ADVANCED 3G AND 4G WIRELESS COMMUNICATIONS

UNIT-1

WIRELESS COMMUNICATIONS AND DIVERSITY

Introduction to 3G/4G standards:-

- **Wireless communication systems have become an integral part of our lives, especially during the last decade.**
- **3G stands for third generation wireless communication systems**
- **4G stands for fourth generation wireless communication systems**

2G WIRELESS SYSTEMS AND STANDARDS:-

GENERATION	STANDARD	DATARATE
2G	GSM=Global Systems for Mobile Communication	10kbps
2G	CDMA= Code Division for Multiple Access	10kbps
2.5G	GPRS= General PacketRadio Service	~50kbps
2.5G for GSM Evolution	EDGE=Edge Standard or Enhanced Data	~200kbps

- The GSM standard has a basic voice data rate of 10kbps.This is used for voice call that you place from one GSM mobile phone to another GSM mobile phone. This is operated in narrow band spectrum.
- The second standard CDMA was developed to enable internet access or is technically known as accessing best effort packet data, over cellular networks. This is operated in wide band spectrum.
- The GSM and CDMA standards belong to 2G.
- Among the two 2.5G standards, the first is GPRS which is used to access packet data or especially to access internet over your GSM mobile phones.
- Another competing 2.5G or sometimes also known as a 2.75G standard is EDGE which has a data rate approximately 200 kbps.
- So this is the family that is GSM,CDMA,GPRS,EDGE these are the family of 2.5G,2.2G,2.5G wireless communication standards ,enable to place voice calls from mobile to mobile ,also some basic internet access packet data access over cellular networks.

ADVANCED 3G AND 4G WIRELESS COMMUNICATIONS

3G WIRELESS SYSTEMS AND STANDARDS:-

GENERATION	STANDARD	DATARATE
3G	WCDMA = Wideband Code Division Multiple Access/ UMTS= Universal Mobile Telecommunication standards	384Kbps
3G	CDMA 2000	384Kbps
3.5G	HSDPA=high speed downlink packet access/ HSUPA=high speed uplink packet access	5-30Mbps
3.5G	1XEVDO=evolution data optimized rev a,b,c	5-30Mbps

- The 3G standard WCDMA is also known as UMTS, which has higher data rates compared to GSM and GPRS, which are data rates around 10kbps for voice call,to 50Kbps for packet or internet access.
- The CDMA 2000 is another 3G standard which has a data rate of 384Kbps. In this standard, 2000 is the year roughly in which it has introduced.
- The 3.5G standard is HSDPA/HSUPA .These are two directions in which communication can take place, one is from the base station to the mobile that is known as downlink , the other is the mobile to the base station that is known as the uplink.
- The another 3.5G standard is 1XEVDO, where EVDO stands for evolution data optimized.

4G WIRELESS SYSTEMS AND STANDARDS:-

GENERATION	STANDARD	DATARATE
4G	LTE=Long Term Evolution	100-200Mbps
4G	WiMAX=World wide Interoperability For Microwave Access	100-200Mbps

These standards have more bandwidth than the previous generation standards

These two standards are most popular standards, currently in development and deployment in different countries in the world.

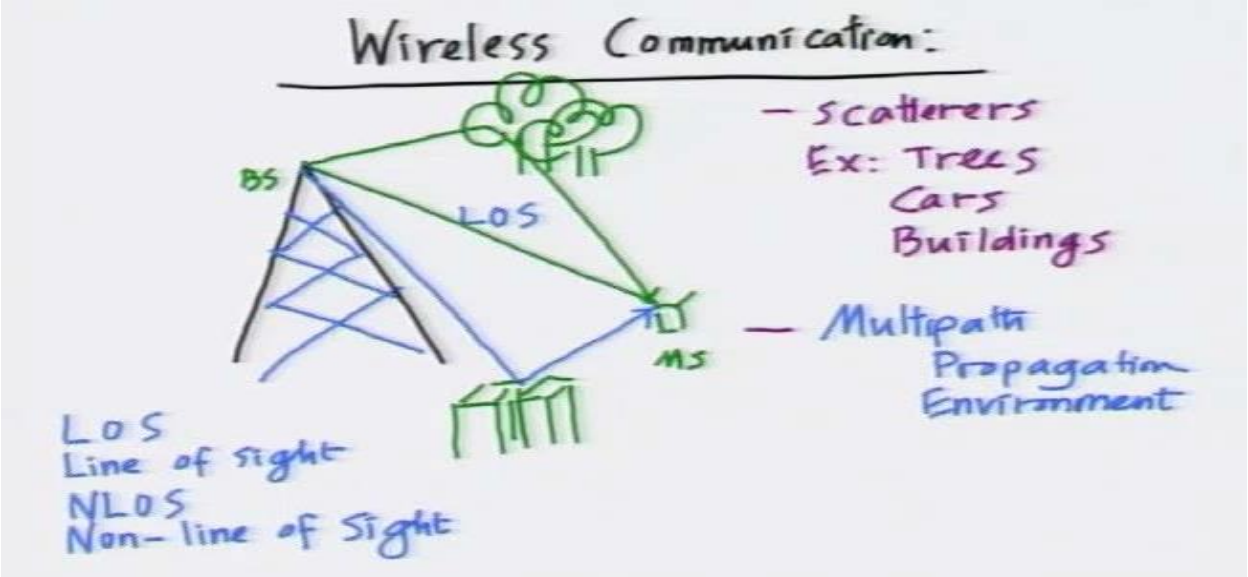
ADVANCED 3G AND 4G WIRELESS COMMUNICATIONS

LIST OF GENERATIONS WITH DATA RATES AND USES OR APPLICATION:-

GENERATION	DATARATE	APPLICATION/USE
2G,2.5G	10-100Kbps	voice+ data
3G,3.5G	30Kbps-30Mbps	voice+ data+ video calling and Conferences
4G	100Mbps-200Mbps	online gaming, HDTV

- So, as the result of the increasing demand for higher bandwidth applications has led to the progressive development of 3G,3.5G and 4G wireless communication systems.

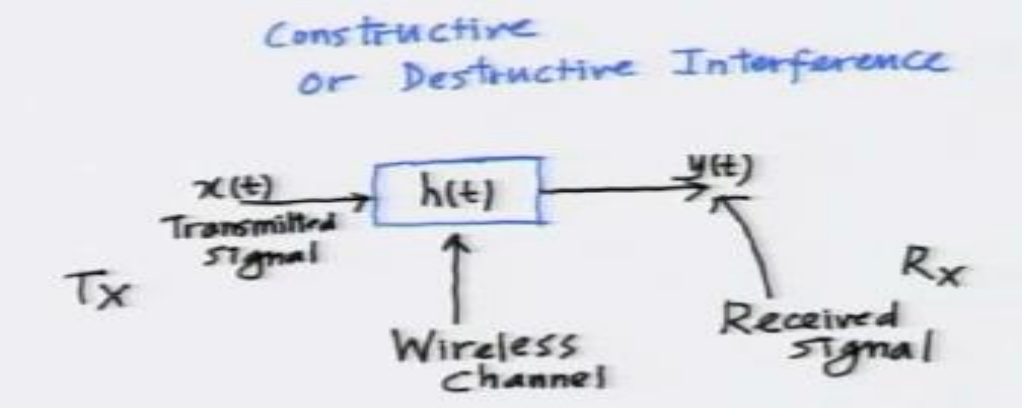
WIRELESS COMMUNICATION:



- A wireless communication systemtypically containsa base stationthat is transmitting to a mobile terminal or mobile phones or that is technically also known as a mobile station.
- Base station, that is mounted typically at a very high at a height on a tower and a mobile station can be seen in figure.
- In a wireless communication system, that the radio environment is open, in addition to the direct propagation, that is the direct line of sight (LOS), propagation between the base station and the mobile station.
- There are also several reflected components, that arise in the environment namely some scatter in the environment, such as trees, buildings etc.. and there might also be other scatters such as moving and which also scatter.

ADVANCED 3G AND 4G WIRELESS COMMUNICATIONS

- The wireless communication environment is very different than the wire line communication environment, because it doesn't have only the direct path between the base station and mobile station, but also have many scattered or reflected paths and these objects which scatter the wireless signal, are known as scatterers.
- These are typically objects such as trees,cars,vehicles,large buildings.In rural scenario large mountains,hill rocks and so on are the scatterers.
- So these are the scatterers, which implies at the receiver not only have a single component, but have multiple components,that you have multiple signal ,arriving at the mobile station, through not just a single path,but multiple paths and hence this is known as a "multipath propagation environment", because unlike wire line channels in a wireless system, there is a direct path, and there are also many scatter paths, and there are multiple paths.
- The direct path is known as the LOS or line of sight and the scatter paths are known as NLOS or non line of sight components.
- In such environment, each wave comes through a different distance is subject to an attenuation. Attenuation because of free space losses and also because the distance is different the delay is different which means the phase that it arrives with at the mobile station is different.
- The signalsdepending on the different delayshave a phase factor and depending on the delays,they can either add constructively to produce constructive interference that is increase in the amplitude of the net signal, or at times they can also add destructively, that is they can cancel out each other,to produce destructive interference.



