UNIT-II (PART – A)

TRANSPORTATION PROBLEMS

INTRODUCTION

F.I. Hitchaxic developed the basic transportation problem in 1941. However it could be solved for optimally as an answer to complex business problem only in 1951, when George B. Dantzig applied the concept of Linear Programming in solving the Transportation models.

Transportation models or problems are primarily concerned with the optimal (best possible) way in which a product produced at different factories or plants (called supply origins) can be transported to a number of warehouses (called demand destinations). The objective in a transportation problem is to fully satisfy the destination requirements within the operating production capacity constraints at the minimum possible cost. Whenever there is a physical movement of goods from the point of manufacture to the final consumers through a variety of channels of distribution (wholesalers, retailers, distributors etc.), there is a need to minimize the cost of transportation so as to increase the profit on sales. Transportation problems arise in all such cases. It aims at providing assistance to the top management in ascertaining how many units

OBJECTIVE:

The objective is to transport various quantities of a single homogeneous product that are initially stored at various origins, to different destinations in such a way that the total transportation cost is minimum.

DEFINITION:

Transportation model is the problem of determining the minimum cost for allocating a product from several supply sources to several destinations.

MATHEMATICAL MODEL OF TRANSPORTATION PROBLEM

Mathematically a transportation problem is nothing but a special linear programming problem in which the objective function is to minimize the cost of transportation subjected to the demand and supply constraints.

Let a_i = quantity of the commodity available at the origin i,

b_j = quantity of the commodity needed at destination j,

c_{ij} = transportation cost of one unit of a commodity from origin i to destination j,

and x_{ij} = quantity transported from origin I to the destination j.

Mathematically, the problem is

Minimize $z = \Sigma\Sigma\, x_{ij}\, c_{ij}$

Subject to

$\Sigma x_{ij} = a_{i,}$ i= 1, 2…m

$\Sigma x_{ij} = b_{j,}$ j= 1, 2… n

and $x_{ij} \geq 0$ for all i and j .

3.5 SOME IMPORTANT DEFINITIONS:

(1) FEASIBLE SOLUTION:

Any set of non negative allocations which satisfies the row and column sum is called a feasible solution.

(2) BASIC FEASIBLE SOLUTION:

A feasible solution is called a basic FEASIBLE SOLUTION if the number of non negative allocations is equal to m+n-1.Where m is the number of rows, n is the number of columns in a transportation table.

(3) OPTIMAL SOLUTION:

A feasible solution (not necessarily basic) is said to be optimal if it minimizes the total transportation cost.

(4) DEGENERATE BASIC FEASIBLE SOLUTION:

When the number of positive allocation at any stage of feasible solution is less than the required number [row (m) + Column (n) – 1] the solution is said to be degenerate.

(5) NON-DEGENERATE BASIC FEASIBLE SOLUTION:

When the number of positive allocation at any stage of feasible solution is equal to the required number [row (m) + Column (n) – 1] the solution is said to be non-degenerate.

(6) OCCUPIED CELL:

Cell in the transportation table having positive allocation will be called occupied cells.

(7) UNOCCUPIED CELL:

Cell in the transportation table having no positive allocation will be called unoccupied cells or empty cells.

(8) BALANCED TRANSPORTATION PROBLEM:

A transportation problem is said to be balanced if the total supply from all the sources equals the total demand in all the destinations.

Total supply = total demand

(9)UN BALANCED TRANSPORTATION PROBLEM:

A transportation problem is said to be unbalanced if the total supply from all the sources not equals the total demand in all the destinations.

Total supply ≠ total demand

1. NORTH-WEST CORNER RULE:

Procedure:

1. Check the given transportation problem is neither balanced nor unbalanced.
2. Write the cost matrix with the cost in the brackets.
3. Give maximum possible supply to the top most left corner cell.
4. If anything remains, move to right hand side other wise move downwards.
5. Repeat the process until all supply and demand is completed.

Advantages:

- This method is very effective as it provide step by step solution.
- It is very simple to obtain optimum solution through this method.

Disadvantages:

- This method does not take into consider the important factor i.e. cost which is sought to be minimized.
- North-west corner rule take more time in obtaining optimal solution.

2. LEAST COST or MATRIX MINIMA METHOD:

Procedure:

1. Check the given transportation problem is neither balanced nor unbalanced.
2. Write the cost matrix with the cost in the brackets.
3. Select the cell lowest cost of transportation and allocate as much units as possible to the cell if the minimum cost appears in more than one cell then select any one cell.
4. From the remaining cells select the lowest cost cell and allocate as much as unit possible
5. Repeat the process until all supply and demand is completed.

Advantages:

- This method provides accurate solution as Transportation cost is consider while making allocation.
- It is very simple and easy to calculate optimum solution under this method.

Disadvantages:

- This method doesn't follow step by step rule for obtaining optimal solution.
- This method is based on the selection through personal observation when there is a tie in the minimum cost.

