N}$$

We know that torque required to overcome friction at the screw,

$$T_1 = P \times \frac{d}{2} = W \tan(\phi - \alpha) \frac{d}{2} = W \left( \frac{\tan\phi - \tan\alpha}{1 + \tan\phi \tan\alpha} \right) \frac{d}{2}$$

$$= 14\ 000 \left( \frac{0.1 - 0.058}{1 + 0.1 \times 0.058} \right) \frac{55}{2} = 16\ 077 \text{ N-mm}$$

and torque required to overcome friction at the washer,

$$T_2 = \mu_1\, W R = 0.12 \times 14\ 000 \times 50 = 84\ 000 \text{ N-mm}$$

∴ Total torque required to overcome friction,

$$T = T_1 + T_2 = 16\ 077 + 84\ 000 = 100\ 077 \text{ N-mm}$$

We know that the torque required at the end of lever ($T$),

$100\ 077 = 2P_1 \times 1000 = 2000\ P_1$ or $P_1 = 100\ 077/2000 = 50.04$ N **Ans.**

***2. Efficiency of the arrangement***

We know that the torque required for raising the load, with no friction,

$$T_0 = W \tan\alpha \times \frac{d}{2} = 22\ 000 \times 0.058 \times \frac{55}{2} = 35\ 090 \text{ N-mm}$$

∴ Efficiency of the arrangement,

$\eta = \dfrac{T_0}{T} = \dfrac{35\ 090}{228\ 148} = 0.154$ or $15.4\%$ **Ans.**

***3. Number of threads and height of nut***

Let $n$ = Number of threads in contact with the nut,

$h$ = Height of nut = $n \times p$, and

$t$ = Thickness of thread = $p / 2 = 10 / 2 = 5$ mm.

We know that the bearing pressure ($p_b$),

$$7 = \frac{W}{\pi . d . t . n} = \frac{22000}{\pi \times 55 \times 5 \times n} = \frac{25.46}{n}$$

∴ $n = 25.46 / 7 = 3.64$ say 4 threads **Ans.**

and $h = n \times p = 4 \times 10 = 40$ mm **Ans.**

**Example 17.10.** *The screw, as shown in Fig. 17.9 is operated by a torque applied to the lower end. The nut is loaded and prevented from turning by guides. Assume friction in the ball bearing to be negligible. The screw is a triple start trapezoidal thread. The outside diameter of the screw is 48 mm and pitch is 8 mm. The coefficient of friction of the threads is 0.15. Find :*

*1. Load which can be raised by a torque of 40 N-m ;*

*2. Whether the screw is overhauling ; and*

*3. Average bearing pressure between the screw and nut thread surface.*

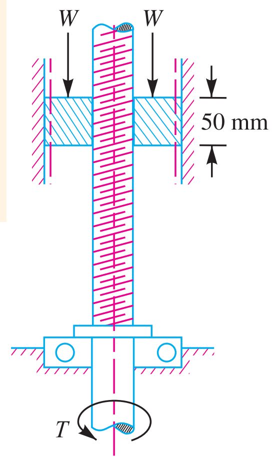



Fig. 17.9

**Solution.** Given : $d_o$ = 48 mm ; $p$ = 8 mm ; $\mu = \tan \phi = 0.15$ ; $T$ = 40 N-m = 40 000 N-mm

**1. *Load which can be raised***

Let $W$ = Load which can be raised.

We know that mean diameter of the screw,

$$d = d_o - p / 2 = 48 - 8 / 2 = 44 \text{ mm}$$

Since the screw is a triple start, therefore lead of the screw

$$= 3\,p = 3 \times 8 = 24 \text{ mm}$$

$$\therefore \quad \tan \alpha = \frac{\text{Lead}}{\pi d} = \frac{24}{\pi \times 44} = 0.174$$

and virtual coefficient of friction,

$$\mu_1 = \tan \phi_1 = \frac{\mu}{\cos \beta} = \frac{0.15}{\cos 15°} = \frac{0.15}{0.9659} = 0.155$$

... ($\because$ For trapezoidal threads, $2\,\beta = 30°$)

We know that the torque required to raise the load,

$$T = P \times \frac{d}{2} = W \tan (\alpha + \phi_1) \frac{d}{2} = W \left[ \frac{\tan \alpha + \tan \phi_1}{1 - \tan \alpha \tan \phi_1} \right] \frac{d}{2}$$

$$40\,000 = W \left( \frac{0.174 + 0.155}{1 - 0.174 \times 0.155} \right) \frac{44}{2} = 7.436\,W$$

$$\therefore \quad W = 40\,000 / 7.436 = 5380 \text{ N}$$ **Ans.**

**2. *Whether the screw is overhauling***

We know that torque required to lower the load,

$$T = W \tan (\phi_1 - \alpha) \frac{d}{2}$$

We have discussed in Art. 17.9 that if $\phi_1$ is less than $\alpha$, then the torque required to lower the load will be ***negative,*** *i.e.* the load will start moving downward without the application of any torque. Such a condition is known as overhauling of screws.

In the present case, $\tan \phi_1 = 0.155$ and $\tan \alpha = 0.174$. Since $\phi_1$ is less than $\alpha$, therefore the screw is overhauling. **Ans.**

**3. *Average bearing pressure between the screw and nut thread surfaces***

We know that height of the nut,

$$h = n \times p = 50 \text{ mm} \quad \text{...(Given)}$$

$\therefore$ Number of threads in contact,

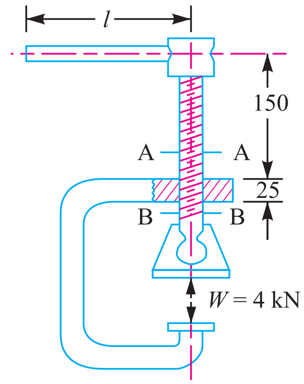
$$n = h / p = 50 / 8 = 6.25$$

and thickness of thread, $t = p / 2 = 8 / 2 = 4$ mm

We know that the average bearing pressure,

$$p_b = \frac{W}{\pi . d . t . n} = \frac{5380}{\pi \times 44 \times 4 \times 6.25} = 1.56 \text{ N/mm}^2 \textbf{ Ans.}$$

**Example 17.11.** *A C-clamp, as shown in Fig. 17.10, has trapezoidal threads of 12 mm outside diameter and 2 mm pitch. The coefficient of friction for screw threads is 0.12 and for the collar is 0.25. The mean radius of the collar is 6 mm. If the force exerted by the operator at the end of the handle is 80 N, find: 1. The length of handle; 2. The maximum shear stress in the body of the screw and where does this exist; and 3. The bearing pressure on the threads.*




All dimensions in mm.

**Fig. 17.10**

**Solution.** Given : $d_o$ = 12 mm ; $p$ = 2 mm ; $\mu = \tan \phi = 0.12$ ; $\mu_2 = 0.25$ ; $R$ = 6 mm ; $P_1$ = 80 N ; $W$ = 4 kN = 4000 N

**1.** ***Length of handle***

Let $l$ = Length of handle.

We know that the mean diameter of the screw,

$$d = d_o - p/2 = 12 - 2/2 = 11 \text{ mm}$$

$$\therefore \quad \tan \alpha = \frac{p}{\pi d} = \frac{2}{\pi \times 11} = 0.058$$

Since the angle for trapezoidal threads is $2\beta = 30°$ or $\beta = 15°$, therefore virtual coefficient of friction,

$$\mu_1 = \tan \phi_1 = \frac{\mu}{\cos \beta} = \frac{0.12}{\cos 15°} = \frac{0.12}{0.9659} = 0.124$$

We know that the torque required to overcome friction at the screw,

$$T_1 = P \times \frac{d}{2} = W \tan(\alpha + \phi_1) \frac{d}{2} = W \left( \frac{\tan \alpha + \tan \phi_1}{1 - \tan \alpha \tan \phi_1} \right) \frac{d}{2}$$

$$= 4000 \left( \frac{0.058 + 0.124}{1 - 0.058 \times 0.124} \right) \frac{11}{2} = 4033 \text{ N-mm}$$

Assuming uniform wear, the torque required to overcome friction at the collar,

$$T_2 = \mu_2 \, W R = 0.25 \times 4000 \times 6 = 6000 \text{ N-mm}$$

$\therefore$ Total torque required at the end of handle,

$$T = T_1 + T_2 = 4033 + 6000 = 10\,033 \text{ N-mm}$$

We know that the torque required at the end of handle ( $T$ ),

$$10\,033 = P_1 \times l = 80 \times l \quad \text{or} \quad l = 10\,033 / 80 = 125.4 \text{ mm} \textbf{ Ans.}$$

**2.** ***Maximum shear stress in the body of the screw***

Consider two sections *A-A* and *B-B*. The section *A-A* just above the nut, is subjected to torque and bending. The section *B-B* just below the nut is subjected to collar friction torque and direct compressive load. Thus, both the sections must be checked for maximum shear stress.

