UNIT- I (PART – A)
INTRODUCTION OF OPERATIONS RESEARCH

1.1 BACKGROUND:

Technology - the application of science to the everyday Physical World - has, on the whole, changed our lives greatly for the better, particularly since the time of Industrial Revolution. Manufacturing industry, which supplies us with and at the same time, creates the wealth, which enables other services to exist, owes its existence to Science and Technology. Some manufactured items, aircraft for example, are both complex and expensive, and there are great difficulties, first of all in organizing their manufacture and then, once made, in using them efficiently. Problems of this latter kind, often called systems problems, were studied rigorously for the first time by scientists, not in industry but in the management of operations in the 1939 - 45 war.

A number of teams of eminent scientists were employed by the British Government to apply their expertise to management and operational problems rather than technical problems. One such team (the Radar Operational Research team - which gave OR its name) was responsible for implementing the installation and operation of radar after the technical development work was complete. Later on another team examined the relative ineffectiveness of the Allied Forces at destroying the German U-boats, which were sinking the food convoy ships supplying Britain. The OR team played an important part in reducing shipping losses and in the ultimate defeat of the U-boats. With no precedent in the application of Science to the management function, the scientists were still able to use their scientific approach-collecting information and developing hypothesis in order to come up with practical plans for the improvement of these wartime operations.

1.2 MEANING AND DEFINATION OF OPERATIONS RESEARCH:

OR has been defined in various ways and it is perhaps too early to define it in some authoritative way. However given below are a few opinions about the definition of OR which have been changed along-with the development of the subject.

"OR is a scientific method of providing executive departments with a quantitative basis for decision regarding the operations under their control"

"OR is a scientific method of providing executives with any analytical and objective basis for decisions"

"OR is the application of scientific methods, techniques and tools to problems involving the operations of systems so as to provide those in control of the operations with optimum solutions to the problem".

1.3 Nature of operation Research:

Operation research can be considered as being the application of scientific method by interdisciplinary teams to problems involving the control of organized systems so as to provide solutions which best serves the purposes of the organization as a whole. Different characteristics constitution the nature of operation research can be discussed below

1) **Inter disciplinary team approach:**
   Operation research has the characteristics that it is done by a team of scientists drawn from various disciplines such as mathematics, statistics, economics, engineering, physics, etc. It is essentially an interdisciplinary team approach.
2) **Systems approach:**
   Operation research emphasizes on the overall approach to the system. This characteristic of operation research is often referred as system orientation. The orientation is based on the observation that in the organized systems the behavior of any part ultimately has some effect on every other part.
3) **Helpful in improving the quality of solution:**
   Operation research cannot give perfect answers or solutions to the problems, it merely gives bad answers to the problems which otherwise have worst answers. Thus operation research simply helps in improving the quality of the solution but does not result into a perfect solution.
4) **Scientific method:**
   Operation research involves scientific and systematic attack of complex problems to arrive at the optimum solution. In other words, operation research uses techniques of scientific research. Thus operation research comprehends both aspects i.e. it includes both scientific research on the phenomena of operating systems and the associated engineering activities aimed at applying the results of research.
5) **Goal oriented optimum solution:**
   Operation research tries to optimize a well defined function subject to given constraints and as such is concerned with the optimization theory

1.4 Scope of operation research in Management:

Operation research is a problem solving and decision making science. It is a kit of scientific and programmable rules providing the management a 'quantitative basis' for decisions regarding the operations under its control. Some of the areas of management where operation research techniques have been successfully applied are:

(1) Allocation and distribution:

- Optimal allocation of limited resources such as men, machines, materials, time and money.
- Location and size of warehouse, distribution centers, retail depots etc.
- Distribution policy.

(2) Production and facility planning:

- Selection, location and design of production plants, distribution centers and retail outlets.
- Project scheduling and allocation of resources
- Determination of the number and size of the items to be produced
- Maintenance policy and preventive maintenance.

(3) Procurement:

- What, how and when to purchase at the minimum procurement cost.
- Bidding and replacement policies
- Transportation planning and vendor analysis.

(4) Marketing:

- Product selection, timing and competitive actions.
- Selection of advertising media.
- Demand forecasts and stock levels.
- Best time to launch a new product.

(5) Finance:

- Capital requirements, cash-flow analysis
- Credit policies, credit risks etc.
- Profit plan for the company.