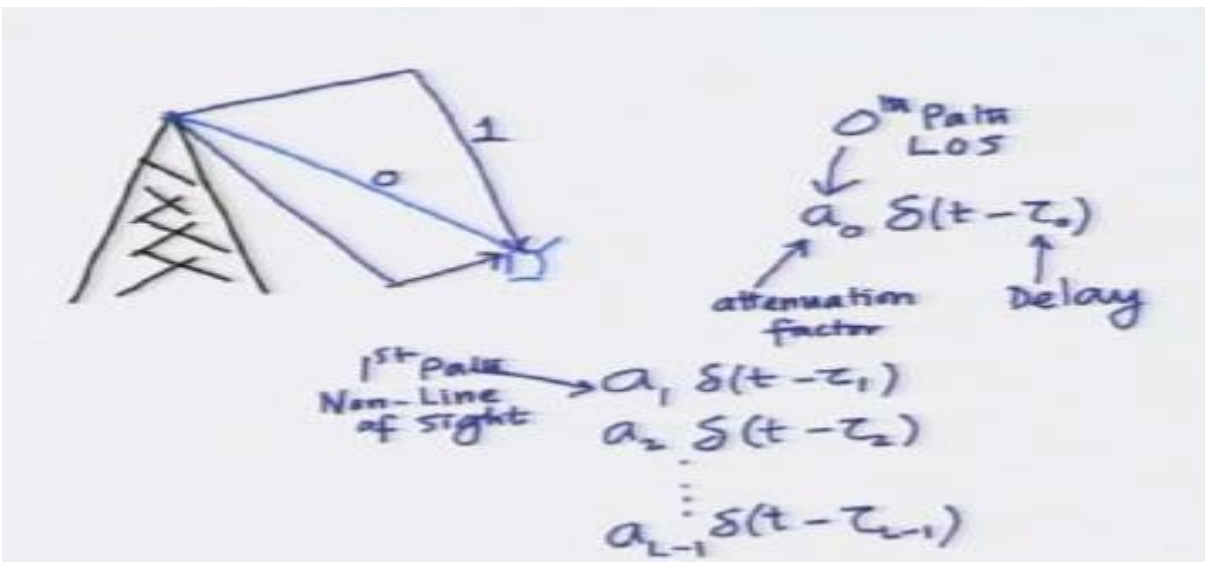
- Wireless communications is multipath components add with different phase factors,because of the delays and different attenuations arising, because of free space losses and scattering, and hence it results in constructive or destructive ,which means if it is constructive interference its good, because the signal strength increase, if is destructive interference strength decreases or signal level goes down
- Wireless communication environment, is an adverse environment , because multipath propagation , or rather multipath interference then the signal level is low, in which implies that there is no reception of signals

ADVANCED 3G AND 4G WIRELESS COMMUNICATIONS

- In the above figure,h(t) is the impulse response ,x(t) is the input signal transmitted by the base station(t) is the output signal that is received by the mobile station .The impulse response of the wireless channel is h(t)

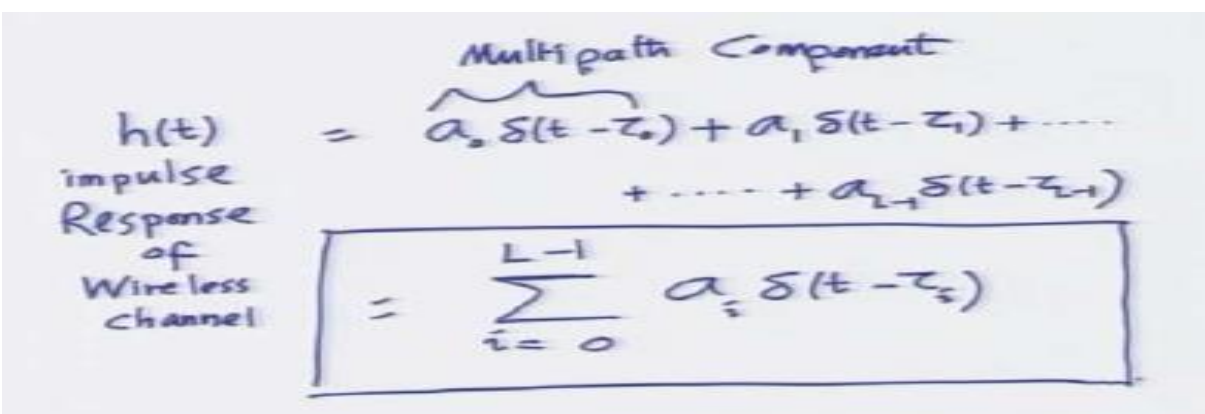
  Y(t)=x(t)*h(t)

- The basic wireless system is redrawn, which contains LOS path indicated by 0 and NLOS path indicated by 1.The system contains direct path and several scatter paths between the base station and the mobile station.



One such path is direct path indicated by 0th path having the attenuation factor (a_0) and delay(τ_0).This is also known as direct line of sight path and delay τ_1 ,that it has an impulse response $a1\delta(t-\tau_1)$.This is also known as non line of site and similarly ,we can have a second path with attenuation factor a_2 and delay τ_2 and so on we can have upto L-1 paths with attenuation factor L-1 and delay τ(L-1)

Impulse response of wireless channel

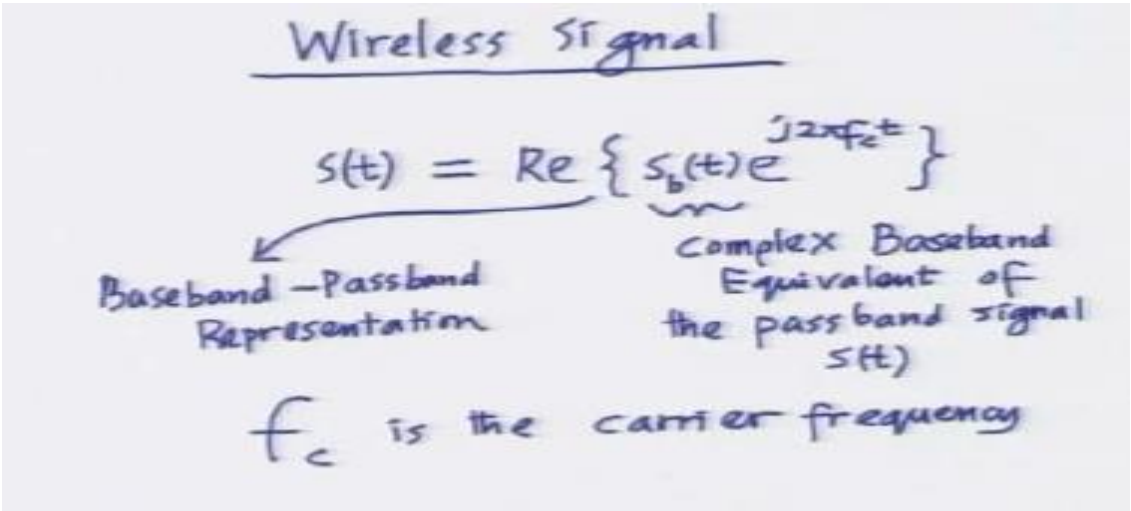


ADVANCED 3G AND 4G WIRELESS COMMUNICATIONS

Each of these path is known as a multipath component.This channel has multipath components;one of them is index 0, is LOS component, the other one is index 1, the NLOS or scatter component.

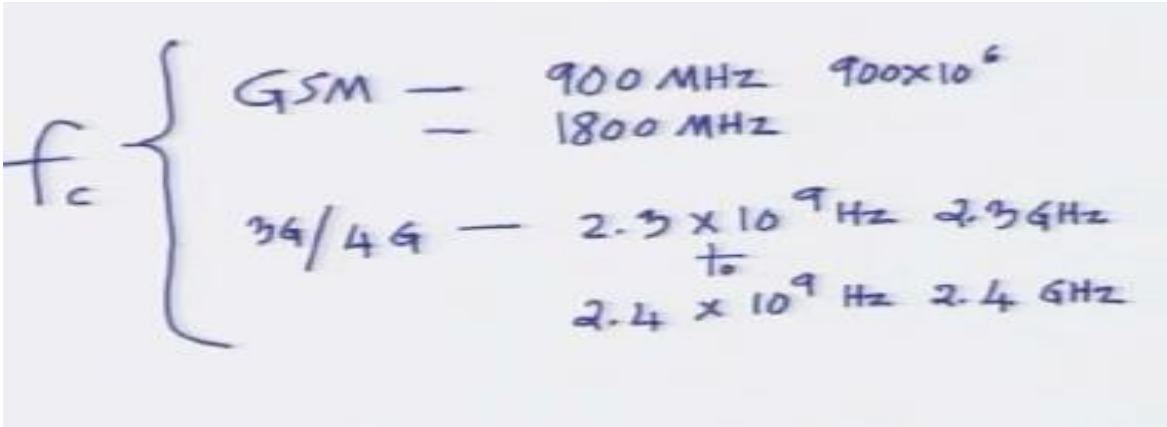
WIRELESS SIGNAL/CHANNEL :-

- Wireless signal is the signal that is transmitted by the base station which is denoted by s(t).It is a pass band signal, that is its transmitted at a carrier frequency



fc is the carrier frequency

fc is the carrier frequency , this is allocated to that particular cellular network operator. There is couple of frequencies allotted, specific allotted carrier frequencies to current systems.



- It is important to allocate different carrier frequencies, for different operators,especially because when transmit over the air, each one needs a unique spectral band, so that these signals do not interfere over there

ADVANCED 3G AND 4G WIRELESS COMMUNICATIONS

- So, spectrum is an important part, so before transmit the base station, up converts the base band signal, to the allocated carrier frequency, and at the receiver at the mobile station , down coverts the received signal, back to the base band.
- The transmitted signal s(t) is represented as

$$s(t) = Re\{ s_b(t) e^{j2\pi f_c t} \}$$

The impulse response of wireless channel h(t) is

$$h(t) = \sum_{i=0}^{L-1} a_i \delta(t - T_i)$$

The received signal at base station y(t) is

$$y(t) = s(t) \times h(t) = Re\{ s_b(t) e^{j2\pi f_c t} \} * \sum_{i=0}^{L-1} a_i \delta(t - T_i)$$

$$y_0(t) = Re\{ a_0 s_b(t - T_0) e^{j2\pi f_c (t - T_0)} \}$$

$$y_1(t) = Re\{ a_1 s_b(t - T_1) e^{j2\pi f_c (t - T_1)} \}$$

$$\vdots$$

$$y_{L-1}(t) = Re\{ a_{L-1} s_b(t - T_{L-1}) e^{j2\pi f_c (t - T_{L-1})} \}$$

Net signal

$$y(t) = Re\{ \sum_{i=0}^{L-1} a_i s_b(t - T_i) e^{j2\pi f_c (t - T_i)}$$

ADVANCED 3G AND 4G WIRELESS COMMUNICATIONS

1. The net signal can be represented as the sum of all signals arriving from the multipath components or the sum of essentially all the signals copies arriving through the different paths.

