UNIT – V

REPLACEMENT MODELS

Introduction: The Replacement Problem The replacement problems are concerned with the situations that arise when some items such as men, machines, electric-light bulbs, etc. need replacement due to their decreased efficiency, failure or breakdown. Such decreased efficiency or complete breakdown may either be gradual or all of a sudden.

The replacement problem arises because of the following factors:

1. The old item has become in worse condition and work badly or requires expensive maintenance.

2. The old item has failed due to accident or otherwise and does not work at alt, or the old item is Expected to fail shortly.

3. A better or more efficient design of machine or equipment has become available in the market.

In the case of items whose efficiency go on decreasing according to their age, it requires to spend more money on account of increased operating cost, increased repair cost, increased scrap, etc. So in such cases, the replacement of an old item with new one is the only alternative to prevent such increased expenses.

Thus the problem of replacement is to decide best policy to determine an age at which the replacement is most economical instead of continuing at increased cost. The need for replacement arises in many situations so that different type of decisions may have to be taken. For example,

i. We may decide whether to wait for complete failure of the item (which might cause some loss), or to replace earlier at the expense of higher cost of the item.

ii. The expensive items may be considered individually to decide whether we should replace now or, if not, when it should be reconsidered for replacement.

iii. It may be decided whether we should replace by the same type of item or by different type (latest model) of item.

The problem of replacement is encountered in the case of both men and machines. Using probability it is possible to estimate the chance of death (or failure) at various ages.

The main objective of replacement is to direct the organization for maximizing its profit (or minimizing the cost).

Failure Mechanism of Items

The term 'failure' has a wider meaning in business than what it has in our daily life. There are two kinds of failure.

1. Gradual Failure: The mechanism under this category is progressive. That is, as the life of an item increases, its efficiency deteriorates, causing:

i. Increased expenditure for operating costs,

ii. decreased productivity of the equipments,

iii. Decrease in the value of the equipment, i.e., the resale of saving value decreases.

For example, mechanical items like pistons, bearings, rings etc. Another example is 'Automobile tyres '.

2. Sudden Failure: This type of failure is applicable to those items that do not deteriorate markedly with service but which ultimately fail after some period of using. The period between installation and failure is not constant for any particular type of equipment but will follow some frequency distribution, which may be progressive, retrogressive or random in nature.

i. Progressive failure: Under this mechanism, probability of failure increases with the increase in the life of an item. For example, electric light bulbs, automobile tubes, etc.

ii. Retrogressive failure: Certain items have more probability of failure in the beginning of their life, and as the time passes the chances of failure become less. That is, the ability of the unit to survive in the initial period of life increases its expected life. Industrial equipments with this type of distribution of life span are exemplified by aircraft engines.

iii. Random failure: Under this failure, constant probability of failure is associated with items that fail from random causes such as physical shocks, not related to age. In such a case, virtually all items fail before aging has any effect. For example, vacuum tubes in air-borne equipment have been shown to fail at a rate independent of the age of the tube.

The replacement situations may be placed into four categories:

1. Replacement of capital equipment that becomes worse with time, e.g. machines tools, buses in a transport organization, planes, etc.

2. Group replacement of items that fail completely, e.g., light bulbs, radio tubes, etc.

3. Problems of mortality and staffing.

4. Miscellaneous Problems.

Replacement of Items that Deteriorate

Costs to be Considered

In general, the costs to be included in considering replacement decisions are all those costs that depend upon the choice or age of machine. In some special problems, certain costs need not be included in the calculations. For example, in considering the optimum decision of replacement for a particular machine, the costs that do not change with the age of the machine need not be considered.

When The Replacement Is Justified?

This question can easily be answered by considering a case of truck owner whose problem is to find the 'best' time at which he should replace the old truck by new one. The truck owner wants to transport goods as cheaply as possible. The associated costs are:

(i) The running costs, and (ii) the capital costs of purchasing a truck

These associated costs can be expressed as average cost per month. Now the truck owner will observe that the average monthly cost will go on decreasing, longer the replacement is postponed. However, there will come an age at which the rate of increase of running costs more than compensates the saving in average capital costs. Thus, at this age the replacement is justified.

Replacement Policy for Items Whose Maintenance Cost:

Increases with Time, and Money Value is Constant:

The cost of maintenance of a machine is given, as a function increasing with time and its scrap value is constant.

a. If time is measured continuously, then the average annual cost will be minimized by replacing the machine when the average cost to date becomes equal to the current maintenance cost.

b. If time is measured in discrete units, then the average annual cost will be minimized by replacing the machine when the next period's maintenance cost becomes greater than the current average cost.

a) Time value of money does not change:

If the value of money does not change with time, then the user of the equipment does not need to pay interest on his investments. We wish to determine the optimal time to replace the equipment.

We make use of the following notations:

C: Capital cost of the equipment

S: Scrap value of the equipment

n: Number of years that the equipment would be in use.

$f(t)$: Maintenance cost function.

$A(n)$: Average total annual cost.

Replace the equipment at the end of n years if the maintenance cost in the (n+1) th year is more than the average total cost in the nth year and the nth year's maintenance cost is less than previous year's average total cost.