(6) Personnel:

- Selection of personnel, determination of retirement age and skills.
- Recruitment policies and assignment of jobs
- Wage or salary administration

1.5 Quantitative Analysis as a Frame Work for Managerial Decision:

Managerial decision-making is a process by which the management, when faced with a problem chooses a specific course of action from a set of possible options. In making a decision, a business manager attempts to choose the most effective course of action in the given circumstances in attaining the goals of the organization. The various types of decision-making situations that a manager might encounter can be listed as follows:

1. Decisions under certainty, where all facts are known fully and for sure, or under uncertainty where the event that would actually occur is not known but probabilities can be assigned to various possible occurrences.
2. Decisions for one time period only, called static decisions, or a sequence of interrelated decisions made either simultaneously or over several time periods, called dynamic decisions.
3. Decisions where the opponent is nature (digging an oil well, for example) or a rational opponent (for instance, setting the advertising strategy when the actions of competitors have to be considered.

4. These classes of decision-making situations are not mutually exclusive and a given situation would exhibit characteristics from each class. Stocking of an item for sale in a certain trade fair, for instance, illustrates a static decision-making situation where uncertainty exists and nature is the opponent.

The elements of any decision are:

1. A decision-maker, who could be an individual, group, organization, or society.
2. A set of possible actions that may be taken to solve the decision problem.
3. A set of possible states that might occur.
4. A set of consequences(pay-off) associated with various combinations of courses of action and the states that may occur and
5. The relationship between pay-off and the values of the decision-maker

In an actual decision-making situation, the definition and identification of alternatives, the states and the consequences, are most difficult, albeit not the most crucial aspects of the decision problem.

In real life, some decision-making situations are simple while others are not. Complexities in decision situations arise due to several factors. These include the complicated manner of interaction of the economic, political, technological, environmental and competitive forces in society, the limited resources of an organization, the values, risk attitudes and knowledge of the decision-makers and the like.

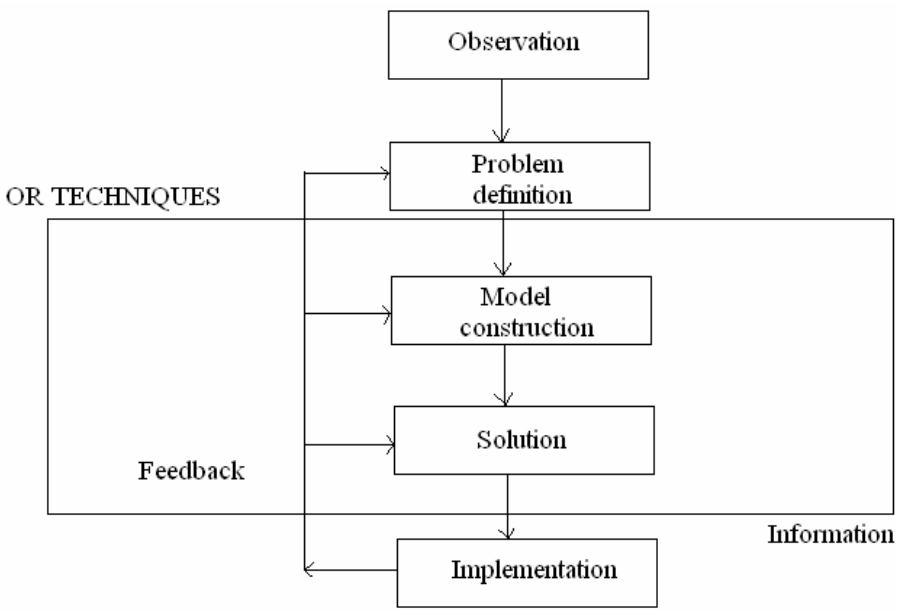
1.6 OR APPROACH TO PROBLEM SOLVING:

OR encompasses a logical systematic approach to problem solving

This approach to problem solving as shown in below fig follows a generally recognized ordered set or steps:

1. Observation,
2. Definition of the problems,
3. Model construction,
4. Model solution, and
5. Implementation of solution results.

OR PROCESS



(1) Observation

The first step in a problem solving exercises in OR is the identification of a problem that exists in the system. This requires that the system be continuously and closely observed so that problems can be identified as soon as they occur.

(2) Definition of the Problem

Once it has determined that a problem exists, it must be clearly and concisely defined. The problem definition includes the limits of the problems and the degree to which it pervades other organs of the system. A requirement of problem definition is that the goals (or objective) must be clearly defined which helps to focus attention on what the problem is.

(3) Model Construction

An OR model is an abstract representation of an existing problem situation. It can be in the form of a graph or chart, but mostly, an OR model consists of a set of mathematical relationship. In OR terminology, these are called objective function and constraints.

(4) Model Solution

Once models are constructed, they are solved using the OR techniques, presented in the next section. Actually it is difficult to separate model construction and solution in most cases, since OR technique usually applies to a specific type of model. Thus, the model type and solution method are both part of the OR technique.