2.The received signal at the mobile station or the mobile phone can be represented as the combination of all the signals corresponding to each path.

$$y(t) = Re\left\{\sum_{i=0}^{L-1} a_i S_b(t-T_i)\, e^{j2\pi f_c (t-T_i)}\right\}$$

$$= Re\left\{\underbrace{\left\{\sum_{i=0}^{L-1} a_i S_b(t-T_i)\, e^{-j2\pi f_c T_i}\right\}}_{\text{Complex signal}} e^{j2\pi f_c t}\right\}$$

Complex base band Rx signal is

$$y_b(t) = \sum_{i=0}^{L-1} a_i S_b(t-T_i)\, \underbrace{e^{-j2\pi f_c T_i}}_{\text{Complex phase factor}}$$

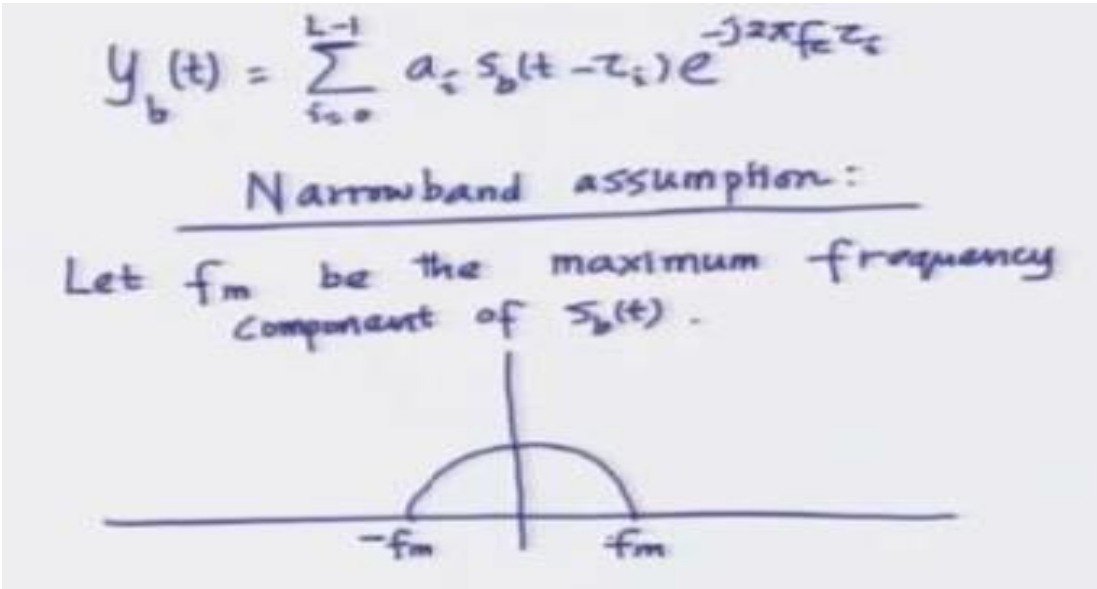
COMPARISON OF WIRELINE AND WIRELESS SYSTEMS:-

- The wire line or wired communication systems, in which the signal propagates on a wire. The output of wired or wire line system is $y_b(t) = S_b(t)$.
- In a wireless communication systems, since the radio environment is open, in addition to direct propagation i.e., direct LOSpropagation between the base station and the mobile station. There are also several reflected components namely some scatter in the environment such as trees,buildings, cars and so on.
  The output of wireless system is $y_b(t) = hS_b(t)$.
  Where h is the complex fading coefficient

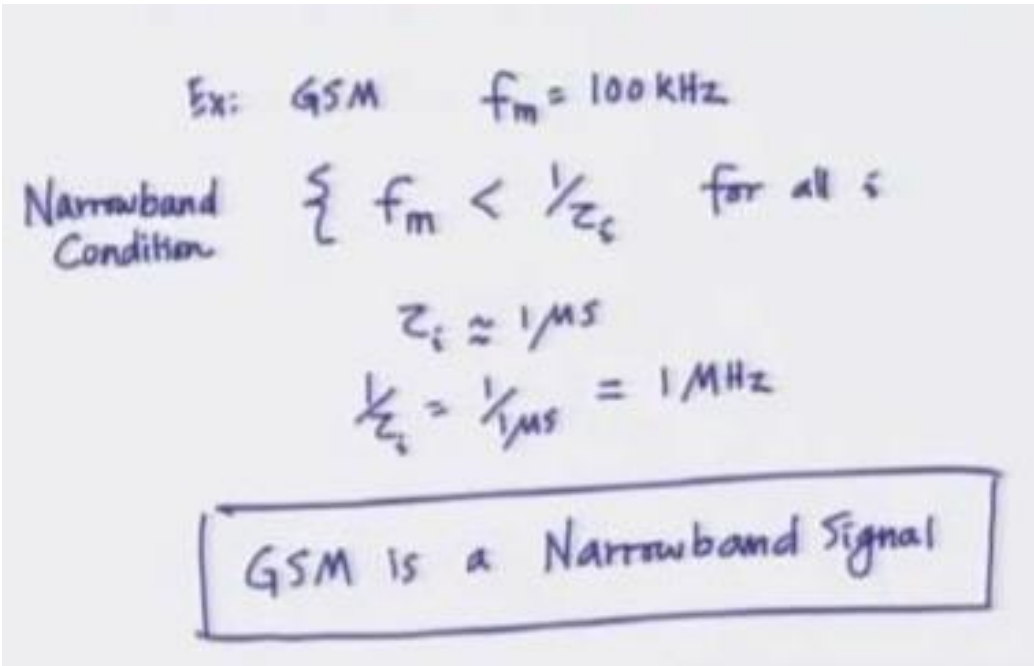
ADVANCED 3G AND 4G WIRELESS COMMUNICATIONS

NARROW BAND ASSUMPTION:-

- Let f_m be the maximum frequency component of $S_b(t)$, i.e., the transmitted base band signal.



- Now ,if for instance for a GSM signal , the total bandwidth is $2f_m$is 200KHz



GSM is a narrow band signal.

- So this is a narrow band signal or in for GSM is essentially is a narrow band. There are many cases where this narrow band assumption is not valid.

- For instance, CDMA is a spread spectrum system; hence it is a wide band signal. So, such as generalization or such as simplification is not possible in CDMA,It holds in the case of some signals.

SYSTEMS MODEL FOR NARROW BAND SIGNALS:-

For a narrow band signal

For a Narrowband Signal:

$$\boxed{s_b(t-\tau_i) \simeq s_b(t)}$$

$$y_b(t) = \underbrace{s_b(t)}_{\text{Transmitted BB Signal}} \underbrace{\sum_{i=0}^{L-1} a_i e^{-j2\pi f_c \tau_i}}_{\text{Complex factor}}$$

Complex coefficient

- The delay is in significant, that the delay does not cause significant distortion in the received signal, because its maximum frequency component f_m is limited.
- The wireless channel as a channel with the multiple propagationpaths consisting of attenuations and delays wireless transmitted signal as a complex base band signal modulating a carrier. If base band signal is a narrow band signal such as GSM what I receive at the output is essentially the transmitted signal $S_b(t)$.
- The different complex numbers can add up to either produces constructive interference or destructive interference.

ADVANCED 3G AND 4G WIRELESS COMMUNICATIONS

→ Let us consider an example $L=2$

$L=2$ ↙ 2 Paths

Attenuation factor: $a_0 = 1$, $a_1 = 1$

delay: $\tau_0 = 0$ = direct Path; $\tau_1 = 1/2f_c$ = another Path

Complex coefficient $h = \sum_{i=0}^{L-1} a_i e^{-j2\pi f_c \tau_i}$

$$= \sum_{i=0}^{2-1} a_i e^{-j2\pi f_c \tau_i}$$

$$= \sum_{i=0}^{1} a_i e^{-j2\pi f_c \tau_i}$$

$$= a_0 e^{-j2\pi f_c \tau_0} + a_1 e^{-j2\pi f_c \tau_1}$$

$$= (1)\, e^{-j2\pi f_c (0)} + (1)\, e^{-j2\pi f_c \left(\frac{1}{2f_c}\right)}$$

$$= (1)\, e^{-0} + (1)\, e^{-j\pi}$$

$$= 1 + [\cos\pi - j\sin\pi]$$

$$= 1 + [-1]$$

$$h = 0$$

- This means the received signal over the above example equals Sb(t) times the coefficient which is 0 equals 0 which means no signal is received, the paths are adding destructively.
- This is the problem here, so even through you are transmitting a signal because both the paths by virtue of one path being delayed compared to other path, they are cancelling each other. As a result we are not getting any signal at the receiver.
- Now, let us consider

$L=2$

$a_0 = 1$, $a_1 = 1$

$\tau_0 = 0$, $\tau_1 = 1/f_c$

Complex coefficient $h = \sum_{i=0}^{L-1} a_i e^{-j2\pi f_c \tau_i}$

$$h = \sum_{i=0}^{2-1} a_i e^{-j2\pi f_c \tau_0} + a_1 e^{-j2\pi f_c \tau_1}$$

$$= 1.e^{-j2\pi f_c (0)} + 1.e^{-j2\pi f_c (1/f_c)}$$

$$= 1.e^{-0} + 1.e^{-j2\pi}$$

$$= 1 + [\cos 2\pi - j\sin 2\pi]$$

$$= 1 + [1 - j(0)]$$

$$= 1 + 1$$

$$h = 2$$

- Received signal, for the above example

Yb(t)=h.Sb(t)