***Considering section A-A***

We know that the core diameter of the screw,

$$d_c = d_o - p = 12 - 2 = 10 \text{ mm}$$

and torque transmitted at *A-A*,

$$T = \frac{\pi}{16} \times \tau \, (d_c)^3$$

$$\therefore \quad \text{Shear stress, } \tau = \frac{16\,T}{\pi\,(d_c)^3} = \frac{16 \times 10033}{\pi \times 10^3} = 51.1 \text{ N/mm}^2$$

Bending moment at $A$-$A$,

$$M = P_1 \times 150 = 80 \times 150 = 12\,000 \text{ N-mm}$$

$$= \frac{\pi}{32} \times \sigma_b\,(d_c)^3$$

$$\therefore \text{ Bending stress, } \sigma_b = \frac{32\,M}{\pi\,(d_c)^3} = \frac{32 \times 12\,000}{\pi\,(10)^3} = 122.2 \text{ N/mm}^2$$

We know that the maximum shear stress,

$$\tau_{max} = \frac{1}{2}\sqrt{(\sigma_b)^2 + 4\tau^2} = \frac{1}{2}\sqrt{(122.2)^2 + 4(51.1)^2} = 79.65 \text{ N/mm}^2$$

$$= 79.65 \text{ MPa}$$

***Considering section B-B***

Since the section $B$-$B$ is subjected to collar friction torque ($T_2$), therefore the shear stress,

$$\tau = \frac{16\,T_2}{\pi\,(d_c)^3} = \frac{16 \times 6000}{\pi \times 10^3} = 30.6 \text{ N/mm}^2$$

and direct compressive stress,

$$\sigma_c = \frac{W}{A_c} = \frac{4W}{\pi(d_c)^2} = \frac{4 \times 4000}{\pi \times 10^2} = 51 \text{ N/mm}^2$$

$\therefore$ Maximum shear stress,

$$\tau_{max} = \frac{1}{2}\sqrt{(\sigma_c)^2 + 4\,\tau^2} = \frac{1}{2}\sqrt{(51)^2 + 4(30.6)^2} = 39.83 \text{ N/mm}^2 = 39.83 \text{ MPa}$$

From above, we see that the maximum shear stress is 79.65 MPa and occurs at section $A$-$A$. **Ans.**

***3. Bearing pressure on the threads***

We know that height of the nut,

$$h = n \times p = 25 \text{ mm} \qquad \text{...(Given)}$$

$\therefore$ Number of threads in contact,

$$n = h/p = 25/2 = 12.5$$

and thickness of threads, $t = p/2 = 2/2 = 1$ mm

We know that bearing pressure on the threads,

$$p_b = \frac{W}{\pi\,d.t.n} = \frac{4000}{\pi \times 11 \times 1 \times 12.5} = 9.26 \text{ N/mm}^2 \quad \textbf{Ans.}$$

**Example 17.12.** *A power transmission screw of a screw press is required to transmit maximum load of 100 kN and rotates at 60 r.p.m. Trapezoidal threads are as under :*

| | | | | |
|---|---|---|---|---|
| *Nominal dia, mm* | *40* | *50* | *60* | *70* |
| *Core dia, mm* | *32.5* | *41.5* | *50.5* | *59.5* |
| *Mean dia, mm* | *36.5* | *46* | *55.5* | *65* |
| *Core area, mm$^2$* | *830* | *1353* | *2003* | *2781* |
| *Pitch, mm* | *7* | *8* | *9* | *10* |

*The screw thread friction coefficient is 0.12. Torque required for collar friction and journal bearing is about 10% of the torque to drive the load considering screw friction. Determine screw dimensions and its efficiency. Also determine motor power required to drive the screw. Maximum permissible compressive stress in screw is 100 MPa.*

**Solution.** Given : $W = 100$ kN $= 100 \times 10^3$ N ; $N = 60$ r.p.m. ; $\mu = 0.12$ ; $\sigma_c = 100$ MPa $= 100$ N/mm²

*Dimensions of the screw*

Let $A_c$ = Core area of threads.

We know that the direct compressive stress ($\sigma_c$),

$$100 = \frac{W}{A_c} = \frac{100 \times 10^3}{A_c}$$

or $$A_c = 100 \times 10^3 / 100 = 1000 \text{ mm}^2$$

Since the core area is 1000 mm², therefore we shall use the following dimensions for the screw (for core area 1353 mm²).

Nominal diameter, $d_o = 50$ mm ;

Core diameter, $d_c = 41.5$ mm ;

Mean diameter, $d = 46$ mm ;

Pitch, $p = 8$ mm. **Ans.**

*This screw press was made in 1735 and installed in the Segovia Mint to strike a new series of coper coins which began in 1772. This press is presently on display in the Alcazar castle of Segovia.*

*Efficiency of the screw*

We know that $\tan \alpha = \dfrac{p}{\pi\, d} = \dfrac{8}{\pi \times 46} = 0.055$

and virtual coefficient of friction,

$$\mu_1 = \tan \phi_1 = \frac{\mu}{\cos \beta} = \frac{0.12}{\cos 15°}$$

$$= \frac{0.12}{0.9659} = 0.124 \qquad \text{... (}\because \text{ For trapezoidal threads, } 2\beta = 30°\text{)}$$

$\therefore$ Force required at the circumference of the screw,

$$P = W \tan(\alpha + \phi_1) = W\left[\frac{\tan \alpha + \tan \phi_1}{1 - \tan \alpha \tan \phi_1}\right]$$

$$= 100 \times 10^3 \left[\frac{0.055 + 0.124}{1 - 0.055 \times 0.124}\right] = 18\,023 \text{ N}$$

and the torque required to drive the load,

$$T_1 = P \times d / 2 = 18\,023 \times 46 / 2 = 414\,530 \text{ N-mm}$$

We know that the torque required for collar friction,

$$T_2 = 10\%\ T_1 = 0.1 \times 414\,530 = 41\,453 \text{ N-mm}$$

$\therefore$ Total torque required,

$$T = T_1 + T_2 = 414\,530 + 41\,453 = 455\,983 \text{ N-mm} = 455.983 \text{ N-m}$$

We know that the torque required with no friction,

$$T_0 = W \tan \alpha \times \frac{d}{2} = 100 \times 10^3 \times 0.055 \times \frac{46}{2} = 126\,500 \text{ N-mm}$$

$\therefore$ Efficiency of the screw,

$$\eta = \frac{T_0}{T} = \frac{126\,500}{455\,983} = 0.278 \text{ or } 27.8\% \textbf{ Ans.}$$

*Power required to drive the screw*

We know that the power required to drive the screw,

$$= T \times \omega = \frac{T \times 2\,\pi N}{60} = \frac{455.683 \times 2\pi \times 60}{60} = 2865 \text{ W}$$
$$= 2.865 \text{ kW } \textbf{Ans.}$$

**Example 17.13.** *A vertical two start square threaded screw of 100 mm mean diameter and 20 mm pitch supports a vertical load of 18 kN. The nut of the screw is fitted in the hub of a gear wheel having 80 teeth which meshes with a pinion of 20 teeth. The mechanical efficiency of the pinion and gear wheel drive is 90 percent. The axial thrust on the screw is taken by a collar bearing 250 mm outside diameter and 100 mm inside diameter. Assuming uniform pressure conditions, find, minimum diameter of pinion shaft and height of nut, when coefficient of friction for the vertical screw and nut is 0.15 and that for the collar bearing is 0.20. The permissible shear stress in the shaft material is 56 MPa and allowable bearing pressure is 1.4 N/mm$^2$.*

**Solution.** Given : $d = 100$ mm ; $p = 20$ mm ; $W = 18$ kN $= 18 \times 10^3$N ; No. of teeth on gear wheel = 80 ; No. of teeth on pinion = 20 ; $\eta_m = 90\% = 0.9$ ; $D_1 = 250$ mm or $R_1 = 125$ mm ; $D_2 = 100$ mm or $R_2 = 50$ mm ; $\mu = \tan \phi = 0.15$ ; $\mu_1 = 0.20$ ; $\tau = 56$ MPa $= 56$ N/mm$^2$ ; $p_b = 1.4$ N/mm$^2$

*Minimum diameter of pinion shaft*

Let $D$ = Minimum diameter of pinion shaft.

Since the screw is a two start square threaded screw, therefore lead of the screw

$$= 2\,p = 2 \times 20 = 40 \text{ mm}$$

$$\therefore \qquad \tan \alpha = \frac{\text{Lead}}{\pi d} = \frac{40}{\pi \times 100} = 0.127$$

and torque required to overcome friction at the screw and nut,

$$T_1 = P \times \frac{d}{2} = W \tan(\alpha + \phi)\frac{d}{2} = W\left(\frac{\tan \alpha + \tan \phi}{1 - \tan \alpha \tan \phi}\right)\frac{d}{2}$$
$$= 18 \times 10^3\left(\frac{0.127 + 0.15}{1 - 0.127 \times 0.15}\right)\frac{100}{2} = 254\ 160 \text{ N-mm}$$
$$= 254.16 \text{ N-m}$$

We know that, for uniform pressure conditions, torque required to overcome friction at the collar bearing,

$$T_2 = \frac{2}{3} \times \mu_1 W\left[\frac{(R_1)^3 - (R_2)^3}{(R_1)^2 - (R_2)^2}\right]$$
$$= \frac{2}{3} \times 0.20 \times 18 \times 10^3\left[\frac{(125)^3 - (50)^3}{(125)^2 - (50)^2}\right] \text{ N-mm}$$
$$= 334\ 290 \text{ N-mm} = 334.29 \text{ N-m}$$

Since the nut of the screw is fixed in the hub of a gear wheel, therefore the total torque required at the gear wheel,

$$T_w = T_1 + T_2 = 254.16 + 334.29 = 588.45 \text{ N-m}$$

Also the gear wheel having 80 teeth meshes with pinion having 20 teeth and the torque is proportional to the number of teeth, therefore torque required at the pinion shaft,

$$= \frac{T_w \times 20}{80} = 588.45 \times \frac{20}{80} = 147.11 \text{ N-m}$$

Since the mechanical efficiency of the pinion and gear wheel is 90%, therefore net torque required at the pinion shaft,

$$T_p = \frac{147.11 \times 100}{90} = 163.46 \text{ N} - \text{m} = 163\ 460 \text{ N-mm}$$