(5) Implementation of Results

The results of an OR technique are information which helps in making a decision. The beauty of OR process lies in obtaining, the results which are implement able or we call it a feasible whole exercise will go waste.

1.7 APPLICATIONS OF OPERATIONS RESEARCH

As mentioned earlier OR can be applied in every field of life. Here are few of the many fields where OR has potential application. This list is by no means comprehensive or exhaustive but definitely will provide an idea of the power of OR as a separate discipline.

1.7.1 Operations Research in the Public Sector

Federal, Provincial and Local Government

- Development of Country Structure Plans
- Manpower Planning and Career Development in Govt. Departments
- Organization of Long-Term planning groups at the National Level
- Corporate Planning in Local Government
- Allocation of Government Houses
- Estimation of Future Requirement of School/College Building
- Placing of Fire Brigade in a City
- Measuring the Effectiveness of Police
- Timetabling in Schools and Colleges for Efficient use of Space

Health

- Management policies for 120-bed nursing units
- Optimum size of general hospitals
- Appointment systems for hospital outpatients
- Stock control for regional and area health units
- National and area planning of health services
- Manpower planning for nurses, radiographers, etc.
- Simulation of pathology laboratories
- Organizing an ambulance service
- Care provided by community nurses

Defense

- Arms control and disarmament studies
- Communications network development
- Logistic support in operations
- Field experimentation
- War games and other models of battle
- Equipment procurement
- Reinforcement and redeployment problems

1.7.2 Operations Research in Industry & Commerce

Finance and Investment

- Developing the five-year plan for a food manufacturer
- Development of the pipeline
- Computer based financial planning
- Structure for the assets of a bank
- Evaluating investment in a new plant
- Corporate planning in the chemical industry
- Financing expansion of a small firm

Production

- Production scheduling in a steel works
- Meeting peak demands for electricity
- Minimization of costs of power station maintenance
- Scheduling newsprint deliveries
- Stock levels of steel plate
- Meeting seasonal demands for products
- Blending scrap metals
- Stock policy for a paint manufacturer
- Allowing for yarn breaks in spinning
- Meeting customer requirements for carpets
- Planning a quarry's output
- Optimum layout for belt coal transport in a colliery

Marketing

- Launching a new product
- Advertising effectiveness and cost
- Planning sales territories
- Measurement of consumer loyalty
- Buyer-seller behavior
- Advertising research and media scheduling
- Most profitable retail brand mix

Personnel

- Personnel shift planning
- Manpower planning
- Manpower for an assembly line
- Effects of flexible working hours

Distribution

- Distribution of Products.
- Returnable bottles: how many?
- Refinery crude tank capacity
- Depot location of pharmaceutical products
- Trucking policy for dairy products
- Distribution of newspapers to newsagents

1.7.3 OR in Transport

Rail

- Rail freight management
- Required fleet size of locomotives and rolling stock
- Forecasting passenger traffic
- Planning reconstruction of main-line termini
- Introduction of freightliners

Road

- Designing urban road networks
- Forecasts of car ownership
- Implementation of bus lanes
- Re-routing bus services
- Purchasing and maintenance of buses
- Introduction of flat-fare buses
- Bus services in rural areas
- Preparation of crew rosters

Air

- Planning the introduction of Boeing 737/Airbus 300
- Allocation of aircraft and crew to routes
- Location of Islamabad Airport
- Karachi-Lahore - Islamabad - Peshawer: aircraft requirements

Sea

- Potential traffic for new container services
- Shipbuilding requirement in the 1990's
- Optimum ship size for given routes
- Construction and management of a container terminal

1.8 QUANTITATIVE SPECIALISTS AND THE MANAGER

Manager gets thing done and with people in an organization. Managers do planning, organizing, staffing, directing and controlling. He plays different roles such as interpersonal role, informational role and divisional role. Management functions and roles don't exist opposite to each other but these are two ways of interpreting what manager also. In order to perform various management functions effectively, managers must possess certain skills. These skills are technical, human and conceptual skills.

Quantitative specialists perform calculations and apply mathematical and statistical tools and methods to process the raw information's are processed and presented in orderly manner, manager uses these data to make logical decisions.