=2. Sb(t)

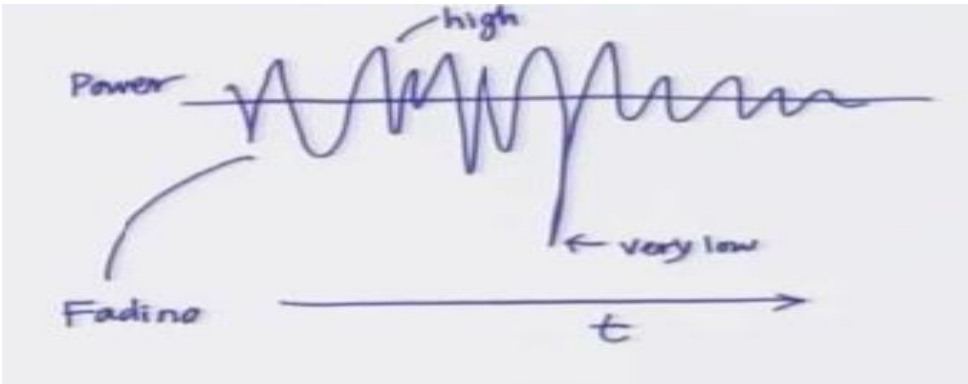
Therefore, Yb(t)= 2Sb(t)

- In this case, the signals from both the paths are adding up constructively in phase to give Sb(t)+Sb(t) i.e., one copy from the direct path, another copy from the scattered path adding up coherently to give 2Sb(t).
- So, the signal amplitude is twice which means, the received power is 4 times the transmitted power.
- If one of the paths is delayed $1/2f_c$ compared to the other path, then the total received signal is 0, because they cancel out each other.
- If one of the paths is delayed $1/f_c$compared to the other path, then the total received signal is twice in amplitude i.e., 2Sb(t), because they add up constructively
- Hence, it is four times in power and for all values of delay between $1/2f_c$ and f_c, the signal amplitude varies between 0 and twice Sb(t)
- So, we receive a range of signal powers at the receiver depending on the random nature of the multi path components in the channel
- So, at receiver if signal is going through a set of various strengths for distance we may receive a signal of very poor power or very high power.

FADING:-

- The below figure shows the plot of quality versus time. It is a curve where the signal has very low and high power. This variation in the signal power is known as “FADING”.
- The signal power waxes and veins and this variation is essentially termed as fading and is is very important characteristics of the wireless propagation environment arising due to the multipath propagation environment
- This does not arise in a wire line propagation environment because in a wire line propagation environment that is a single path between transmitter and receiver which means there is no constructive or destructive interference at the receiver. Because there is only a single path and signal i.e., transmitted is the signal that is essentially received.

ADVANCED 3G AND 4G WIRELESS COMMUNICATIONS



STATISTICS OF THE FADING COEFFICIENT:-

Statistics of the Fading Coefficient:

$$h = \sum_{i=0}^{L-1} a_i e^{-j2\pi f_c \tau_i} = x + jy = a e^{j\phi}$$

$$= \sum_{i=0}^{L-1} a_i \cos(2\pi f_c \tau_i) - j a_i \sin(2\pi f_c \tau_i)$$

$$x = \sum_{i=0}^{L-1} a_i \cos 2\pi f_c \tau_i$$

$$y = -\sum_{i=0}^{L-1} a_i \sin 2\pi f_c \tau_i$$

- In general, it is very difficult to explicitly estimate or explicitly arrive at values of each of a_i and each of τ_i in a real time wireless communication systems or explicitly characterized each of these.
- Instead of characterizing each of them separately, we can characterize the properties of the complex fading coefficient as a whole. In order to characterize the behaviour of the complex fading coefficient, we need the theory of random processes and statistics and probability.

GAUSSIAN RANDOM VARIABLE (GRV)

A Gaussian random variable X is a variable with

mean (μ) and variance (σ^2).

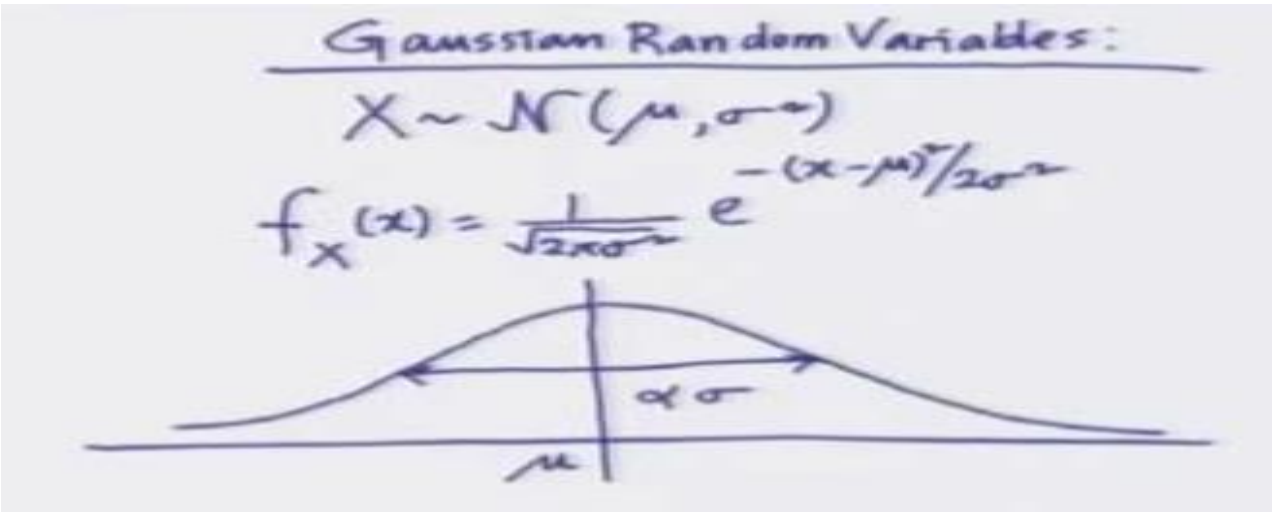
$$X \sim N(\mu, \sigma^2)$$

ADVANCED 3G AND 4G WIRELESS COMMUNICATIONS

The probability density function is given as

$$f_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \quad e^{-(x-\mu)^2/2\sigma^2}$$

The approximate shape, probability density function of a Gaussian random variable is centered at mean and has a spread which is essentially proportional to sigma. The variance is square the spread is proportional or the deviation is proportional to sigma.



STANDARD NORMAL:-

There is a very specific kind of a Gaussian random variable which is the standard normal or standard GRV

N(0,1) , 0 indicates mean and 1 indicates variance

$$f_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \quad e^{-(x-\mu)^2/2\sigma^2}$$

ADVANCED 3G AND 4G WIRELESS COMMUNICATIONS

$$\mu = 0, \quad \sigma^2 = 1$$

$$f_X(x) = \frac{1}{\sqrt{2\pi(1)}} e^{-(x-0)^2/2(1)}$$

$\therefore$ PDF of the Standard Gaussian $\longleftarrow$ $\boxed{f_X(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}}$

RAYLEIGH FADING WIRELESS CHANNEL:-

We know that, fading coefficient h, is the sum of large number of random components i.e., x and y

h=x+jy

wherex,y are to be Gaussian in nature

$$\text{Real part, } x = \sum_{i=0}^{L-1} a_i \cos 2\pi f_c T_i$$

$$\text{Imaginary Part, } y = \sum_{i=0}^{L-1} a_i \sin 2\pi f_c T_i$$

In advanced property, central limit theorem gives more information about a large number of random quantities when they add up result in a Gaussian random variable

h=x+jy

x ~ N(0,1/2), y ~ N(0,1/2)

Where x and y are independent random variables which means the probability distribution, the joint distribution of x and y is the marginal distribution of x and y

$\mu = 0, \sigma^2 = 1/2$

$$f_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(x-\mu)^2/2\sigma^2}$$

$$= \frac{1}{\sqrt{2\pi(\frac{1}{2})}} e^{-(x-0)^2/2(\frac{1}{2})}$$

$$\boxed{f_X(x) = \frac{1}{\sqrt{\pi}} e^{-x^2}}$$

Similarly, $f_Y(y) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(y-\mu)^2/2\sigma^2}$

$$= \frac{1}{\sqrt{2\pi(\frac{1}{2})}} e^{-(y-0)^2/2(\frac{1}{2})}$$

$$\boxed{f_Y(y) = \frac{1}{\sqrt{\pi}} e^{-y^2}}$$

$\therefore$ The Joint distribution is given by

$$f_{X,Y}(x,y) = f_X(x) \cdot f_Y(y)$$

$$= \left[\frac{1}{\sqrt{\pi}} e^{-x^2}\right]\left[\frac{1}{\sqrt{\pi}} e^{-y^2}\right]$$

$$= \frac{1}{\sqrt{\pi}} \frac{1}{\sqrt{\pi}} e^{-x^2} e^{-y^2}$$

$$\boxed{f_{X,Y}(x,y) = \frac{1}{\pi} e^{-(x^2+y^2)}}$$

The distribution is converted into magnitude and phase of the fading coefficient as

$$h = x + jy = a e^{j\phi}$$

$$= a[\cos\phi + j\sin\phi]$$

$$= a\cos\phi + ja\sin\phi$$

$$= x + jy$$

where $x = a\cos\phi$

$y = a\sin\phi$

$$x^2+y^2 = (a\cos\phi)^2 + (a\sin\phi)^2$$

$$= a^2\cos^2\phi + a^2\sin^2\phi$$

$$= a^2[1]$$

$$x^2+y^2 = a^2$$

a^2 is the gain of the communication system

$$f_{A,\phi} = \frac{1}{\pi} e^{-(x^2+y^2)} \det(J_{xy})$$

$$= \frac{1}{\pi} e^{-a^2} \det(J_{xy})$$

$\therefore$ $\det(J_{xy})$ is termed as the determinant of the Jacobian matrix of xy, this is a scaling term

$$J_{xy} = \begin{bmatrix} \frac{\partial x}{\partial a} & \frac{\partial y}{\partial a} \\ \frac{\partial x}{\partial \phi} & \frac{\partial y}{\partial \phi} \end{bmatrix}$$