We know that the torque required at the pinion shaft ($T_p$),

$$163\ 460 = \frac{\pi}{16} \times \tau \times D^3 = \frac{\pi}{16} \times 56 \times D^3 = 11\ D^3$$

$\therefore$ $D^3 = 163\ 460 / 11 = 14\ 860$ or $D = 24.6$ say 25 mm **Ans.**

***Height of nut***

Let $h$ = Height of nut,

$n$ = Number of threads in contact, and

$t$ = Thickness or width of thread $= p/2 = 20/2 = 10$ mm

We know that the bearing pressure ($p_b$),

$$1.4 = \frac{W}{\pi\, d.t.n} = \frac{18 \times 10^3}{\pi \times 100 \times 10 \times n} = \frac{5.73}{n}$$

$\therefore$ $n = 5.73 / 1.4 = 4.09$ say 5 threads

and height of nut, $h = n \times p = 5 \times 20 = 100$ mm **Ans.**

**Example 17.14.** *A screw press is to exert a force of 40 kN. The unsupported length of the screw is 400 mm. Nominal diameter of screw is 50 mm. The screw has square threads with pitch equal to 10 mm. The material of the screw and nut are medium carbon steel and cast iron respectively. For the steel used take ultimate crushing stress as 320 MPa, yield stress in tension or compression as 200 MPa and that in shear as 120 MPa. Allowable shear stress for cast iron is 20 MPa and allowable bearing pressure between screw and nut is 12 N/mm². Young's modulus for steel = 210 kN/mm². Determine the factor of safety of screw against failure. Find the dimensions of the nut. What is the efficiency of the arrangement? Take coefficient of friction between steel and cast iron as 0.13.*

**Solution.** Given : $W = 40$ kN $= 40 \times 10^3$ N ; $L = 400$ mm $= 0.4$ m ; $d_o = 50$ mm ; $p = 10$ mm ; $\sigma_{cu} = 320$ MPa $= 320$ N/mm² ; $\sigma_y = 200$ MPa $= 200$ N/mm² ; $\tau_y = 120$ MPa $= 120$ N/mm² ; $\tau_c = 20$ MPa $= 20$ N/mm² ; $p_b = 12$ N/mm² ; $E = 210$ kN/mm² $= 210 \times 10^3$ N/mm² ; $\mu = \tan \phi = 0.13$

We know that the inner diameter or core diameter of the screw,

$$d_c = d_o - p = 50 - 10 = 40 \text{ mm}$$

and core area of the screw,

$$A_c = \frac{\pi}{4}(d_c)^2 = \frac{\pi}{4}(40)^2 = 1257 \text{ mm}^2$$

$\therefore$ Direct compressive stress on the screw due to axial load,

$$\sigma_c = \frac{W}{A_c} = \frac{40 \times 10^3}{1257} = 31.8 \text{ N/mm}^2$$

We know that the mean diameter of the screw,

$$d = \frac{d_o + d_c}{2} = \frac{50 + 40}{2} = 45 \text{ mm}$$

and

$$\tan \alpha = \frac{p}{\pi d} = \frac{10}{\pi \times 45} = 0.07$$

$\therefore$ Torque required to move the screw,

$$T = P \times \frac{d}{2} = W \tan(\alpha + \phi)\frac{d}{2} = W\left[\frac{\tan \alpha + \tan \phi}{1 - \tan \alpha \tan \phi}\right]\frac{d}{2}$$

$$= 40 \times 10^3 \left[\frac{0.07 + 0.13}{1 - 0.07 \times 0.13}\right]\frac{45}{2} = 181.6 \times 10^3 \text{ N-mm}$$

We know that torque transmitted by the screw ($T$),

$$181.6 \times 10^3 = \frac{\pi}{16} \times \tau\,(d_c)^3 = \frac{\pi}{16} \times \tau\,(40)^3 = 12\ 568\ \tau$$

$\therefore$ $\tau = 181.6 \times 10^3/12\ 568 = 14.45$ N/mm²

According to maximum shear stress theory, we have

$$\tau_{max} = \frac{1}{2}\sqrt{(\sigma_c)^2 + 4\tau^2} = \frac{1}{2}\sqrt{(31.8)^2 + 4\,(14.45)^2} = 21.5 \text{ N/mm}^2$$

***Factor of safety***

We know that factor of safety

$$= \frac{\tau_y}{\tau_{max}} = \frac{120}{21.5} = 5.58$$

Now considering the screw as a column, assuming one end fixed and other end free. According to J.B. Johnson's formula, critical load,

$$W_{cr} = A_c \times \sigma_y \left[1 - \frac{\sigma_y}{4C\,\pi^2\,E}\left(\frac{L}{k}\right)^2\right]$$

For one end fixed and other end free, $C = 0.25$.

$$\therefore \quad W_{cr} = 1257 \times 200 \left[1 - \frac{200}{4 \times 0.25 \times \pi^2 \times 210 \times 10^3}\left(\frac{400}{10}\right)^2\right] \text{N}$$

... ($\because$ $k = d_c / 4 = 40 / 4 = 10$ mm)

$$= 212\,700 \text{ N}$$

$$\therefore \quad \text{Factor of safety} = \frac{W_{cr}}{W} = \frac{212\,700}{40 \times 10^3} = 5.3$$

We shall take larger value of the factor of safety.

$\therefore$ Factor of safety = 5.58 say 6 **Ans.**

***Dimensions of the nut***

Let $n$ = Number of threads in contact with nut, and

$h$ = Height of nut = $p \times n$

Assume that the load is uniformly distributed over the threads in contact.

We know that the bearing pressure ( $p_b$),

$$12 = \frac{W}{\frac{\pi}{4}\left[(d_o)^2 - (d_c)^2\right] n} = \frac{40 \times 10^3}{\frac{\pi}{4}\left[(50)^2 - (40)^2\right] n} = \frac{56.6}{n}$$

$\therefore$ $n = 56.6 / 12 = 4.7$ say 5 threads **Ans.**

and $h = p \times n = 10 \times 5 = 50$ mm **Ans.**

Now let us check for the shear stress induced in the nut which is of cast iron. We know that

$$\tau_{nut} = \frac{W}{\pi n.d_o\,t} = \frac{40 \times 10^3}{\pi \times 5 \times 50 \times 5} = 10.2 \text{ N/mm}^2 = 10.2 \text{ MPa}$$

...($\because$ $t = p / 2 = 10 / 2 = 5$ mm)

This value is less than the given value of $\tau_c$ = 20 MPa, hence the nut is safe.

***Efficiency of the arrangement***

We know that torque required to move the screw with no friction,

$$T_0 = W \tan \alpha \times \frac{d}{2} = 40 \times 10^3 \times 0.07 \times \frac{45}{2} = 63 \times 10^3 \text{ N-mm}$$

$\therefore$ Efficiency of the arrangement

$$\eta = \frac{T_0}{T} = \frac{63 \times 10^3}{181.6 \times 10^3} = 0.347 \text{ or } 34.7\%$$ **Ans.**

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## 17.14 Design of Screw Jack

A bottle screw jack for lifting loads is shown in Fig. 17.11. The various parts of the screw jack are as follows:

1. Screwed spindle having square threaded screws,
2. Nut and collar for nut,
3. Head at the top of the screwed spindle for handle,
4. Cup at the top of head for the load, and
5. Body of the screw jack.

In order to design a screw jack for a load *W*, the following procedure may be adopted:

1. First of all, find the core diameter ($d_c$) by considering that the screw is under pure compression, *i.e.*

$$W = \sigma_c \times A_c = \sigma_c \times \frac{\pi}{4} (d_c)^2$$

The standard proportions of the square threaded screw are fixed from Table 17.1.

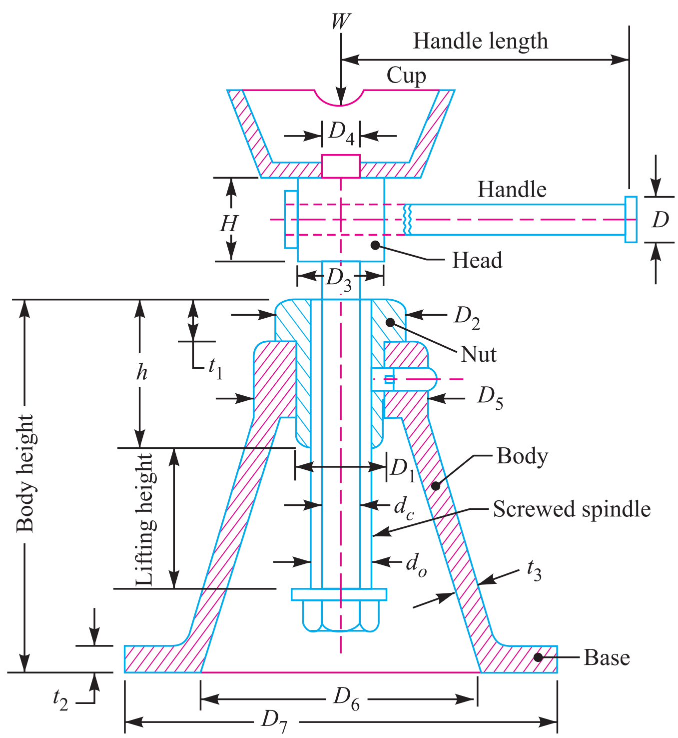


**Fig. 17.11.** Screw jack.

2. Find the torque ($T_1$) required to rotate the screw and find the shear stress (τ) due to this torque.

We know that the torque required to lift the load,

$$T_1 = P \times \frac{d}{2} = W \tan(\alpha + \phi) \frac{d}{2}$$

where $P$ = Effort required at the circumference of the screw, and

$d$ = Mean diameter of the screw.