RELATIONSHIP BETWEEN THE QUANTITATIVE SPECIALISTS AND THE MANAGER

There exists a strong bond between Quantitative specialists and the manager. Quantitative specialists provide valuable data to manager, on the basis of these data manager takes important decisions for the organization. Information is processed by Quantitative specialists. He applies various books and techniques to process the raw information. Manager uses these data for important decision making. Quantitative techniques provide round fundamental base for decision making. Quantitative specialists and manager both work for the benefit of the organization. Every important decision is based on conclusion drawn by Quantitative specialists. Although, manager is the person who makes the decision but Quantitative specialists is the person who supports the manager in the decision-making process. Quantitative specialists are concerned with only mathematical and statistical tools and techniques, while manager is the overall head of the organization. Manager performs execution of operations and managing the company.

Finally, we can say that there exists a strong bond between Quantitative specialists and manager. Position of both of them is very vital for the organization.

UNIT- II (PART – B)
LINEAR PROGRAMMING PROBLEMS

2.1 INTRODUCTION:

Linear Programming is that branch of mathematical programming which is designed to solve optimization problems where all the constraints as well as the objectives are expressed as linear function .It was developed by George B. Denting in 1947. Its earlier application was solely related to the activities of the second' World War. However soon its importance was recognized and it came to occupy a prominent place in the industry and trade.

Linear Programming is a technique for making decisions under certainty i.e.; when all the courses of options available to an organization are known & the objective of the firm along with its constraints are quantified. That course of action is chosen out of all possible alternatives which yield the optimal results. Linear Programming can also be used as a verification and checking mechanism to ascertain the accuracy and the reliability of the decisions which are taken solely on the basis of manager's experience-without the aid of a mathematical model.

Linear equality function:

A function $f(x_1, x_2 \ldots x_n)$ of $x_1, x_2 \ldots x_n$ is a linear function if and only if for some set of constants $c_1, c_2 \ldots c_n$

$$f(x_1, x_2 \ldots x_n) = c_1x_1+c_2x_2+\ldots+c_nx_n$$

Linear inequality function:

For any linear function $f(x_1, x_2 \ldots x_n)$ any number b, the inequalities

$$f(x_1, x_2 \ldots x_n) \leq b$$

$$f(x_1, x_2 \ldots x_n) \geq b$$

2.2 Linear programming Definition:

Linear programming is an optimization problem for which we do the following

- We attempt to maximize (or minimize) a linear function of the decision variables. The function that is to be maximized or minimized is called the objective function.
- The values of the decision variables must satisfy a set of constraints. Each constraint must be a linear equation or linear inequality.
- A sign restriction is associated with each variable for any variable x_i the sign restriction specifies that x_i must be either nonnegative ($x_i \geq 0$) or unrestricted in sign.

Example: 1

Max $Z = 3x_1 + 2x_2$ (Objective function)

Subjected to

$2x_1 + x_2 \leq 100$ (constraints)

$x_1 + x_2 \leq 40$

$x_1, x_2 \geq 0$ (sign restriction)

Example: 2

Min $z = 4x_1 + 3x_2$ (objective function)

Subjected to

$3x_1 + 4x_2 \geq 20$(constraints)

$x_1 - x_2 \geq 15$

$x_1, x_2 \geq 0$ (sign restriction)

"Subjected to" means that the values of the decision variables x_1 and x_2 must satisfy all constraints and all sign restrictions.

2.3 Linear Programming problem Formation

Steps in Formulating a Linear Programming Model

Linear programming is one of the most useful techniques for effective decision making. It is an optimization approach with an emphasis on providing the optimal solution for resource allocation. How best to allocate the scarce organizational or national resources among different competing and conflicting needs (or uses) forms the core of its working. The scope for application of linear programming is very wide and it occupies a central place in many diversified decisional problems. The effective use and application of linear programming requires the formulation of a realistic model which represents accurately the objectives of the decision making subject to the constraints in which it is required to be made.

The basic steps in formulating a linear programming model are as follows:

Step I.

Identification of the decision variables: The decision variables (parameters) having a bearing on the decision at hand shall first be identified, and then expressed or determined in the form of linear algebraic functions or in equations.

Step II.

Identification of the constraints: All the constraints in the given problem which restrict the operation of a firm at a given point of time must be identified in this stage. Further these constraints should be broken down as linear functions in terms of the pre-defined decision variables.

Step III.

Identification of the objective: In the last stage, the objective which is required to be optimized (i.e., maximized or memorized) must be dearly identified and expressed in terms of the pre-defined decision variables.

2.4 SOME IMPORTANT DEFINITIONS

(1)Feasible solution:

All such solutions which can be worked out under given constraints are called "feasible solution" and region comprising such solution is called the "feasible region".

(2) Optimal solution:

Optimum means either maximum or minimum. The object of obtaining the feasible optimum solution may be maximization of profit or minimization of cost. Optimum solution is the best of all feasible solution.