ADVANCED 3G AND 4G WIRELESS COMMUNICATIONS

$$= \begin{bmatrix} \frac{\partial}{\partial a}(a\cos\phi) & \frac{\partial}{\partial a}(a\sin\phi) \\ \frac{\partial}{\partial \phi}(a\cos\phi) & \frac{\partial}{\partial \phi}(a\sin\phi \end{bmatrix}$$

$$J_{xy} = \begin{bmatrix} \frac{\partial a}{\partial a}\cos\phi & \frac{\partial a}{\partial a}\sin\phi \\ a\frac{\partial \cos\phi}{\partial \phi} & a\frac{\partial \sin\phi}{\partial \phi} \end{bmatrix}$$

$$= \begin{bmatrix} \cos\phi & \sin\phi \\ a(-\sin\phi) & a\cos\phi \end{bmatrix}$$

$$J_{xy} = \begin{bmatrix} \cos\phi & \sin\phi \\ -a\sin\phi & a\cos\phi \end{bmatrix}$$

$$\therefore \det(J_{xy}) = \det \begin{vmatrix} \cos\phi & \sin\phi \\ -a\sin\phi & a\cos\phi \end{vmatrix}$$

$$= a\cos\phi\cos\phi - [-a\sin\phi\sin\phi]$$

$$= a\cos^2\phi + a\sin^2\phi$$

$$= a[\cos^2\phi + \sin^2\phi]$$

$$\boxed{\det(J_{xy}) = a}$$

$$\therefore f_{A,\phi} = \frac{1}{\pi} e^{-a^2} \det(J_{xy})$$

$$= \frac{1}{\pi} e^{-a^2}(a)$$

$$= \frac{a}{\pi} e^{-a^2}$$

$\therefore$ The Joint distribution $\boxed{f_{A,\phi} = \frac{a}{\pi} e^{-a^2}}$

ADVANCED 3G AND 4G WIRELESS COMMUNICATIONS

$$f_A(a) = \int_{-\pi}^{\pi} f_{A,\phi}(a,\phi)\,d\phi$$

$$= \int_{-\pi}^{\pi} \frac{a}{\pi} e^{-a^2}\,d\phi$$

$$= \frac{a}{\pi} e^{-a^2} \int_{-\pi}^{\pi} d\phi$$

$$= \frac{a}{\cancel{\pi}} e^{-a^2} (2\cancel{\pi})$$

$$f_A(a) = 2a e^{-a^2}$$

The marginal distribution of the amplitude of the fading wireless channel, also known as the envelope of the wireless channel

$$a = \sqrt{x^2 + y^2}$$

Where a is the envelope of the fading channel

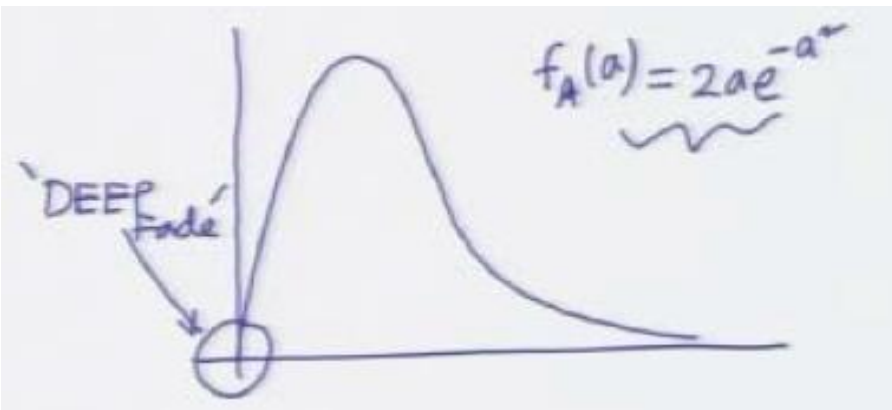
$$f_A(a) = 2a e^{-a^2}, \quad 0 \le a \le \infty$$

- This is the Rayleigh distribution and it is fading coefficient which has the distribution, is the Rayleigh fading distribution or the Rayleigh density function.
- The below figure shows a plot of the Rayleigh distribution

ADVANCED 3G AND 4G WIRELESS COMMUNICATIONS

The Rayleigh distribution i.e.,$2ae^{-a^2}$ is plotted.

- Let us consider the interval (0.5,1) ,the probability that the magnitude lies in the interval, is simply the integral of the density function. There is very low probability, that the coefficient takes very high values. Also low probability, that it takes very low values, close to zero and with high probability, it rise in a range somewhere in between.
- The magnitude of the fading coefficient is close to zero, it means that the fading coefficient has very low gain, which inturn means the signal power received is attenuated, so the signal power received is extremely low, and such a scenario is know typically as a deep fade i.e, the channel magnitude of the channel coefficient is very low.
- Then signal power is almost zero and this can be the signal cannot be distinguished from noise if the received signal power is zero, then essentially there is no signal.



- The marginal distribution of ϕ, the phase of the fading wireless channel is

$$f_\phi(\phi) = \int_0^\infty \frac{a}{\pi} e^{-a^2} da$$

$$= \int_0^\infty \frac{1}{2\pi} (2ae^{-a^2}) da$$

$$= \frac{1}{2\pi}\left(-e^{-a^2}\Big|_0^\infty\right)$$

$$\boxed{f(\phi) = \frac{1}{2\pi}} \longleftarrow \text{Uniform distribution in } [-\pi, \pi]$$

$$f_{A,\phi}(a,\phi) = \frac{a}{\pi} e^{-a^2}$$

$$= \underbrace{\left(\frac{1}{2\pi}\right)}_{f_\phi(\phi)} \underbrace{(2ae^{-a^2})}_{f_A(a)}$$

A, ϕ are INDEPENDENT RVs.

where, A,ϕ are independent random variables.

$Y_b(t)=hS_b(t)$

where$S_b(t)$=Transmitted base band signal

h=complex fading coefficient

$$y_b(t) = h\, s_b(t)$$

$$h = ae^{j\phi}$$

complex fading coeff

Dist. of a : $2ae^{-a^2}$ $\quad 0 \le a \le \infty$

Density of ϕ : $\frac{1}{2\pi}$ $\quad -\pi \le \phi \le \pi$

ADVANCED 3G AND 4G WIRELESS COMMUNICATIONS

EXAMPLE 1: What is the probability that the attenuation is worse than 20db?

Note:This example illustrates the use of wireless channel or analyze the use of characterization of the statistical properties of the wireless channel coefficient.

Solution:-

g is the gain of the channel and let us consider the signal power in db. The probability that this attenuation is worse, the db.

Example:

Problem: What is the probability that the attenuation is worse than 20 dB?

g is the gain of the channel

$$10\log_{10} g \leq -20$$

$$\log_{10} g \leq -2$$

$$g \leq 10^{-2} = 0.01$$

- The probability is given by

$$g = a^2$$

$$a^2 \leq 0.01$$

$$\Rightarrow a \leq 0.1$$

$$P(a \leq 0.1) = \int_0^{0.1} 2a e^{-a^2} da$$

$$= -e^{-a^2} \Big|_0^{0.1}$$

$$= 1 - e^{-0.01}$$

$$= 0.01$$

- Probability that attenuation is worse than -20db is 0.01 or 1%.
- By knowing the statistical behavior of the complex fading coefficient, one can predict probability with received signal is attenuated greater than -20db.

Example:What is the probability that the phase $\phi \in [-\pi/3, \pi/3]$?

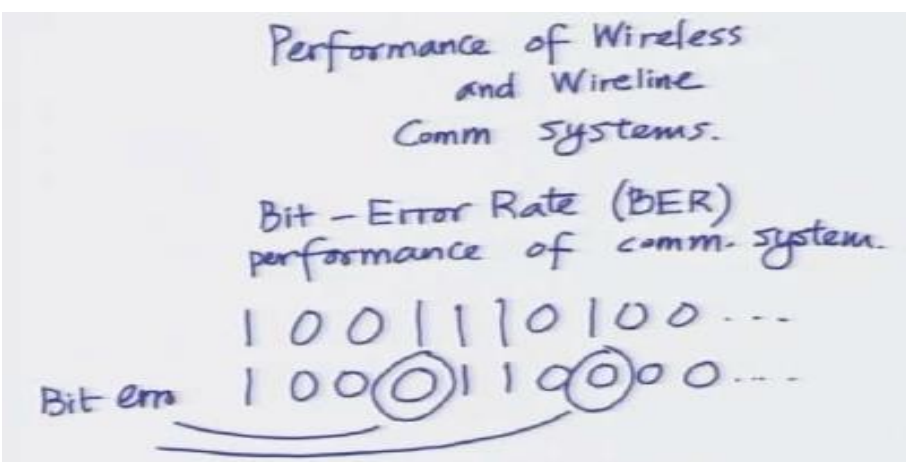
ADVANCED 3G AND 4G WIRELESS COMMUNICATIONS

$$\Pr\left(-\pi/3 \le \phi \le \pi/3\right)$$
$$= \int_{-\pi/3}^{\pi/3} \frac{1}{2\pi}\, d\phi$$
$$= \frac{1}{2\pi}\left(\frac{\pi}{3} - \left(-\frac{\pi}{3}\right)\right)$$
$$\boxed{\Pr\left(\frac{\pi}{3} \le \phi \le \frac{\pi}{3}\right) = \frac{1}{2\pi}\,\frac{2\pi}{3} = \frac{1}{3}}$$

- Thus by knowing the statistical properties, or having characterized statistical properties of statistical behavior of these complex fading coefficients, and wireless communication system especially in terms of magnitude, which describes the attenuation of the received signal, and also phase which describes the phase of set of the received signal.