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∴ Shear stress due to torque $T_1$,

$$\tau = \frac{16\,T_1}{\pi (d_c)^3}$$

Also find direct compressive stress ($\sigma_c$) due to axial load, *i.e.*

$$\sigma_c = \frac{W}{\frac{\pi}{4}(d_c)^2}$$

**3.** Find the principal stresses as follows:

Maximum principal stress (tensile or compressive),

$$\sigma_{c(max)} = \frac{1}{2}\left[\sigma_c + \sqrt{(\sigma_c)^2 + 4\tau^2}\right]$$

and maximum shear stress,

$$\tau_{max} = \frac{1}{2}\sqrt{(\sigma_c)^2 + 4\,\tau^2}$$

These stresses should be less than the permissible stresses.

**4.** Find the height of nut ($h$), considering the bearing pressure on the nut. We know that the bearing pressure on the nut,

$$p_b = \frac{W}{\frac{\pi}{4}\left[(d_o)^2 - (d_c)^2\right] n}$$

where $n$ = Number of threads in contact with screwed spindle.

∴ Height of nut, $h = n \times p$

where $p$ = Pitch of threads.

**5.** Check the stressess in the screw and nut as follows :

$$\tau_{(screw)} = \frac{W}{\pi\, n.d_c.t}$$

$$\tau_{(nut)} = \frac{W}{\pi\, n.d_o.t}$$

where $t$ = Thickness of screw $= p / 2$

**6.** Find inner diameter ($D_1$), outer diameter ($D_2$) and thickness ($t_1$) of the nut collar.

The inner diameter ($D_1$) is found by considering the tearing strength of the nut. We know that

$$W = \frac{\pi}{4}\left[(D_1)^2 - (d_o)^2\right]\sigma_t$$

The outer diameter ($D_2$) is found by considering the crushing strength of the nut collar. We know that

$$W = \frac{\pi}{4}\left[(D_2)^2 - (D_1)^2\right]\sigma_c$$

The thickness ($t_1$) of the nut collar is found by considering the shearing strength of the nut collar. We know that

$$W = \pi\, D_1.t_1.\tau$$

**7.** Fix the dimensions for the diameter of head ($D_3$) on the top of the screw and for the cup. Take $D_3$ = 1.75 $d_o$. The seat for the cup is made equal to the diameter of head and it is chamfered at the top. The cup is fitted with a pin of diameter $D_4 = D_3 / 4$ approximately. This pin remains a loose fit in the cup.

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**8.** Find the torque required ($T_2$) to overcome friction at the top of screw. We know that

$$T_2 = \frac{2}{3} \times \mu_1 W \left[ \frac{(R_3)^3 - (R_4)^3}{(R_3)^2 - (R_4)^2} \right] \quad \text{... (Assuming uniform pressure conditions)}$$

$$= \mu_1 W \left[ \frac{R_3 + R_4}{2} \right] = \mu_1 \, W \, R \quad \text{... (Assuming uniform wear conditions)}$$

where $R_3$ = Radius of head, and

$R_4$ = Radius of pin.

**9.** Now the total torque to which the handle will be subjected is given by

$$T = T_1 + T_2$$

Assuming that a person can apply a force of 300 – 400 N intermittently, the length of handle required

$$= T / 300$$

The length of handle may be fixed by giving some allowance for gripping.

**10.** The diameter of handle ($D$) may be obtained by considering bending effects. We know that bending moment,

$$M = \frac{\pi}{32} \times \sigma_b \times D^3 \quad \text{... } (\because \sigma_b = \sigma_t \text{ or } \sigma_c)$$

**11.** The height of head ($H$) is usually taken as twice the diameter of handle, *i.e.* $H = 2D$.

**12.** Now check the screw for buckling load.

Effective length or unsupported length of the screw,

$$L = \text{Lift of screw} + \frac{1}{2} \text{ Height of nut}$$

We know that buckling or critical load,

$$W_{cr} = A_c . \sigma_y \left[ 1 - \frac{\sigma_y}{4 \, C \, \pi^2 \, E} \left( \frac{L}{k} \right)^2 \right]$$

where $\sigma_y$ = Yield stress,

$C$ = End fixity coefficient. The screw is considered to be a strut with lower end fixed and load end free. For one end fixed and the other end free, $C = 0.25$

$k$ = Radius of gyration = $0.25 \, d_c$

The buckling load as obtained by the above expression must be higher than the load at which the screw is designed.

**13.** Fix the dimensions for the body of the screw jack.

**14.** Find efficiency of the screw jack.

**Example 17.15.** *A screw jack is to lift a load of 80 kN through a height of 400 mm. The elastic strength of screw material in tension and compression is 200 MPa and in shear 120 MPa. The material for nut is phosphor-bronze for which the elastic limit may be taken as 100 MPa in tension, 90 MPa in compression and 80 MPa in shear. The bearing pressure between the nut and the screw is not to exceed 18 N/mm*$^2$*. Design and draw the screw jack. The design should include the design of 1. screw, 2. nut, 3. handle and cup, and 4. body.*

*Screw jack*

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**Solution.** Given : $W = 80$ kN $= 80 \times 10^3$N ; $H_1 = 400$ mm $= 0.4$ m ; $\sigma_{et} = \sigma_{ec} = 200$ MPa $= 200$ N/mm$^2$ ; $\tau_e = 120$ MPa $= 120$ N/mm$^2$ ; $\sigma_{et(nut)} = 100$ MPa $= 100$ N/mm$^2$ ; $\sigma_{ec\ (nut)} = 90$ MPa $= 90$ N/mm$^2$ ; $\tau_{e(nut)} = 80$ MPa $= 80$N/mm$^2$ ; $p_b = 18$ N/mm$^2$

The various parts of a screw jack are designed as discussed below:

***1. Design of screw for spindle***

Let $d_c$ = Core diameter of the screw.

Since the screw is under compression, therefore load ($W$),

$$80 \times 10^3 = \frac{\pi}{4}(d_c)^2 \times \frac{\sigma_{ec}}{F.S.} = \frac{\pi}{4}(d_c)^2 \frac{200}{2} = 78.55\ (d_c)^2$$

... (Taking factor of safety, *F.S.* = 2)

$\therefore$ $(d_c)^2 = 80 \times 10^3 / 78.55 = 1018.5$ or $d_c = 32$ mm

For square threads of normal series, the following dimensions of the screw are selected from Table 17.2.

*Core diameter, $d_c = 38$ mm **Ans.**

Nominal or outside diameter of spindle,

$d_o = 46$ mm **Ans.**

Pitch of threads, $p = 8$ mm **Ans.**

Now let us check for principal stresses:

We know that the mean diameter of screw,

$$d = \frac{d_o + d_c}{2} = \frac{46 + 38}{2} = 42 \text{ mm}$$

and

$$\tan\alpha = \frac{p}{\pi d} = \frac{8}{\pi \times 42} = 0.0606$$

Assuming coefficient of friction between screw and nut,

$\mu = \tan\phi = 0.14$

$\therefore$ Torque required to rotate the screw in the nut,

$$T_1 = P \times \frac{d}{2} = W \tan(\alpha + \phi)\frac{d}{2} = W\left[\frac{\tan\alpha + \tan\phi}{1 - \tan\alpha \tan\phi}\right]\frac{d}{2}$$

$$= 80 \times 10^3 \left[\frac{0.0606 + 0.14}{1 - 0.0606 \times 0.14}\right]\frac{42}{2} = 340 \times 10^3 \text{ N-mm}$$

Now compressive stress due to axial load,

$$\sigma_c = \frac{W}{A_c} = \frac{W}{\frac{\pi}{4}(d_c)^2} = \frac{80 \times 10^3}{\frac{\pi}{4}(38)^2} = 70.53 \text{ N/mm}^2$$

and shear stress due to the torque,

$$\tau = \frac{16\,T_1}{\pi(d_c)^3} = \frac{16 \times 340 \times 10^3}{\pi\,(38)^3} = 31.55 \text{ N/mm}^2$$

$\therefore$ Maximum principal stress (tensile or compressive),

$$\sigma_{c(max)} = \frac{1}{2}\left[\sigma_c + \sqrt{(\sigma_c)^2 + 4\,\tau^2}\right] = \frac{1}{2}\left[70.53 + \sqrt{(70.53)^2 + 4\,(31.55)^2}\right]$$

$$= \frac{1}{2}[70.53 + 94.63] = 82.58 \text{ N/mm}^2$$

* From Table 17.2, we see that next higher value of 32 mm for the core diameter is 33 mm. By taking $d_c = 33$ mm, gives higher principal stresses than the permissible values. So core diameter is chosen as 38 mm.

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The given value of $\sigma_c$ is equal to $\dfrac{\sigma_{ec}}{F.S.}$, *i.e.* $\dfrac{200}{2} = 100 \text{ N/mm}^2$

We know that maximum shear stress,

$$\tau_{max} = \frac{1}{2}\left[\sqrt{(\sigma_c)^2 + 4\tau^2}\right] = \frac{1}{2}\left[\sqrt{(70.53)^2 + 4\,(31.55)^2}\right]$$

$$= \frac{1}{2} \times 94.63 = 47.315 \text{ N/mm}^2$$

The given value of $\tau$ is equal to $\dfrac{\tau_e}{F.S.}$, *i.e.* $\dfrac{120}{2} = 60 \text{ N/mm}^2$.

Since these maximum stresses are within limits, therefore design of screw for spindle is safe.