(3) Slack variables:

If the constraint has the sign '≤' to make that inequality into equality a positive variable is added to L.H.S of the constraints this positive variable is called a slack variable.

Example:

$$\text{Max } Z = 100x_1+125x_2$$

Subjected to

$$4x_1+6x_2 \le 24$$
$$4x_1+2x_2 \le 16$$
$$x_1, x_2 \ge o$$

Introducing slack variable

$$\text{Max } Z = 100x_1+125x_2+0s_1+0s_2$$

Subjected to

$$4x_1+6x_2+1s_1+0s_2 = 24$$
$$4x_1+2x_2 +0s_1+1s_2 = 16$$
$$x_1, x_2, s_1, s_2 \ge o$$

(3) Surplus variables:

If the constraint has the sign '≥' to make that inequality into equality a positive variable is subtracted from the L.H.S of the constraints this positive variable is called a surplus variable.

Example:

$$\text{Min } Z = 100x_1+125x_2$$

Subjected to

$$4x_1+6x_2 \geq 24$$
$$4x_1+2x_2 \geq 16$$
$$x_1, x_2 \geq 0$$

Introducing surplus variable

$$\text{Max } Z = 100x_1+125x_2-0s_1-0s_2$$

Subjected to

$$4x_1+6x_2-1s_1-0s_2 = 24$$
$$4x_1+2x_2 -0s_1-s_2 = 16$$
$$x_1, x_2, s_1, s_2 \geq 0$$

2.5 SOLUTION OF LINEAR PROGRAMMING PROBLEMS

Linear programming problems can be solved by following methods:

(1) Graphical method

(2) Simplex method

2.5.1 GRAPHICAL METHOD

Graphical method can be used only for two or three variables; whereas the simplex method can be used for any number of variables.

The various steps involved in the Graphical method are as follows

(1) The constraints are converted temporarily into equalities (into equations) and plotted on the graph

(2) The region satisfying each constraint is determined and then the region common to the entire constraint region is located. The region is known as feasible region, as all the points lying in this region will simultaneously satisfy the given constraints.

If the feasible region is shaded it takes the shape of some form of polygon.

(3) Compute the coordinates of all corner points of the feasible region. Any points inside the feasible region is called feasible solution and provides values of x1 and x2 (decision variables) that satisfy all the constraints.

(4) The objective function is also plotted on the same graph paper by taking some arbitrary value of Z

(5) The next step is to use trial and error method to locate the optimal solution inside the feasible region so that the objective function is optimized. It is experienced that the

optimal solution is located near the extreme points of the feasible region. It may be noted that the feasible region may be bounded or unbounded. Linear programming problem may give infeasible solution if there is inconsistency between two constraints.

2.5.2 SIMPLEX METHOD:

Procedure:

Step 1:

Check the objective function is in maximization or minimization. If it is in minimization convert it into maximization as follows

$$\text{Min } Z = -\text{ Max } Z$$

$$= \text{Max } Z^1$$

Step 2:

Check all b_i's are non-negative. If any one of b_i is negative convert it into non-negative by multiplying the constraints by -1

Step 3:

Convert the inequalities into equalities by introducing slack or surplus variables according to the conditions.

Step 4:

Find the Initial basic feasible solution

Step 5:

(a) Simplex table

		c_j	c_1	c_2	0	0	
Basic variable	C_B	X_B	x_1	x_2	s_1	s_2	Min$\{\frac{x_B}{x_j}\}$
s_1	0	b_1	a_{11}	a_{12}	1	0	
s_2	0	b_2	a_{21}	a_{22}	0	1	
		Z_j	$c_B.x_j$				
		Δ	$Z_j - c_j$				

(b) Calculate the value $\Delta = Z_j - c_j$ and also calculate $Z_j = c_B.x_j$

Where c_B denotes the cost of the basic variables in the objective function.

c_j denotes the cost of all variables in the objective function.

x_j denotes the cost of variables in the given constraints.

(c) If allΔ's ≥ 0 then initial basic feasible solution is optimal

(d) If atleast one $\Delta < 0$ the solution is not optimal then go to next step.

Step 5:

Find key column and key row the intersection of key column and key row is the key element.

Step 6:

Make the key element as '1' and all other elements in that column as zero's by elementary row operations.

Key column = Min $\{\Delta_1, \Delta_2, \ldots\}$

Key row = Min $\{\frac{x_B}{x_j}, x_i > 0\}$

Here negative ratios are not considered

Entering vector = Key column vector

Leaving Vector = Key row vector

Step 8:

Find the values of the and test the solution for optimality as in step 5 and step 6

Step 9:

Find the optimal solution