# UNIT- IV (PART – B)
# PERT & CPM (THEORY)

## 4.1 INTRODUCTION:

Network Scheduling is a technique used for planning, and scheduling large projects in the field of construction, maintenance, fabrication etc. The technique is a method of minimizing the trouble spots such as production, delays and interruptions, by determining critical factors and coordinating various parts of the overall job.

A project is defined as a combination of interrelated activities all of which must be executed in a certain order for its completion.

The work involved in a project can be divided into three phases corresponding to the management functions of planning, scheduling and control.

**Planning:**

This phase involves setting the objectives of the project and the assumptions to be made. Also it involves listing of tasks or jobs that must be performed to complete a project under consideration. In this phase, men, machines and materials required for the project in addition to the estimates of costs and duration of the various activities of the project are also determined.

**Scheduling:**

This consists of laying the activities according to the precedence order and determining,

- The start and finish times for each activity
- The critical path on which the activity require special attention and
- The slack and float for the non critical paths

**Controlling:**

This phase is exercised after the planning and scheduling, which involves the following;

- Making periodical progress reports
- Reviewing the progress
- Analyzing the status of the project and
- Management decisions regarding updating, crashing.

## 4.2 BASIC TERMS:

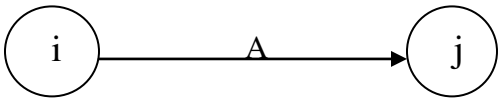
**Network:**

It is the graphic representation of logically and sequentially connected arrows and nodes representing activities and events of a project. Networks are also called arrow diagram.

**Activity:**

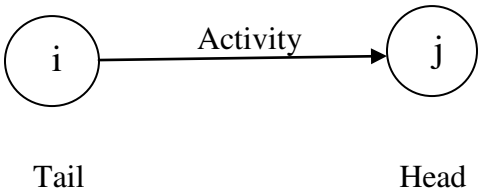
An activity represents some action and is a time consuming effort necessary to complete a particular part of the overall project. Thus, each and every activity has a point of time where it begins and a point where it ends.

It is represented in the network by an arrow



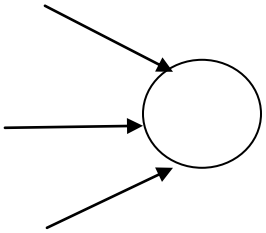

**Event:**

The beginning and end points of a activity are called events or nodes. Event is a point in the time and does not consume any resources. It is represented by a numbered circle. The head event called the jth event has always a number higher than the tail event called the ith event.

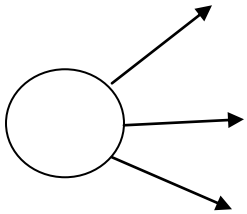



**Merge and burst events:**

It is not necessary for an event to be the ending event of only one activity but can be the ending event of two or more activities. Such event is defined as a merge event.



If the event happens to be the beginning event of two or more activities it is defined as a Burst event

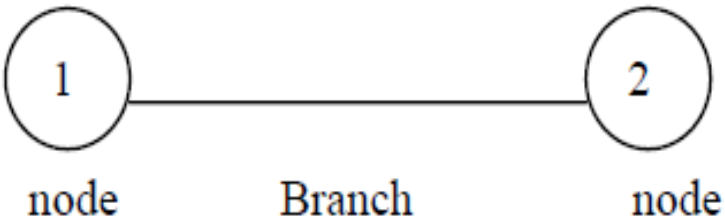


## 8.3 CONCEPT OF NETWORK

A 'network' is defined as a graphic representation with a flow of some type in its branches. It represents nodes and branches. Below in table, we represent different systems satisfying the definition of network in the physical world.

| Physical situation | Nodes | Branches | Flow |
|---|---|---|---|
| Highway systems | Intersections | Roads | Vehicles |
| Communication systems | Switching points | Wires | Messages |
| Fluid supply systems | Pumping stations | Pipes | Fluid |
| Production systems | Work centres | Handling routes | Jobs |
| Project Management | Decision points | Activities | Time |
| Airway systems | Airports | Airlines | Aircraft |

A node is the intersection of the two branch lines. It is denoted by a circle. Each branch represents an activity. Each node represents an event, which is a specific definable accomplishment recognizable at a particular instant of time. The arrowheads indicate the sequence in which events must be achieved.