### *2. Design for nut*

Let $n$ = Number of threads in contact with the screwed spindle,

$h$ = Height of nut = $n \times p$, and

$t$ = Thickness of screw = $p / 2 = 8 / 2 = 4$ mm

Assume that the load is distributed uniformly over the cross-sectional area of nut.

We know that the bearing pressure ($p_b$),

$$18 = \frac{W}{\frac{\pi}{4}\left[(d_o)^2 - (d_c)^2\right] n} = \frac{80 \times 10^3}{\frac{\pi}{4}\left[(46)^2 - (38)^2\right] n} = \frac{151.6}{n}$$

$\therefore$ $n = 151.6 / 18 = 8.4$ say 10 threads **Ans.**

and height of nut, $h = n \times p = 10 \times 8 = 80$ mm **Ans.**

Now, let us check the stresses induced in the screw and nut.

We know that shear stress in the screw,

$$\tau_{(screw)} = \frac{W}{\pi\, n.d_c.t} = \frac{80 \times 10^3}{\pi \times 10 \times 38 \times 4} = 16.15 \text{ N/mm}^2$$

... ($\because$ $t = p / 2 = 4$ mm)

and shear stress in the nut,

$$\tau_{(nut)} = \frac{W}{\pi\, n.d_o.t} = \frac{80 \times 10^3}{\pi \times 10 \times 46 \times 4} = 13.84 \text{ N/mm}^2$$

Since these stresses are within permissible limit, therefore design for nut is safe.

Let $D_1$ = Outer diameter of nut,

$D_2$ = Outside diameter for nut collar, and

$t_1$ = Thickness of nut collar.

First of all considering the tearing strength of nut, we have

$$W = \frac{\pi}{4}\left[(D_1)^2 - (d_o)^2\right]\sigma_t$$

$$80 \times 10^3 = \frac{\pi}{4}\left[(D_1)^2 - (46)^2\right]\frac{100}{2} = 39.3\left[(D_1)^2 - 2116\right] \quad ...\left[\because \sigma_t = \frac{\sigma_{et(nut)}}{F.S.}\right]$$

or $(D_1)^2 - 2116 = 80 \times 10^3 / 39.3 = 2036$

$\therefore$ $(D_1)^2 = 2036 + 2116 = 4152$ or $D_1 = 65$ mm **Ans.**

Now considering the crushing of the collar of the nut, we have

$$W = \frac{\pi}{4}\left[(D_2)^2 - (D_1)^2\right]\sigma_c$$

$$80 \times 10^3 = \frac{\pi}{4}\left[(D_2)^2 - (65)^2\right]\frac{90}{2} = 35.3\left[(D_2)^2 - 4225\right] \qquad \ldots\left[\sigma_c = \frac{\sigma_{ec(nut)}}{F.S.}\right]$$

or $\quad (D_2)^2 - 4225 = 80 \times 10^3 / 35.3 = 2266$

$\therefore \quad (D_2)^2 = 2266 + 4225 = 6491 \quad$ or $\quad D_2 = 80.6$ say 82 mm **Ans.**

Considering the shearing of the collar of the nut, we have

$$W = \pi D_1 \times t_1 \times \tau$$

$$80 \times 10^3 = \pi \times 65 \times t_1 \times \frac{80}{2} = 8170\, t_1 \qquad \ldots\left[\tau = \frac{\tau_{e(nut)}}{F.S.}\right]$$

$\therefore \quad t_1 = 80 \times 10^3/8170 = 9.8$ say 10 mm **Ans.**

**3.** ***Design for handle and cup***

The diameter of the head ($D_3$) on the top of the screwed rod is usually taken as 1.75 times the outside diameter of the screw ($d_o$).

$\therefore \quad D_3 = 1.75\, d_o = 1.75 \times 46 = 80.5$ say 82 mm **Ans.**

The head is provided with two holes at the right angles to receive the handle for rotating the screw. The seat for the cup is made equal to the diameter of head, *i.e.* 82 mm and it is given chamfer at the top. The cup prevents the load from rotating. The cup is fitted to the head with a pin of diameter $D_4$ = 20 mm. The pin remains loose fit in the cup. Other dimensions for the cup may be taken as follows :

Height of cup = 50 mm **Ans.**

Thickness of cup = 10 mm **Ans.**

Diameter at the top of cup = 160 mm **Ans.**

Now let us find out the torque required ($T_2$) to overcome friction at the top of the screw.

Assuming uniform pressure conditions, we have

$$T_2 = \frac{2}{3} \times \mu_1\, W\left[\frac{(R_3)^3 - (R_4)^3}{(R_3)^2 - (R_4)^2}\right]$$

$$= \frac{2}{3} \times 0.14 \times 80 \times 10^3\left[\frac{\left(\frac{82}{2}\right)^3 - \left(\frac{20}{2}\right)^3}{\left(\frac{82}{2}\right)^2 - \left(\frac{20}{2}\right)^2}\right] \qquad \ldots (\text{Assuming } \mu_1 = \mu)$$

$$= 7.47 \times 10^3\left[\frac{(41)^3 - (10)^3}{(41)^2 - (10)^2}\right] = 321 \times 10^3 \text{ N-mm}$$

$\therefore$ Total torque to which the handle is subjected,

$$T = T_1 + T_2 = 340 \times 10^3 + 321 \times 10^3 = 661 \times 10^3 \text{ N-mm}$$

Assuming that a force of 300 N is applied by a person intermittently, therefore length of handle required

$$= 661 \times 10^3 / 300 = 2203 \text{ mm}$$

Allowing some length for gripping, we shall take the length of handle as 2250 mm.

A little consideration will show that an excessive force applied at the end of lever will cause bending. Considering bending effect, the maximum bending moment on the handle,

$$M = \text{Force applied} \times \text{Length of lever}$$

$$= 300 \times 2250 = 675 \times 10^3 \text{ N-mm}$$

Let $D$ = Diameter of the handle.

Assuming that the material of the handle is same as that of screw, therefore taking bending stress $\sigma_b = \sigma_t = \sigma_{et} / 2 = 100 \text{ N/mm}^2$.

We know that the bending moment ($M$),

$$675 \times 10^3 = \frac{\pi}{32} \times \sigma_b \times D^3 = \frac{\pi}{32} \times 100 \times D^3 = 9.82\ D^3$$

$\therefore$ $D^3 = 675 \times 10^3 / 9.82 = 68.74 \times 10^3$ or $D = 40.96$ say 42 mm **Ans.**

The height of head ($H$) is taken as $2D$.

$\therefore$ $H = 2\,D = 2 \times 42 = 84$ mm **Ans.**

Now let us check the screw for buckling load.

We know that the effective length for the buckling of screw,

$$L = \text{Lift of screw} + \frac{1}{2}\ \text{Height of nut} = H_1 + h / 2$$

$$= 400 + 80 / 2 = 440 \text{ mm}$$

When the screw reaches the maximum lift, it can be regarded as a strut whose lower end is fixed and the load end is free. We know that critical load,

$$W_{cr} = A_c \times \sigma_y \left[1 - \frac{\sigma_y}{4C\ \pi^2\ E}\left(\frac{L}{k}\right)^2\right]$$

For one end fixed and other end free, $C = 0.25$.

Also $k = 0.25\ d_c = 0.25 \times 38 = 9.5$ mm

$$\therefore \quad W_{cr} = \frac{\pi}{4}\,(38)^2\ 200\left[1 - \frac{200}{4 \times 0.25 \times \pi^2 \times 210 \times 10^3}\left(\frac{440}{9.5}\right)^2\right]$$

... (Taking $\sigma_y = \sigma_{et}$)

$$= 226\ 852\ (1 - 0.207) = 179\ 894 \text{ N}$$

Since the critical load is more than the load at which the screw is designed (*i.e.* $80 \times 10^3$ N), therefore there is no chance of the screw to buckle.

### 4. *Design of body*

The various dimensions of the body may be fixed as follows:

Diameter of the body at the top,

$D_5 = 1.5\ D_2 = 1.5 \times 82 = 123$ mm **Ans.**

Thickness of the body,

$t_3 = 0.25\ d_o = 0.25 \times 46 = 11.5$ say 12 mm **Ans.**

Inside diameter at the bottom,

$D_6 = 2.25\ D_2 = 2.25 \times 82 = 185$ mm **Ans.**

Outer diameter at the bottom,

$D_7 = 1.75\ D_6 = 1.75 \times 185 = 320$ mm **Ans.**

Thickness of base, $t_2 = 2\,t_1 = 2 \times 10 = 20$ mm **Ans.**

Height of the body = Max. lift + Height of nut + 100 mm extra

= 400 + 80 + 100 = 580 mm **Ans.**

The body is made tapered in order to achieve stability of jack.