<u>Performance of wireless and wire line communication systems/Bit-Error Rate(BER) performance of communication system:-</u>

- Bit Error Rate performance of communication system, which is abbreviated as BER performance. Every communication system transmits digital information i.e., in terms of binary information.
- So, every communication system transmits a string of 1's and 0's.These are coded and transmitted over the information channel. Now, when these bits decoded, there are errors in the decoded or detected information stream.



- The error occurs in two places in the bit stream. The rate at which these bit errors in the information stream is known as the bit error rate. So the bit error rate is simply probability of bit error in information stream.

ADVANCED 3G AND 4G WIRELESS COMMUNICATIONS

EXAMPLE:-

- Let us consider an example of transmitting 10,000 bits and a hundreds of them are received error, the bit error rate is given as

BER=Received error bits/Transmitted bits

BER=100/10,000

BER=1/100

$\boxed{\text{BER=0.01}}$

BER arises due to 2 effects

- Noise at the transmitter.
- There is another effect that arises in a wireless communication system.

1) BER of wireline communication system:-

y = 1x + n

$$y = x \oplus n$$

Additive

- x coefficient is 1, because there is no multipath propagation hence there is no multipath interference.

n is white Gaussian noise and $n \sim \mathcal{N}(0, \sigma_n^2)$

- This also has another name as AWGN channel, let us consider the information symbol 1, which is coded as the level +1 and the information symbol 0, which is coded as the level -1.

1 : +1*√p

0 : -1*√p

Where p is transmit power

$$1 : +1 \times \sqrt{P}$$
$$0 : -1 \times \sqrt{P}$$

BPSK Binary Phase Shift Keyed System $\left\{ \begin{matrix} 1 & \sqrt{P} \\ 0 & -\sqrt{P} \end{matrix} \right\}$ Digital Comm systems

ADVANCED 3G AND 4G WIRELESS COMMUNICATIONS

- consider the case where the bit zero(0) is transmitted

  $y = -\sqrt{p} + n$ [$y=x+n$, $x=-\sqrt{p}$ for bit=0]

  Bit error occurs if $y>0$ then, $-\sqrt{p}+n> 0$

  $n-\sqrt{p}> 0$

  $n >\sqrt{p}$

- consider the case where the bit one(1) is transmitted

  $y = \sqrt{p} + n$ [$y=x+n$, $x=\sqrt{p}$ for bit =1]

  Bit error occurs if $y<1$ then, $\sqrt{p}+n< 1$

  $n+\sqrt{p} < 1$

  $n < -\sqrt{p}$

$n \sim N(0,\sigma_n^2)$ where mean=0 and variance= σ_n^2

where n is additive white guassian noise.

- The probability density function is given by

$$n \sim N(0,\sigma_n^2)$$

$$P(n > \sqrt{P})$$

$$= \int_{\sqrt{P}}^{\infty} \frac{1}{\sqrt{2\pi\sigma_n^2}} e^{-x^2/2\sigma_n^2} dx$$

$$\frac{x}{\sigma_n} = t$$

$$dx = \sigma_n dt$$

$$= \int_{\sqrt{P/\sigma_n^2}}^{\infty} \frac{1}{\sqrt{2\pi\sigma_n^2}} e^{-t^2/2} \sigma_n dt$$

$$\underbrace{P(n > \sqrt{P})}_{\text{Probability of bit error}} = \int_{\sqrt{P/\sigma_n^2}}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-t^2/2} dt = Q\left(\sqrt{\frac{P}{\sigma_n^2}}\right)$$

$$\frac{1}{\sqrt{2\pi}} e^{-t^2/2}$$

$$Q(v) = \int_{v}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-t^2/2} dt$$

ADVANCED 3G AND 4G WIRELESS COMMUNICATIONS

Therefore Bit error rate of wireline communication system is $Q\left(\sqrt{\frac{p}{\sigma_n^2}}\right)$

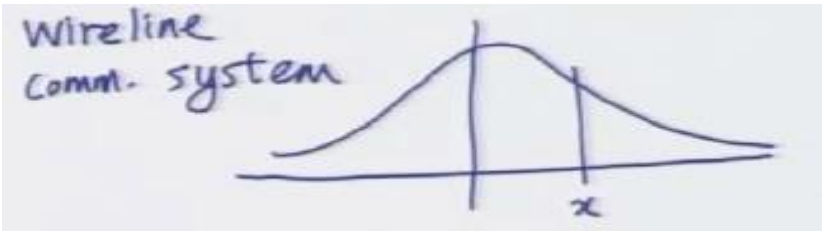
y=x + n where p=signal power and σ_n^2 is noise power.

BER of a Wire line Comm. System $\left\}\right.$ $\boxed{Q\left(\sqrt{\frac{p}{\sigma_n^2}}\right) = Q\left(\sqrt{SNR}\right)}$

Where SNR=signal to noise ratio

=signal power/noise power

=p/ σ_n^2



Q function decreases a0 is the function of SNR.

It is a cumulative distribution function of the Gaussian.

ADVANCED 3G AND 4G WIRELESS COMMUNICATIONS

EXAMPLE:-

1)AtSNRdb=10db, what is the BER of wire line communication system?

Example:

Prob: At $SNR_{dB} = 10$ dB, what is the BER of wireline comm. system?

$$SNR_{dB} = 10\log_{10} SNR$$

$$10\log_{10} SNR = 10 \text{ dB}$$

$$\log_{10} SNR = 1$$

$$SNR = 10$$

$$BER = Q(\sqrt{10})$$

$$= 7.82 \times 10^{-4}$$

bits in error in 10,000 bits

$$= 7.82 \times 10^{-4} \times 10{,}000$$

$$= 7.82$$

@ $SNR_{dB} = 10$ dB

Therefore, the bit error rate (BER) of a wireline communication systems is 7.82 at SNRdb = 10db

ADVANCED 3G AND 4G WIRELESS COMMUNICATIONS

The below figure shows the plot of the bit error rate i.e, plot of Q function.

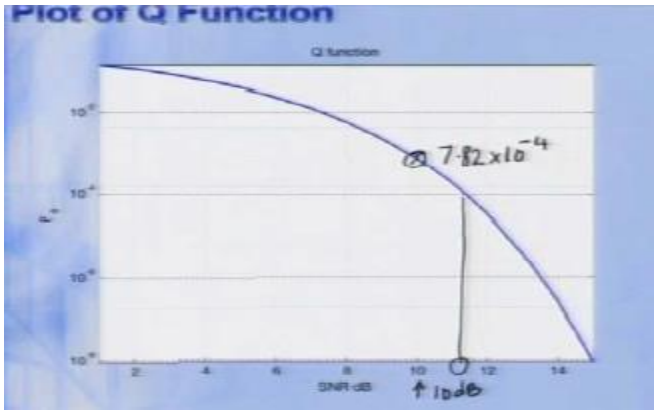


fig: Plot of Q-function

- For bit error rate of 10^-4 ,the SNR required is essentially something around between 11 and 12db, probability at 11.2db or something like that.
- For bit error rate at SNR = 10db, the bit error rate is computed as being 7.84*10^-4.

<u>EXAMPLE</u>:-For the performance of a wired communication system.

<u>Problem</u>:Compute the SNRdb required for a probability of bit error(BER)=10^-6

<u>Sol</u>:Given data Bit-error(BER)= 10^-6
We know that BER = Q(√SNR)Therefore, substitute the given BER value.

ADVANCED 3G AND 4G WIRELESS COMMUNICATIONS

Example of Performance of a wired comm system:

Problem: Compute the SNR_{dB} required for a probability of bit-error (BER) = 10^{-6}.

$$10^{-6} = Q(\sqrt{SNR})$$

$$\sqrt{SNR} = Q^{-1}(10^{-6})$$

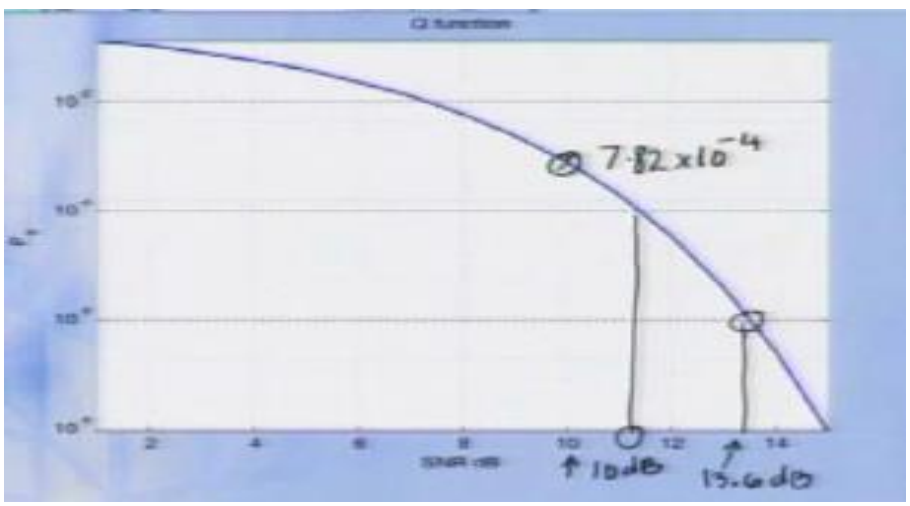
$$SNR = (Q^{-1}(10^{-6}))^2$$

$$SNR = (4.7534)^2$$

$$= 22.595$$

$$SNR_{dB} = 10\log_{10}(22.595)$$

$$= 13.6 \text{ dB}$$



- The above two examples will give the complete explanation of the bit error rate of a wired or wireline communication system.