3. VOGEL'S APPROXIMATION METHOD OR PENALTY METHOD

Penalty: in the cost matrix the difference between two lower costs is called penalty of that row or column

Procedure:

1. Check the given transportation problem is neither balanced nor unbalanced.
2. Write the cost matrix with the cost in the brackets.
3. Find the penalties for each row and each column.
4. Choose the maximum penalty row or column and allocate maximum possible supply or demand to the lowest cost cell if there are more than one maximum penalty allocate to the cell associated with the lowest cost.
5. Repeat the process until all supply and demand is completed.

Advantages:

- This method is very systematic
- This method takes lesser time in solving transportation problem.
- Less computation are involved in this method.

Disadvantages:

- This method provides approximation solution to the given problem.
- This method is tedious when the given matrix is a large one.

SPECIAL CASES IN TRANSPORTATION PROBLEM:

UNBALANCED TRANSPORTATION PROBLEM:

The given transportation problem is said to be unbalanced if the total supply is not equal to the total demand.

There are two possible cases

<u>Case-i:</u>

$$\sum_{i=1}^{m} a_i < \sum_{j=1}^{n} b_j$$

If the total supply is less than the total demand a dummy row is added to the cost matrix with zero cost. Hence the unbalanced transportation problem can be converted in to a balanced transportation problem.

<u>Case-ii:</u>

$$\sum_{i=1}^{m} a_i > \sum_{j=1}^{n} b_j$$

If the total supply is greater than the total demand a dummy column is added to the cost matrix with zero cost. Hence the unbalanced transportation problem can be converted in to a balanced transportation problem.

UNIT – II (PART – B)

ASSIGNMENT PROBLEMS

OBJECTIVE:

Assignment problem is to assign a number of origins (jobs) to the equal number of destinations (persons) at a minimum cost or maximum profit.

DEFINITION:

Suppose there are n jobs to be performed and n persons are available for doing these jobs. Assume that each person can do each job at a time, though with varying degree of efficiency. Let c_{ij} be the cost if the ith person is assigned to the jth job. The problem is to find an assignment (which job should be assigned to which person on one-one basis) so that the total cost of performing all jobs is minimum. Problems of this kind are known as assignment problem.

ASSIGNMENT PROBLEM MODEL:

The assignment problem can be stated in the form of n×n cost matrix $[c_{ij}]$ of real numbers as given in the following table

Jobs

		1	2	……	n
Persons	1	c_{11}	c_{12}	…….	c_{1n}
	2	c_{21}	c_{22}	…….	c_{2n}
	⋮	⋮	⋮	⋮	⋮
	n	c_{n1}	c_{n2}	……..	c_{nn}

HUNGARIAN METHOD:

The Hungarian method (developed by Hungarian Mathematician D.Konig) is the efficient method of finding optimal solution to the assignment problems. It works on the principle of reducing the given cost matrix to a matrix of opportunity costs. Hence, it is also called reduced matrix method. It involves a rapid reduction of the original matrix and finding at least one zero in each row and column which results in optimal solution.

The various steps involved in Hungarian method can be summarized as follows:

1. Prepare a cost matrix .If the cost matrix is not a square matrix then add a dummy row (column) with zero cost element.
2. Subtract the minimum element in each row from all the elements of the respective rows.
3. Further modify the resulting matrix by subtracting the minimum element of each column from all the elements of the respective columns. Thus, obtain the modified matrix.
4. Then, draw minimum number of horizontal and vertical lines to cover all zeros in the resulting matrix. Let the minimum number of lines N. Now there are two possible cases.
   - ✓ Case i: If N=n where n is the order of matrix, then an optimal assignment can be made. So make the assignment to get the required solution.
   - ✓ Case ii: If N<n, then proceed to step 5.
5. Determine the smallest uncovered element in the matrix. Subtract this minimum element from all uncovered elements and add the same element at the intersection of horizontal and vertical lines. Thus, the second modified matrix is obtained.
6. Repeat step (3) and (4) until we get the case (i) of step 3.
7. (To make zero assignment) Examine the rows successively until a row-wise exactly single zero is found. Circle (O) this zero to make the assignment. Then make a cross (×) over all zeros if lying in the column of the circled zero. Showing that they can't be considered for future assignment. Continue in this manner until all the zeros have been examined. Repeat the same procedure for column also.
8. Repeat the step 6 successively until one of the following situation arises----
   - ✓ If no unmarked zero is left, then the process ends or
   - ✓ If there lies more than one of the unmarked zero in any column or row then, circle one of the unmarked zeros arbitrarily and mark a cross in the cells of remaining zeros in its row or column. Repeat the process until no marked zero is left in the matrix.
9. Thus exactly one marked circled zero in each row and each column of the matrix is obtained. The assignment corresponding to these marked circled zeros will give the optimal assignment.