Let us now find out the efficiency of the screw jack. We know that the torque required to rotate the screw with no friction,

$$T_0 = W \tan\alpha \times \frac{d}{2} = 80 \times 10^3 \times 0.0606 \times \frac{42}{2} = 101\,808 \text{ N-mm}$$

∴ Efficiency of the screw jack,

$$\eta = \frac{T_0}{T} = \frac{101808}{661 \times 10^3} = 0.154 \text{ or } 15.4\% \quad \textbf{Ans.}$$

**Example 17.16.** *A toggle jack as shown in Fig. 17.12, is to be designed for lifting a load of 4 kN. When the jack is in the top position, the distance between the centre lines of nuts is 50 mm and in the bottom position this distance is 210 mm. The eight links of the jack are symmetrical and 110 mm long. The link pins in the base are set 30 mm apart. The links, screw and pins are made from mild steel for which the permissible stresses are 100 MPa in tension and 50 MPa in shear. The bearing pressure on the pins is limited to 20 N/mm².*

*Assume the pitch of the square threads as 6 mm and the coefficient of friction between threads as 0.20.*

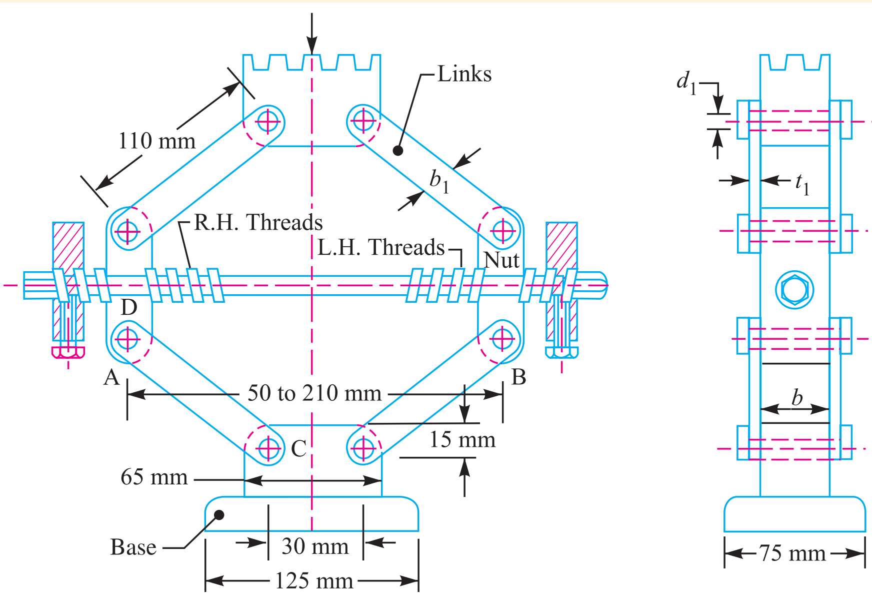


**Fig. 17.12**

**Solution.** Given : $W = 4$ kN = 4000 N ; $l = 110$ mm ; $\sigma_t = 100$ MPa = 100 N/mm² ; $\tau = 50$ MPa = 50 N/mm² ; $p_b = 20$ N/mm² ; $p = 6$ mm ; $\mu = \tan\phi = 0.20$

The toggle jack may be designed as discussed below :

**1.** ***Design of square threaded screw***

A little consideration will show that the maximum load on the square threaded screw occurs when the jack is in the bottom position. The position of the link *CD* in the bottom position is shown in Fig. 17.13 (*a*).

Let θ be the angle of inclination of the link *CD* with the horizontal.

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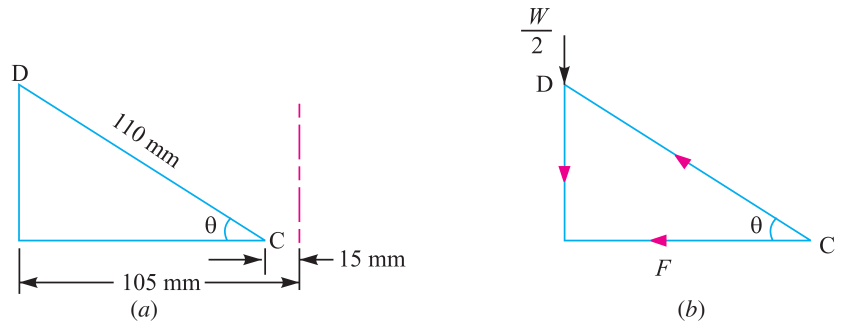


**Fig. 17.13**

From the geometry of the figure, we find that

$$\cos\theta = \frac{105 - 15}{110} = 0.8112 \text{ or } \theta = 35.1°$$

Each nut carries half the total load on the jack and due to this, the link *CD* is subjected to tension while the square threaded screw is under pull as shown in Fig. 17.13 (*b*). The magnitude of the pull on the square threaded screw is given by

*The rotational speed of the lead screw relative to the spindle speed can be adjusted manually by adding and removing gears to and from the gear*

$$F = \frac{W}{2\tan\theta} = \frac{W}{2\tan 35.1°}$$

$$= \frac{4000}{2 \times 0.7028} = 2846 \text{ N}$$

Since a similar pull acts on the other nut, therefore total tensile pull on the square threaded rod,

$$W_1 = 2F = 2 \times 2846 = 5692 \text{ N}$$

Let $d_c$ = Core diameter of the screw,

We know that load on the screw ($W_1$),

$$5692 = \frac{\pi}{4}(d_c)^2 \sigma_t = \frac{\pi}{4}(d_c)^2\, 100$$

$$= 78.55\,(d_c)^2$$

$\therefore$ $(d_c)^2 = 5692 / 78.55 = 72.5$ or $d_c = 8.5$ say 10 mm

Since the screw is also subjected to torsional shear stress, therefore to account for this, let us adopt

$$d_c = 14 \text{ mm} \quad \textbf{Ans.}$$

$\therefore$ Nominal or outer diameter of the screw,

$$d_o = d_c + p = 14 + 6 = 20 \text{ mm} \quad \textbf{Ans.}$$

and mean diameter of the screw,

$$d = d_o - p/2 = 20 - 6/2 = 17 \text{ mm}$$

Let us now check for principal stresses. We know that

$$\tan\alpha = \frac{p}{\pi d} = \frac{6}{\pi \times 17} = 0.1123 \quad \text{... (where } \alpha \text{ is the helix angle)}$$

We know that effort required to rotate the screw,

$$P = W_1 \tan(\alpha + \phi) = W_1 \left(\frac{\tan\alpha + \tan\phi}{1 - \tan\alpha\tan\phi}\right)$$

$$= 5692\left(\frac{0.1123 + 0.20}{1 - 0.1123 \times 0.20}\right) = 1822 \text{ N}$$

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∴ Torque required to rotate the screw,

$$T = P \times \frac{d}{2} = 1822 \times \frac{17}{2} = 15\ 487 \text{ N-mm}$$

and shear stress in the screw due to torque,

$$\tau = \frac{16\,T}{\pi\,(d_c)^3} = \frac{16 \times 15\ 487}{\pi\,(14)^3} = 28.7 \text{ N/mm}^2$$

We know that direct tensile stress in the screw,

$$\sigma_t = \frac{W_1}{\frac{\pi}{4}\,(d_c)^2} = \frac{W_1}{0.7855\,(d_c)^2} = \frac{5692}{0.7855\,(14)^2} = 37 \text{ N/mm}^2$$

∴ Maximum principal (tensile) stress,

$$\sigma_{t(max)} = \frac{\sigma_t}{2} + \frac{1}{2}\sqrt{(\sigma_t)^2 + 4\,\tau^2} = \frac{37}{2} + \frac{1}{2}\sqrt{(37)^2 + 4\,(28.7)^2}$$

$$= 18.5 + 34.1 = 52.6 \text{ N/mm}^2$$

and maximum shear stress,

$$\tau_{max} = \frac{1}{2}\sqrt{(\sigma_t)^2 + 4\tau^2} = \frac{1}{2}\sqrt{(37)^2 + 4(28.7)^2} = 34.1 \text{ N/mm}^2$$

Since the maximum stresses are within safe limits, therefore the design of square threaded screw is satisfactory.

***2. Design of nut***

Let $n$ = Number of threads in contact with the screw (*i.e.* square threaded rod).

Assuming that the load $W_1$ is distributed uniformly over the cross-sectional area of the nut, therefore bearing pressure between the threads ( $p_b$ ),

$$20 = \frac{W_1}{\frac{\pi}{4}\left[(d_o)^2 - (d_c)^2\right] n} = \frac{5692}{\frac{\pi}{4}\left[(20)^2 - (14)^2\right] n} = \frac{35.5}{n}$$

$$\therefore \quad n = 35.5 / 20 = 1.776$$

In order to have good stability and also to prevent rocking of the screw in the nut, we shall provide $n = 4$ threads in the nut. The thickness of the nut,

$$t = n \times p = 4 \times 6 = 24 \text{ mm}$$ **Ans.**

The width of the nut ($b$) is taken as 1.5 $d_o$.

$$\therefore \quad b = 1.5\,d_o = 1.5 \times 20 = 30 \text{ mm}$$ **Ans.**

To control the movement of the nuts beyond 210 mm (the maximum distance between the centre lines of nuts), rings of 8 mm thickness are fitted on the screw with the help of set screws.

∴ Length of screwed portion of the screw

$$= 210 + t + 2 \times \text{Thickness of rings}$$

$$= 210 + 24 + 2 \times 8 = 250 \text{ mm}$$ **Ans.**

The central length (about 25 mm) of screwed rod is kept equal to core diameter of the screw *i.e.* 14 mm. Since the toggle jack is operated by means of spanners on both sides of the square threaded rod, therefore the ends of the rod may be reduced to 10 mm square and 15 mm long.

∴ Toal length of the screw

$$= 250 + 2 \times 15 = 280 \text{ mm}$$ **Ans.**

Assuming that a force of 150 N is applied by each person at each end of the rod, therefore length of the spanner required

$$= \frac{T}{2 \times 150} = \frac{15487}{300} = 51.62 \text{ mm}$$

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We shall take the length of the spanner as 200 mm in order to facilitate the operation and even a single person can operate it.

**3. *Design of pins in the nuts***

Let $d_1$ = Diameter of pins in the nuts.