<u>BER Analysis of a wireless communication system</u>:

- To derive the bit error rate of a wireless communication system, so that for the same SNR, we can compare the performance of the wired and wireless communication system and see how each of the system performs, in terms of bit error rate at a given SNR.
- A wireless communication system model, can be represented as follows $y = hx + n$

where x is the transmitted symbol
n is noise at the receiver

ADVANCED 3G AND 4G WIRELESS COMMUNICATIONS

h is the fading coefficient

- The difference between the wire line and the wireless communication channel essentially is the presence of this fading coefficient (h) in the wireless communication system.
- The fading coefficient(h) can be represented as the

$h=ae^{j\phi}$

Where a is the Rayleigh distribution magnitude

ϕ is the phase

- For bit error rate analysis rate analysis, we need only information of the magnitude, because the gain depends only on the magnitude will not aid or support information about the factor (ϕ).
- In a wireless communication system , the power in the signal and noise power are represented with

Power in the signal = p

Noise power = σ_n^2

- As we know that wireless communication system represented as
  y = hx + n
- Now the power in the signal or transmitted power is multiplied by a fading coefficient. So the received power in a wireless system is simply the transmitted power $_p$times in h square and represented as

  Received power = P* $|h|^2$

  Here the magnitude of h i.e, |h| is a

Therefore Received power=p*a^2

- Hence the received SNR is the ratio of the received power to the Noise Power(σ_n^2)

Received SNR = Received power/Noise power

$$\text{Received SNR} = \frac{Pa^2}{\sigma_n^2} = a^2\frac{P}{\sigma_n^2}$$

<u>Comparision of wired and wireless communication system:</u>

Wired	Wireless
$SNR = \frac{P}{\sigma_n^2}$	$SNR = a^2\frac{P}{\sigma_n^2}$
$Q\left(\sqrt{P/\sigma_n^2}\right)$	$Q\left(\sqrt{a^2P/\sigma_n^2}\right)$

ADVANCED 3G AND 4G WIRELESS COMMUNICATIONS

- And hence the bit error rate simply as we define the q-function, as the cumulative distribution function of the standard Gaussian random variable. So the BER for the wireless channel is represented as

$$BER = \int_{\sqrt{a^2 P/\sigma_n^2}}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-x^2/2} \, dx$$

<u>Derivation fpr BER</u>:-

- Deriving the expression for the bit error rate of a wireless channel as a function of q.
- Now q function,is the cumulative distribution function of the Gaussian random standard , Gaussian random variable.
- This random variable has a Rayleigh distribution, which means this random gain of Rayleigh fading channel.
- Hence to get the average performance of this Rayleigh fading channel, we have to take this bit error rate. Which is the function of this random quantity, and average it over the distribution of that random variable and its average is represented as

Average of $g(a)$ $\int_0^{\infty} g(a) f_A(a) \, da$

we know that

$$BER = \int_{\sqrt{a^2 P/\sigma_n^2}}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-x^2/2} \, dx \quad \text{——— (1)}$$

$$\text{BER of a wireless System} = \int_0^{\infty} \underbrace{Q\left(\sqrt{a^2 \frac{P}{\sigma_n^2}}\right)}_{\text{average BER}} 2a e^{-a^2} \, da \rightarrow (2)$$

We know that BER as shown in equ 1 substitute $\frac{P}{\sigma_n^2}$=μ in eua 1 limits subeq 1 in eq 2 we get,

ADVANCED 3G AND 4G WIRELESS COMMUNICATIONS

$$BER = \int_0^{\infty}\int_{\sqrt{a^2\mu}}^{\infty} \underbrace{\frac{1}{\sqrt{2\pi}} e^{-x^2/2}}_{\text{standard Normal}} dx \; 2ae^{-a^2} da \quad \text{—— (3)}$$

$$u = \frac{x}{a\sqrt{\mu}}$$

$$dx = a\sqrt{\mu}\, du$$

when $x = \sqrt{a^2\mu}$ therefore $u = \frac{a\sqrt{\mu}}{a\sqrt{\mu}} = 1$

→ ∴ The Lower limit for $u = 1$

similarly when $x = \infty$; $u = \infty$

→ ∴ The upper Limit for $u = \infty$

$$BER = \frac{1}{\sqrt{2\pi}} \int_0^{\infty}\int_1^{\infty} 2ae^{-a^2} [a\sqrt{\mu}\, du]\, e^{-\frac{(u(\sqrt{a^2\mu}))^2}{2}} da$$

$$= \frac{1}{\sqrt{2\pi}} \int_0^{\infty}\int_1^{\infty} 2ae^{-a^2} a\sqrt{\mu}\, e^{-\mu a^2 u^2/2} du\, da$$

$$= \sqrt{\mu} \underbrace{\int_1^{\infty} \Bigg(\underbrace{\int_0^{\infty} \frac{2a^2}{\sqrt{2\pi}} e^{-\frac{a^2}{2}(2+\mu u^2)} da}_{\text{Inner Part}} \Bigg) du}_{\text{outer part}} \quad \text{—— (4)}$$

$$\text{Inner Part} = \int_0^{\infty} \frac{2a^2}{\sqrt{2\pi}} e^{-\frac{a^2}{2}(2+\mu u^2)} da \quad \text{—— (4a)}$$

$$\frac{1}{\sigma^2} = 2+\mu u^2 \Rightarrow \sigma = \left(\frac{1}{2+\mu u}\right)^{1/2}$$

$$\sigma^3 = \left(\frac{1}{2+\mu u}\right)^{3/2}$$

Substitute (2+µu^2) value as 1/ $\frac{1}{\sigma^2}$ in eq 4a

$$\text{Inner part} = \int_0^{\infty} \frac{2a^2}{\sqrt{2\pi}} e^{-\frac{a^2}{2}\left(\frac{1}{\sigma^2}\right)} da \quad \text{—— (4a}_0\text{)}$$

- In the random process, the variance of a zero mean Gaussian random variable with parameter sigma square, and the variance of this is nothing but σ^2 .

ADVANCED 3G AND 4G WIRELESS COMMUNICATIONS

i.e $\int_0^\infty \frac{2y^2}{\sqrt{2\pi}\,\sigma^2} e^{-y^2/2\sigma^2}\, dy = \sigma^2$ [formulae]

if we multiply with 'σ' we get

$$\int_0^\infty \frac{2y^2}{\sqrt{2\pi}\,\cancel{\sigma^2}} (\cancel{\sigma})\, e^{-y^2/2\sigma^2}\, dy = \sigma^2(\sigma)$$

$$\int_0^\infty \frac{2y^2}{\sqrt{2\pi}} e^{-y^2/2\sigma^2}\, dy = \sigma^3 \quad \text{—— (4a)}$$

Now compare (4a) and (4a) equations

$\therefore$ Innerpart $= \sigma^3$

As we know the value of $\sigma = \left(\frac{1}{2+\mu u^2}\right)^{1/2}$

Innerpart $= \left(\frac{1}{2+\mu u^2}\right)^{3/2}$

Now sub this Inner part value in equ (4)

$$BER = \sqrt{\mu} \int_1^\infty \left(\frac{1}{2+\mu u^2}\right)^{3/2} du \quad \text{—— (5)}$$

changing the limit values

→ when $u = 1$; $1 = \sqrt{\frac{2}{\mu}} \tan\theta$ therefore $\theta = \tan^{-1}\left(\sqrt{\frac{\mu}{2}}\right)$

$\therefore$ lower limit for $\theta = \tan^{-1}\left(\sqrt{\frac{\mu}{2}}\right)$

→ when $u = \infty$; $\infty = \sqrt{\frac{2}{\mu}} \tan\theta$ therefore $\theta = \pi/2$

$\therefore$ upper limit for $\theta = \pi/2$

Sub all the 'limits' and 'u' value in equ (5)

$$BER = \sqrt{\mu} \int_{\tan^{-1}\left(\sqrt{\frac{\mu}{2}}\right)}^{\pi/2} \left(\frac{1}{2+\mu\left(\sqrt{\frac{2}{\mu}}\tan\theta\right)^2}\right)^{3/2} \sqrt{\frac{2}{\mu}} \sec^2\theta\, d\theta$$

$$= \sqrt{\mu} \int_{\tan^{-1}\left(\sqrt{\frac{\mu}{2}}\right)}^{\pi/2} \left[\frac{1}{(2+2\tan^2\theta)}\right]^{3/2} \sqrt{\frac{2}{\mu}} \sec^2\theta\, d\theta$$

$$= \sqrt{\mu} \int_{\tan^{-1}\left(\sqrt{\frac{\mu}{2}}\right)}^{\pi/2} \left[\frac{1}{2(1+\tan^2\theta)}\right]^{3/2} \sqrt{\frac{2}{\mu}} \sec^2\theta\, d\theta$$

ADVANCED 3G AND 4G WIRELESS COMMUNICATIONS

$$= \sqrt{\mu} \int_{\tan^{-1}\left(\sqrt{\frac{\mu}{2}}\right)}^{\pi/2} \left[\frac{1}{2\sec^2\theta}\right]^{3/2} \sqrt{\frac{2}{\mu}}\, \sec^2\theta \, d\theta$$

$$= \frac{\sqrt{\mu}}{2\sqrt{2}} \times \sqrt{\frac{2}{\mu}} \int_{\tan^{-1}\left(\sqrt{\frac{\mu}{2}}\right)}^{\pi/2} \left[\frac{\sec^2\theta}{\sec^3\theta}\right] d\theta$$

$$= \frac{1}{2} \int_{\tan^{-1}\left(\sqrt{\frac{\mu}{2}}\right)}^{\pi/2} \left[\frac{1}{\sec\theta}\right] d\theta \qquad \left[\because \frac{1}{\sec\theta} = \cos\theta\right]$$

$$= \frac{1}{2} \int_{\tan^{-1}\left(\sqrt{\frac{\mu}{2}}\right)}^{\pi/2} \cos\theta \, d\theta \qquad \left[\because \int \cos\theta \, d\theta = \sin\theta\right]$$

$$BER = \frac{1}{2}\left[\sin\theta\right]_{\tan^{-1}\left(\sqrt{\frac{\mu}{2}}\right)}^{\pi/2}$$

$$BER = \frac{1}{2}\left[1 - \sin\left[\tan^{-1}\left(\sqrt{\frac{\mu}{2}}\right)\right]\right] \longrightarrow (6)$$

we know that,

$$\sin\theta = \sqrt{\frac{\tan^2\theta}{1+\tan^2\theta}} \longrightarrow [\text{Trignometric formulae}]$$

similarly $\sin\left[\tan^{-1}\left(\sqrt{\frac{\mu}{2}}\right)\right] = \sqrt{\frac{\tan^2\tan^{-1}\left(\sqrt{\frac{\mu}{2}}\right)}{1+\tan^2\tan^{-1}\left(\sqrt{\frac{\mu}{2}}\right)}}$

$$= \sqrt{\frac{\mu/2}{1+\mu/2}}$$

$$\sin\left[\tan^{-1}\left(\sqrt{\frac{\mu}{2}}\right)\right] = \sqrt{\frac{\mu}{2+\mu}}$$