Thus an event is the completion of all the activities leading into that node and this event must precede the initiation of the activity leading out of the node

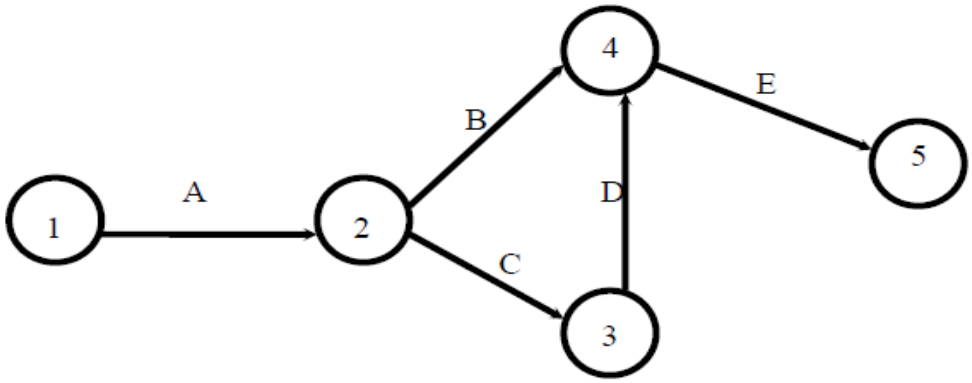
An arrow diagram represents a project graph. An arrow connecting two nodes, representing two events, represents each activity. The head of the arrow identifies the start of the activity. Let us take an example and illustrate the construction of arrow diagram for a project.

**Example 1**: A company is interested in preparing a budget. The details of the activities and the departments involved are given in the Table

**The project of preparation of a Production budget.**

| Job Identification | Alternate | Job Description | Department |
|---|---|---|---|
| A | 1-2 | Forecasting sales | Sales |
| B | 2-4 | Pricing Sales | Sales |
| C | 2-3 | Preparing production Schedule | Engineering |
| D | 3-4 | Costing the production | Costing |
| E | 4-5 | Preparation of budget | President |

The project of budgeting can be displayed in a network or project graph by an arrow diagram. Jobs are shown as arrows leading from one node to the other as in Figure




From the arrow diagram, we infer that activity A is the first job. Jobs B and C start only after A is over. A is called the predecessor of B and C and B and C the successors of A.

**RULES FOR CONSTRUCTION OF NETWORK**

1. Each activity is represented by one and only one arrow. This means that no single activity can be represented twice in a network.
2. No two activities can be identified by the same end events. This means that there should not be loops in the network.
3. Time follows from left to right. All the arrows point in one direction. Arrows pointing in opposite direction must be avoided.
4. Arrows should not cross each other.
5. Every node must have at least one activity preceding it and at least one activity following it, except for the nodes at the very beginning and at the very end of the network.

## 4.4 CRITICAL PATH METHOD (CPM)

**Determination of Floats and Slack Times:**

Float is defined as the difference between the latest and the earliest activity time.

Slack is defined as the difference between the latest and the earliest event time.

Hence the basic difference between the slack and float is that slack is used for events only; whereas float is used for activities.

There are mainly three kinds of floats as given below.

**Total float:** It refers to the amount of time by which the completion of an activity could be delayed beyond the earliest expected completion time without affecting the overall project duration time.

Mathematically, the total float of an activity (i, j) is the difference between the latest start time and the earliest start time of that activity.

Hence the total float for an activity (i, j) denoted by $(Tf)_{ij}$ is calculated by the formula

$$(TF)_{ij} = (\text{Latest start} - \text{Earliest start}) \text{ for activity } (i,j)$$

$$\text{i.e. } (TF)_{ij} = (LS)_{ij} - (ES)_{ij}$$

$$\text{Or } (TF)_{ij} = (L_j - E_i) - t_{ij}$$

Where $E_i$, $L_j$ are the earliest time and latest time for the tail event I and head event j and $t_{ij}$ is the normal time for the activity (i, j). This is the most important type of float as it concerns with the overall project duration.

**Free float:** The time by which the completion of an activity can be delayed beyond the earliest finish time without affecting the earliest start of a subsequent succeeding activity.

Mathematically, the free float for activity (i, j) denoted by $(Ff)_{ij}$ can be calculated by the formula.

$$Ff_{ij} = (E_j - E_i) - t_{ij}$$

$$Ff_{ij} = \text{Total float} - \text{Head event slack}$$

$$\text{Head event slack} = L_j - E_j$$

This float is concerned with the commencement of subsequent activity.

The free float can take values from zero up to total float, but it can't exceed total float. This float is very useful for rescheduling the activities with minimum disruption of earlier plans.

**Independent float:** The amount of time by which the start of an activity can be delayed without affecting the earliest start time of any immediately following activities assuming that the preceding activity has finished at its latest finish time.