Since the pins are in double shear, therefore load on the pins ($F$),

$$2846 = 2 \times \frac{\pi}{4}(d_1)^2 \tau = 2 \times \frac{\pi}{4}(d_1)^2\, 50 = 78.55\,(d_1)^2$$

$$\therefore \quad (d_1)^2 = 2846 / 78.55 = 36.23 \text{ or } d_1 = 6.02 \text{ say } 8 \text{ mm} \quad \textbf{Ans.}$$

The diameter of pin head is taken as 1.5 $d_1$ (*i.e.* 12 mm) and thickness 4 mm. The pins in the nuts are kept in position by separate rings 4 mm thick and 1.5 mm split pins passing through the rings and pins.

**4. *Design of links***

Due to the load, the links may buckle in two planes at right angles to each other. For buckling in the vertical plane (*i.e.* in the plane of the links), the links are considered as hinged at both ends and for buckling in a plane perpendicular to the vertical plane, it is considered as fixed at both ends. We know that load on the link

$$= F / 2 = 2846 / 2 = 1423 \text{ N}$$

Assuming a factor of safety = 5, the links must be designed for a buckling load of

$$W_{cr} = 1423 \times 5 = 7115 \text{ N}$$

Let $t_1$ = Thickness of the link, and

$b_1$ = Width of the link.

Assuming that the width of the link is three times the thickness of the link, *i.e.* $b_1 = 3\,t_1$, therefore cross-sectional area of the link,

$$A = t_1 \times 3t_1 = 3(t_1)^2$$

and moment of inertia of the cross-section of the link,

$$I = \frac{1}{12} \times t_1\,(b_1)^3 = \frac{1}{12} \times t_1\,(3t_1)^3 = 2.25\,(t_1)^4$$

We know that the radius of gyration,

$$k = \sqrt{\frac{I}{A}} = \sqrt{\frac{2.25\,(t_1)^4}{3(t_1)^2}} = 0.866\,t_1$$

Since for buckling of the link in the vertical plane, the ends are considered as hinged, therefore equivalent length of the link,

$$L = l = 110 \text{ mm}$$

and Rankine's constant, $a = \dfrac{1}{7500}$

According to Rankine's formula, buckling load ($W_{cr}$),

$$7115 = \frac{\sigma_c \times A}{1 + a\left(\dfrac{L}{k}\right)^2} = \frac{100 \times 3(t_1)^2}{1 + \dfrac{1}{7500}\left(\dfrac{110}{0.866\,t_1}\right)^2} = \frac{300\,(t_1)^2}{1 + \dfrac{2.15}{(t_1)^2}}$$

or $$\frac{7115}{300} = \frac{(t_1)^4}{(t_1)^2 + 2.15}$$

$(t_1)^4 - 23.7\,(t_1)^2 - 51 = 0$

$$\therefore \quad (t_1)^2 = \frac{23.7 \pm \sqrt{(23.7)^2 + 4 \times 51}}{2} = \frac{23.7 + 27.7}{2} = 25.7$$

or $t_1 = 5.07$ say 6 mm ... (Taking + ve sign)

and $b_1 = 3\,t_1 = 3 \times 6 = 18$ mm

Now let us consider the buckling of the link in a plane perpendicular to the vertical plane.

Moment of inertia of the cross-section of the link,

$$I = \frac{1}{12} \times b_1\,(t_1)^3 = \frac{1}{12} \times 3t_1\,(t_1)^3 = 0.25\,(t_1)^4$$

and cross-sectional area of the link,

$$A = t_1.b_1 = t_1 \times 3\,t_1 = 3\,(t_1)^2$$

$\therefore$ Radius of gyration,

$$k = \sqrt{\frac{I}{A}} = \sqrt{\frac{0.25\,(t_1)^4}{3\,(t_1)^2}} = 0.29\,t_1$$

Since for buckling of the link in a plane perpendicular to the vertical plane, the ends are considered as fixed, therefore

Equivalent length of the link,

$$L = l / 2 = 110 / 2 = 55 \text{ mm}$$

Again according to Rankine's formula, buckling load,

$$W_{cr} = \frac{\sigma_c \times A}{1 + a\left(\frac{L}{k}\right)^2} = \frac{100 \times 3(t_1)^2}{1 + \frac{1}{7500}\left(\frac{55}{0.29\,t_1}\right)^2} = \frac{300\,(t_1)^2}{1 + \frac{4.8}{(t_1)^2}}$$

Substituting the value of $t_1 = 6$ mm, we have

$$W_{cr} = \frac{300 \times 6^2}{1 + \frac{4.8}{6^2}} = 9532 \text{ N}$$

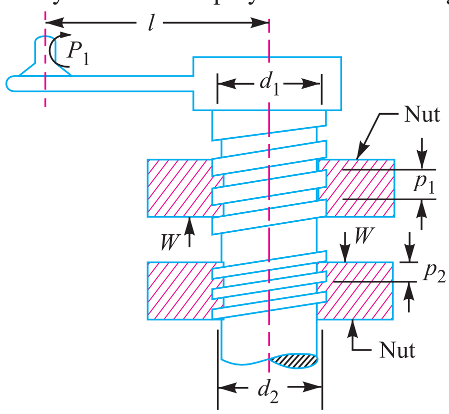
Since this buckling load is more than the calculated value (*i.e.* 7115 N), therefore the link is safe for buckling in a plane perpendicular to the vertical plane.

$\therefore$ We may take $t_1 = 6$ mm ; and $b_1 = 18$ mm **Ans.**

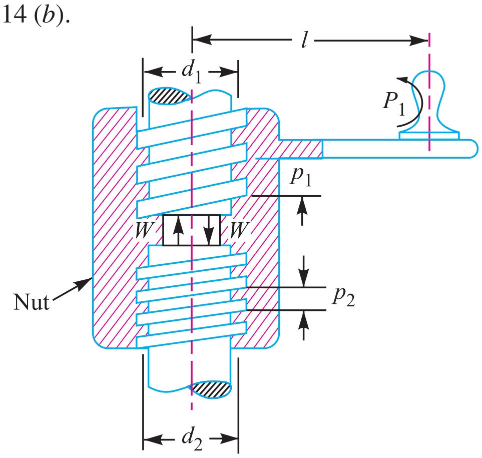
## 17.15 Differential and Compound Screws

There are certain cases in which a very slow movement of the screw is required whereas in other cases, a very fast movement of the screw is needed. The slow movement of the screw may be obtained by using a small pitch of the threads, but it results in weak threads. The fast movement of the screw may be obtained by using multiple-start threads, but this method requires expensive machning and the loss of self-locking property. In order to overcome these difficulties, differential or compound screws, as discussed below, are used.

**1.** ***Differential screw.*** When a slow movement or fine adjustment is desired in precision equipments, then a differential screw is used. It consists of two threads of the same hand (*i.e.* right handed or left handed) but of different pitches, wound on the same cylinder or different cylinders as shown in Fig. 17.14. It may be noted that when the threads are wound on the same cylinder, then two nuts are employed as shown in Fig. 17.14 (*a*) and when the threads are wound on different cylinders, then only one nut is employed as shown in Fig. 17.14 (*b*).




(*a*) Threads wound on the same cylinder.




(*b*) Threads wound on the different cylinders.

**Fig. 17.14**

In this case, each revolution of the screw causes the nuts to move towards or away from each other by a distance equal to the difference of the pitches.

Let $p_1$ = Pitch of the upper screw,

$d_1$ = Mean diameter of the upper screw,

$\alpha_1$ = Helix angle of the upper screw, and

$\mu_1$ = Coefficient of friction between the upper screw and the upper nut

= $\tan \phi_1$, where $\phi_1$ is the friction angle.

$p_2$, $d_2$, $\alpha_2$ and $\mu_2$ = Corresponding values for the lower screw.

We know that torque required to overcome friction at the upper screw,

$$T_1 = W \tan(\alpha_1 + \phi_1)\frac{d_1}{2} = W\left[\frac{\tan\alpha_1 + \tan\phi_1}{1 - \tan\alpha_1 \tan\phi_1}\right]\frac{d_1}{2} \qquad ...(i)$$

Similarly, torque required to overcome friction at the lower screw,

$$T_2 = W \tan(\alpha_2 + \phi_2)\frac{d_2}{2} = W\left[\frac{\tan\alpha_2 + \tan\phi_2}{1 - \tan\alpha_2 \tan\phi_2}\right]\frac{d_2}{2} \qquad ...(ii)$$

∴ Total torque required to overcome friction at the thread surfaces,

$$T = P_1 \times l = T_1 - T_2$$

When there is no friction between the thread surfaces, then $\mu_1 = \tan\phi_1 = 0$ and $\mu_2 = \tan\phi_2 = 0$. Substituting these values in the above expressions, we have

$$\therefore \quad T_1' = W \tan\alpha_1 \times \frac{d_1}{2}$$

and

$$T_2' = W \tan\alpha_2 \times \frac{d_2}{2}$$

∴ Total torque required when there is no friction,

$$T_0 = T_1' - T_2'$$

$$= W \tan\alpha_1 \times \frac{d_1}{2} - W \tan\alpha_2 \times \frac{d_2}{2}$$

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$$= W\left[\frac{p_1}{\pi d_1}\times\frac{d_1}{2} - \frac{p_2}{\pi d_2}\times\frac{d_2}{2}\right] = \frac{W}{2\pi}(p_1 - p_2)$$

$$\left[\because \tan\alpha_1 = \frac{p_1}{\pi d_1};\ \text{and } \tan\alpha_2 = \frac{p_2}{\pi d_2}\right]$$

We know that efficiency of the differential screw,

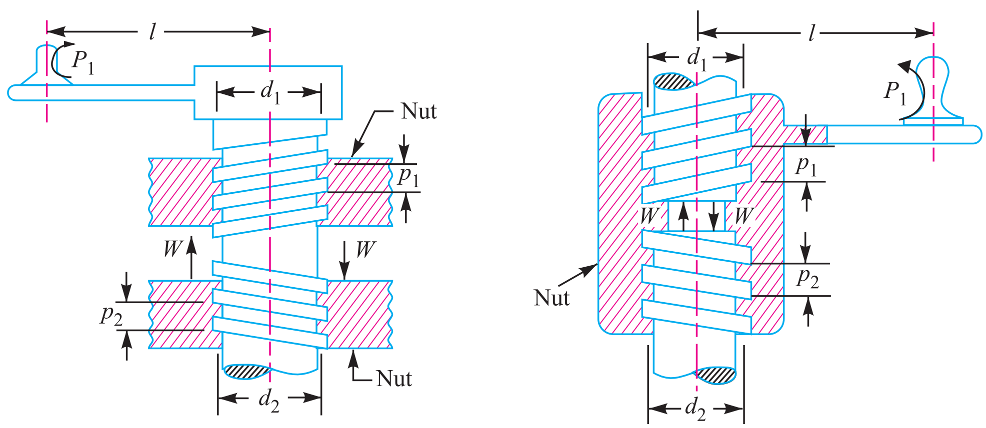
$$\eta = \frac{T_0}{T}$$

**2. *Compound screw.*** When a fast movement is desired, then a compound screw is employed. It consists of two threads of opposite hands (*i.e.* one right handed and the other left handed) wound on the same cylinder or different cylinders, as shown in Fig. 17.15 (*a*) and (*b*) respectively.