Example: A firm is considering replacement of a machine, whose cost price is Rs, 1,20,000, and the scrap value is Rs. 20,000. The running (maintenance and operating) costs of the machine are as follows:

Year	1	2	3	4	5	6	7	8
Running cost (Rs.)	2,000	5,000	8,000	12,000	18,000	25,000	32,000	40,000

When should the machine be replaced?

Sol:

C = Rs. 1, 20,000

S = Rs. 20,000

f (n) = Running cost in the nth year

We prepare the following table

Year of service (n)	Running cost (Rs.) $f(n)$	Cumulative running cost (Rs.) $\sum_{n} f(n)$	$TC = C - S + \sum_{0}^{n} f(t)$ (Rs.)	$A(n) = \frac{1}{n} TC$ (Rs.)
1	2,000	2,000	1,02,000	1,02,000
2	5,000	7,000	1,07,000	53,500
3	8,000	15,000	1,15,000	38,334
4	12,000	27,000	1,27,000	31,750
5	18,000	45,000	1,45,000	29,000
6	25,000	70,000	1,70,000	**28,334**
7	32,000	1,02,000	2,02,000	28,857
8	40,000	1,42,000	2,42,000	30,250

Thus A (6) = Rs. 28,334 is minimum. Hence replacement should be done after every sixth year.

b) Time value of money changes:

In this case, the investor is paying interest on the money that has been invested. We assume that

- The maintenance costs are incurred at the beginning of time periods; and
- The maintenance costs increase with time.

Let the money carry an interest rate of r% per year, i.e., one rupee in n years time is equivalent to Rs. $(1+r)^{-n}$ today.

$(1+r)^{-n}$ is called the **present worth factor of rupee one spent *n* years after now**. $(1+r)^{n}$ is the **payment compound amount factor of rupee one** spent in *n* years time.

Let

$\boldsymbol{C}$: Initial cost of the equipment

$\boldsymbol{R_n}$: Running cost of the equipment in year *n*.

$\boldsymbol{v}$: Rate of interest = $(1+r)^{-1}$

$\boldsymbol{V_n}$: Future discounted costs associated with the policy of replacing the Equipment at the end of each *n* years.

Thus the optimal replacement policy is:

a) Do not replace the equipment if the next period's operating cost is less than the weighted average of previous costs.

b) Replace the equipment if the next period's operating cost is greater than the weighted average of previous costs.

$$For\ v = 1, \qquad R_{n-1} < V_n < R_n$$

Example: A manufacturer is to make a choice between two machines, say, *A* and *B*, which are priced at Rs. 50,000 and Rs. 25,000 respectively. The annual running costs for machine *A* are Rs. 8,000 for first five years after which the costs increase per year by Rs. 2,000. Machine *B*, which has the same capacity as the machine *A*, will have a running cost of Rs. 12,000 for first six years, and after that would increase by Rs. 2,000 per year

If the money is worth 10% per year, which machine should be purchased? Assume that the scrap value of the two machines is nil.

Solution: r = 0.1

$$v = \frac{1}{1+0.1} = 0.9091$$

We prepare the following tables:

For machine A

Year (n)	R_{n-1}	v^{n-1}	$v^{n-1}R_{n-1}$	$C+\sum v^{k-1}R_{k-1}$	$\sum v^{k-1}$	W_n
1	8000	1.0000	8000	58000	1.0000	58000
2	8000	0.9091	7272.80	57272.80	1.9091	34190.35
3	8000	0.8264	6611.20	56611.20	2.7355	26278.19
4	8000	0.7513	6010.40	56010.40	3.4868	22339.8
5	8000	0.6830	5464	55464	4.1698	19990.98
6	10000	0.6209	6209	56209	4.7907	18696.1
7	12000	0.5645	6774	56774	5.3552	17990.25
8	14000	0.5132	7184.80	57184.80	5.8684	17641.3
9	16000	0.4665	7464	57464	6.3349	**17520.43**
10	18000	0.4241	8482	58482	6.7590	17525.53

For machine B

Year (n)	R_{n-1}	v^{n-1}	$v^{n-1}R_{n-1}$	$C+\sum v^{k-1}R_{k-1}$	$\sum v^{k-1}$	W_n
1	12000	1.0000	12000	37000	1.0000	37000
2	12000	0.9091	10909.20	47909.20	1.9091	25095.18
3	12000	0.8264	9916.80	57826	2.7355	21139.10
4	12000	0.7513	9015.60	66841.60	3.4868	19169.90
5	12000	0.6830	8196	75037.60	4.1698	17995.49
6	14000	0.6209	8692.60	83730.20	4.7907	17477.65
7	12000	0.5645	6774	90504.20	5.3552	16900.25
8	16000	0.5132	8211.20	98715.40	5.8684	**16821.52**
9	18000	0.4665	8397	107112.40	6.3349	16908.30
10	20000	0.4241	8482	115594.40	6.7590	17102.29

For machine *A*, the running cost in the ninth year of operations is the least so it should be replaced after every ten years.

For machine *B*, the running cost in the eighth year of operations is the least so it should be replaced after every eight years.

Further the weighted average cost in ten years of machine *A* is Rs. 17505.53, whereas the weighted average cost in eight years of machine *B* is Rs. 16821.52. So machine *B* should be purchased.