Mathematically, independent float of an activity (i,j) denoted **by** $If_{ij}$ can be calculated by the formula

$$If_{ij} = (E_j - L_i) - t_{ij}$$

Or

$$If_{ij} = \text{Free float} - \text{Tail event slack}$$

Where tail event slack is given by

$$\text{Tail event slack} = L_i - E_i$$

The negative independent float is always taken as zero. This float is concerned with prior and subsequent activities.

$$IF_{ij} \leq FF_{ij} \leq TF_{ij}$$

The iterative procedure of determining the critical path is as follows.

**Step 1:** List all the jobs and then draw arrow (network) diagram. Each job is indicated by an arrow with the direction of the arrow showing the sequence of jobs. The length of the arrows has no significance. The arrows are placed based on the predecessor, successor, and concurrent relation within the job.

**Step 2:** Indicate the normal time ($t_{ij}$) for each activity (i, j) above the arrow which is deterministic.

**Step 3:** Calculate the earliest start time and the earliest finish time for each event and write the earliest time $E_i$ for each event i in the ▭

Also calculate the latest finish and latest start time. From this we calculate the latest time $L_j$ for each event j and put in the △ .

**Step 4:** Tabulate the various times namely normal time, earliest time and latest time on the arrow diagram.

**Step 5:** Determine the total float for each activity by taking the difference between the earliest start and the latest start time.

**Step 6:** Identify the critical activities and connect them with the beginning event and the ending event in the network diagram by double line arrows. Which gives the critical path?

**Step 7:** Calculate the total project duration.

## 4.5 PROGRAMME EVALUATION AND REVIEW TECHNIQUE (PERT)

The network methods discussed so far may be termed as deterministic, since estimated activity times are assumed to be known with certainty. However, in research project or design of gear box or a new machine various activities are based on judgment. It is difficult to obtain a reliable time estimate due to the changing technology. Time values are subject to chance variations. For such cases where the activities are non-deterministic in nature, PERT was developed. Hence, PERT is a probabilistic method where the activity times are represented by a probability distribution. This probability distribution of activity times is based upon three different time estimates made for each activity. These are as follows

1. Optimistic time estimate
2. Most likely time estimate
3. Pessimistic time estimate

1. **Optimistic time estimate:** It is the smallest time taken to complete the activity if everything goes on well. There is very little chance that activity can be done in time less than the optimistic time. It is denoted by $t_0$ or a.
2. **Most likely time estimate:** It refers to the estimate of the normal time the activity would take. This assumes normal delays. It is the mode of the probability distribution. It is denoted by
3. **Pessimistic time estimate:** It is the longest time that an activity would take if everything goes wrong. It is denoted by $t_p$ or b. These three time values are shown in the following figure.

From these three time estimates, we have to calculate the expected time of on activity. It is given by the weighted average of the three time estimates.

$$\mathbf{t_e} = \frac{to+4tm+tp}{6}$$

[β distribution with weights of 1,4,1, for $t_0$,$t_m$ and $t_p$ estimates respectively]

Variance of the activity is given by

$$\boldsymbol{\sigma}^2 = \left[\frac{tp-t0}{6}\right]^2$$

The expected length (duration), denoted by to $T_e$ of the entire project is the length of the critical path (i.e.) the sum of the $t_e$'s of all the activities along the critical path.

The main objective in the analysis through PERT is to find the completion for a particular event within specified date $T_0$ given by P (Z≤D)

Where

$$\mathbf{D} = \frac{Due\ date - Expected\ date\ of\ completion}{\sqrt{Project\ variance}}$$

Where Z stands for standard normal variable.

**PERT Procedure:**

**Step 1:** Draw the project network.

**Step 2:** Compute the expected duration of each activity using the formula.

$$\mathbf{t_e} = \frac{t0+4tm+tp}{6}$$

Also calculate the expected variance $\sigma^2$ of each activity.

**Step 3:** Compute the earliest start, earliest finish, latest start, latest finish and total float of each activity.

**Step 4:** Find the critical path and identify the critical activities.

**Step 5:** Compute the project length variance $\sigma^2$ which is the sum of the variance of all the critical activities and hence find the standard deviation of the project length σ.

**Step 6:** Calculate the standard normal variable $\mathbf{Z=\frac{Ts-Te}{\sigma}}$ where $T_s$ is the scheduled time to complete the project.

$T_e$ = Normal expected project length duration.

σ= Expected standard deviation of the project length.

Using the normal curve, we can estimate the probability of completing the project within a specified time.

### 4.6 Applications of PERT and CPM

The following are the suggested applications when PERT or CPM is found useful.

1. The construction of a building or of a highway.
2. Planning and launching a new product.
3. Scheduling maintenance for a project.
4. The manufacture and assembly of a large machine tool.
5. To conduct a music or drama festival.
6. Preparation of budget for a company.