In this case, each revolution of the screw causes the nuts to move towards one another equal to the sum of the pitches of the threads. Usually the pitch of both the threads are made equal.

We know that torque required to overcome friction at the upper screw,

$$T_1 = W\tan(\alpha_1 + \phi_1)\frac{d_1}{2} = W\left[\frac{\tan\alpha_1 + \tan\phi_1}{1 - \tan\alpha_1\tan\phi_1}\right]\frac{d_1}{2} \quad \ldots(i)$$




(*a*) Threads wound on the same cylinder. (*b*) Threads wound on the different cylinders.

**Fig. 17.15**

Similarly, torque required to overcome friction at the lower screw,

$$T_2 = W\tan(\alpha_2 + \phi_2)\frac{d_2}{2} = W\left[\frac{\tan\alpha_2 + \tan\phi_2}{1 - \tan\alpha_2\tan\phi_2}\right]\frac{d_2}{2} \quad \ldots(ii)$$

∴ Total torque required to overcome friction at the thread surfaces,

$$T = P_1 \times l = T_1 + T_2$$

When there is no friction between the thread surfaces, then $\mu_1 = \tan\phi_1 = 0$ and $\mu_2 = \tan\phi_2 = 0$. Substituting these values in the above expressions, we have

$$T_1' = W\tan\alpha_1 \times \frac{d_1}{2}$$

$$T_2' = W\tan\alpha_2 \times \frac{d_2}{2}$$

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∴ Total torque required when there is no friction,

$$T_0 = T_1' + T_2'$$

$$= W \tan \alpha_1 \times \frac{d_2}{2} + W \tan \alpha_2 \times \frac{d_2}{2}$$

$$= W\left[\frac{p_1}{\pi\, d_1} \times \frac{d_1}{2} + \frac{p_2}{\pi\, d_2} \times \frac{d_2}{2}\right] = \frac{W}{2\,\pi}\,(p_1 + p_2)$$

We know that efficiency of the compound screw,

$$\eta = \frac{T_0}{T}$$

**Example 17.17.** *A differential screw jack is to be made as shown in Fig. 17.16. Neither screw rotates. The outside screw diameter is 50 mm. The screw threads are of square form single start and the coefficient of thread friction is 0.15.*

*Determine : 1. Efficiency of the screw jack; 2. Load that can be lifted if the shear stress in the body of the screw is limited to 28 MPa.*

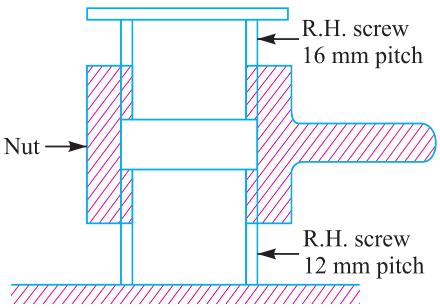


**Fig. 17.16.** Differential screw.

**Solution.** Given : $d_o = 50$ mm ; $\mu = \tan \phi = 0.15$ ; $p_1 = 16$ mm ; $p_2 = 12$ mm ; $\tau_{max} = 28$ MPa $= 28$ N/mm$^2$

**1.** ***Efficiency of the screw jack***

We know that the mean diameter of the upper screw,

$$d_1 = d_o - p_1 / 2 = 50 - 16 / 2 = 42 \text{ mm}$$

and mean diameter of the lower screw,

$$d_2 = d_o - p_2 / 2 = 50 - 12 / 2 = 44 \text{ mm}$$

∴ $$\tan \alpha_1 = \frac{p_1}{\pi d_1} = \frac{16}{\pi \times 42} = 0.1212$$

and $$\tan \alpha_2 = \frac{p_2}{\pi d_2} = \frac{12}{\pi \times 44} = 0.0868$$

Let $W$ = Load that can be lifted in N.

We know that torque required to overcome friction at the upper screw,

$$T_1 = W \tan (\alpha_1 + \phi) \frac{d_1}{2} = W\left[\frac{\tan \alpha_1 + \tan \phi}{1 - \tan \alpha_1 \tan \phi}\right]\frac{d_1}{2}$$

$$= W\left[\frac{0.1212 + 0.15}{1 - 0.1212 \times 0.15}\right]\frac{42}{2} = 5.8\, W \text{ N-mm}$$

Similarly, torque required to overcome friction at the lower screw,

$$T_2 = W \tan (\alpha_2 - \phi) \frac{d_2}{2} = W\left[\frac{\tan \alpha_2 - \tan \phi}{1 + \tan \alpha_2 \tan \phi}\right]\frac{d_2}{2}$$

$$= W\left[\frac{0.0868 - 0.15}{1 + 0.0868 \times 0.15}\right]\frac{44}{2} = -\,1.37\, W \text{ N-mm}$$

∴ Total torque required to overcome friction,

$$T = T_1 - T_2 = 5.8\, W - (-\,1.37\, W) = 7.17\, W \text{ N-mm}$$

We know that the torque required when there is no friction,

$$T_0 = \frac{W}{2\pi}(p_1 - p_2) = \frac{W}{2\pi}(16 - 12) = 0.636\, W \text{ N-mm}$$

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∴ Efficiency of the screw jack,

$$\eta = \frac{T_0}{T} = \frac{0.636\ W}{7.17\ W} = 0.0887 \text{ or } 8.87\% \quad \textbf{Ans.}$$

***2. Load that can be lifted***

Since the upper screw is subjected to a larger torque, therefore the load to be lifted ($W$) will be calculated on the basis of larger torque ($T_1$).

We know that core diameter of the upper screw,

$$d_{c1} = d_o - p_1 = 50 - 16 = 34 \text{ mm}$$

Since the screw is subjected to direct compressive stress due to load $W$ and shear stress due to torque $T_1$, therefore

Direct compressive stress,

$$\sigma_c = \frac{W}{A_{c1}} = \frac{W}{\frac{\pi}{4}(d_{c1})^2} = \frac{W}{\frac{\pi}{4}(34)^2} = \frac{W}{908} \text{ N/mm}^2$$

and shear stress,

$$\tau = \frac{16\ T_1}{\pi\,(d_{c1})^3} = \frac{16 \times 5.8\ W}{\pi(34)^3} = \frac{W}{1331} \text{ N/mm}^2$$

We know that maximum shear stress ($\tau_{max}$),

$$28 = \frac{1}{2}\sqrt{(\sigma_c)^2 + 4\tau^2} = \frac{1}{2}\sqrt{\left(\frac{W}{908}\right)^2 + 4\left(\frac{W}{1331}\right)^2}$$

$$= \frac{1}{2}\sqrt{1.213 \times 10^{-6}\ W^2 + 2.258 \times 10^{-6}\ W^2} = \frac{1}{2}\,1.863 \times 10^{-3}\ W$$

$$\therefore \quad W = \frac{28 \times 2}{1.863 \times 10^{-3}} = 30\ 060 \text{ N} = 30.06 \text{ kN} \quad \textbf{Ans.}$$

## EXERCISES

1. In a hand vice, the screw has double start square threads of 24 mm outside diameter. If the lever is 200 mm long and the maximum force that can be applied at the end of lever is 250 N, find the force with which the job is held in the jaws of the vice. Assume a coefficient of friction of 0.12. **[Ans. 17 420 N]**
2. A square threaded bolt of mean diameter 24 mm and pitch 5 mm is tightened by screwing a nut whose mean diameter of bearing surface is 50 mm. If the coefficient of friction for the nut and bolt is 0.1 and for the nut and bearing surfaces 0.16, find the force required at the end of a spanner 0.5 m long when the load on the bolt is 10 kN. **[Ans. 120 N]**
3. The spindle of a screw jack has a single start square thread with an outside diameter of 45 mm and a pitch of 10 mm. The spindle moves in a fixed nut. The load is carried on a swivel head but is not free to rotate. The bearing surface of the swivel head has a mean diameter of 60 mm. The coefficient of friction between the nut and screw is 0.12 and that between the swivel head and the spindle is 0.10. Calculate the load which can be raised by efforts of 100 N each applied at the end of two levers each of effective length of 350 mm. Also determine the efficiency of the lifting arrangement. **[Ans. 9945 N ; 22.7%]**

*Lead screw supported by collar bearing.*