Sub this value in equation (6)

$$BER = \frac{1}{2}\left[1 - \sqrt{\frac{\mu}{2+\mu}}\right]$$

where $\mu = \frac{P}{\sigma_n^2} = SNR$

$$\boxed{\therefore \text{ BER of a wireless system} = \frac{1}{2}\left(1 - \sqrt{\frac{SNR}{2+SNR}}\right)}$$

A little further simplification of this BER is

ADVANCED 3G AND 4G WIRELESS COMMUNICATIONS

$$BER = \frac{1}{2}\left(1-\sqrt{\frac{SNR}{2+SNR}}\right)$$
$$= \frac{1}{2}\left(1-\sqrt{\frac{1}{\frac{2}{SNR}+1}}\right)$$
$$= \frac{1}{2}\left(1-\frac{1}{\sqrt{\frac{2}{SNR}+1}}\right)$$

Similarly $\frac{1}{\sqrt{1+2/SNR}} \approx (1+2/SNR)^{-1/2}$

$$\approx \left(1-\frac{1}{2}\times\frac{2}{SNR}\right)$$

Sub this value in BER

$$BER = \frac{1}{2}\left(1-\left(1-\frac{1}{2}\times\frac{2}{SNR}\right)\right)$$
$$= \frac{1}{2}\times\frac{1}{SNR}$$
$$BER \approx \frac{1}{2SNR}$$

- This is a wireless channel bit error of a. wireless channel at high SNR.

<u>Comparision of wired channel and wireless channel</u>

Wired channel	wireless channel
$y = x+n$	$y = hx+n$
$BER = Q\sqrt{SNR}$	$BER = \frac{1}{2}\left(1-\sqrt{\frac{SNR}{2+SNR}}\right)$

<u>EXAMPLE :-</u> (for wireless channel)

<u>Problem:</u> Compute the bit error rate of a wireless communication system at SNR=20db.

<u>Solution:</u> Given data SNR=20db

$$20\,dB = 10\log_{10}(SNR)$$
$$\log_{10}(SNR) = 2$$
$$SNR = 10^2 = 100$$
$$BER = \frac{1}{2SNR} = \frac{1}{200}$$
$$BER = 5\times10^{-3}$$

(This is wireless system)

- When compared with wire line system SNR=10db and BER=7.8*10

- If we compare both the wireless channel is having very high BER

EXAMPLE 2:

Problem: Compute SNRdb of a wireless communication system for BER $=10^{-6}$.

Sol:Given data

Example 2:

Problem: Compute SNR_{dB} of a wireless Communication system for BER $= 10^{-6}$?

$$10^{-6} = \frac{1}{2\,SNR}$$

$$SNR = \frac{1}{2 \times 10^{-6}}$$

$$= \frac{10^{+6}}{2}$$

$$SNR_{dB} = 10\log_{10}\left(\frac{10^{6}}{2}\right)$$

$$= 60\text{ dB} - 3\text{ dB}$$

$$\boxed{SNR = 57\text{ dB}}$$

Difference:-

- The Difference of SNRdb between the wireless and wired communication system.
  Difference=wireless system-wired system
  =57db-13.6db
  =43db
- Wireless system has high BER and poor performance, this is because of fading.
- This difference is due to bit error rate(BER)$=10^{-6}$.

ADVANCED 3G AND 4G WIRELESS COMMUNICATIONS

i.e. BER of 10^{-6} for both wired & wireless system

$$\left[10\log_{10} SNR_{wireless} - 10\log_{10} SNR_{wired}\right] = 43$$

we know that $\log\left(\frac{a}{b}\right) = \log a - \log b$

$$10\log_{10}\left[\frac{SNR_{wireless}}{SNR_{wired}}\right] = 43\,dB$$

$$\log_{10}\left[\frac{SNR_{wireless}}{SNR_{wired}}\right] = 4.3$$

$$\frac{SNR_{wireless}}{SNR_{wired}} = 10^{4.3} \approx 10^{4} = 10{,}000$$

$$SNR_{wireless} = 10{,}000 \times SNR_{wired}$$

$$P_{wireless} = 10{,}000 \times P_{wired}$$

- This tells that transmitted power of a wireless system equals to 10,000 times the transmitted power of a wired communication system.
- So, that means in a wireless communication system we need a huge amount of power to achieve the same bit error rate of a wireless communication system.

<u>BER expression of wired and wireless systems</u>:-

- Let us look at the expressions of the bit error rate of a wired line,that is a wired and a wireless communication system.

BER Expressions of Wired and Wireless Systems:

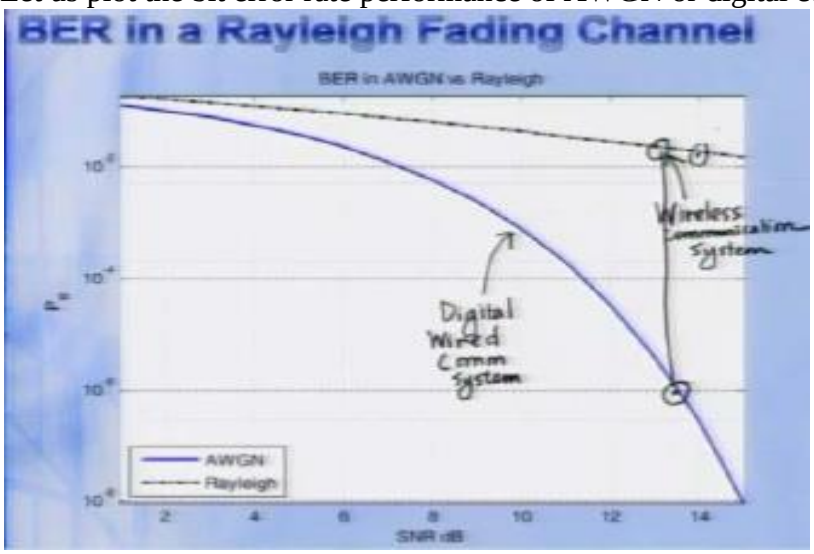
Wireless System $\quad BER = \frac{1}{2}\left(1 - \sqrt{\frac{SNR}{2+SNR}}\right) \approx \frac{1}{2\,SNR}$

Wired Communication System $\quad BER = Q(\sqrt{SNR}) \approx e^{-SNR/2}$

$$Q(\sqrt{x}) \approx e^{-x/2}$$

- Now you can see the difference between the bit error rates performances of a wired communication system compared with a wireless communication.
- In a wireless communication system, the bit error rate is 1/SNR, so it is decreasing only as 1/SNR, but in a wired communication system the bit error rate is e^-SNR/2 .

- So, it is decreasing exponentially in SNR, which is the reason the bit error rate in a wired communication system is much lower than that of a wireless communication system.
- Let us plot the bit error rate performance of AWGN or digital communication channel.



- The curve in the plot represents the digital wired communication system.
- This is q function of square root of SNR and this curve here is for a Rayleigh fading channel or a wireless communication system.
- From the plot or graph we can say how slowly this is decreasing wrt to SNR compared to the wired communication system. It decreases to 10^{-8} bit rate at almost 14db.
- As we know that wireless communication system is represented as

y = hx+n, where h is fading coefficient($|h|^2$) , n is noise(σ_n^2)

- The performance of wireless system is poor when received power is lower than the noise power.
- At times, there is destructive interference in the channel due to the multiple path propagation environment and the signal reception or the power of the received signal is small when the destructive interference is $|h|^2$ times transmit power i.e,

$$|h|^2 P = a^2 P$$

➤ Performance is BAD, when $a^2 P$ is less than the Noise i.e.,

$$a^2 P < \sigma_n^2$$

$$a^2 < \frac{\sigma_n^2}{P} = \frac{1}{SNR}$$

$$\boxed{a < \frac{1}{\sqrt{SNR}}}$$ deep fade event

ADVANCED 3G AND 4G WIRELESS COMMUNICATIONS

- If received power is much less than the noise power at the receiver which means that bit error ratio is going to be high because the signal is very low.
- So, the bit error rate is going to be very high and this is known as a deep fade event .this is happening because of destructive interference is so large.
- If the destructive interference is to an extent that almost very little signal power received such that to happen i.e deep fade event.
- The magnitude of the fading coefficient has to be $<1/\sqrt{SNR}$ this is known as a deep fade event.
- WKT the probability of fading coefficient is the Rayleigh fading distributionwhich is given as $f_A(a)=2ae^{-(a)2}$
- The probability of a deep fade event is nothing but

$$f_A(a) = 2ae^{-a^2}$$

Probability of a deep fade event

$$P\left(a < \frac{1}{\sqrt{SNR}}\right) = \int_0^{\frac{1}{\sqrt{SNR}}} f_A(a)\, da$$

$$\int_0^{\frac{1}{\sqrt{SNR}}} 2ae^{-a^2}\, da$$

$$e^{-a^2} \approx 1$$

$$\approx \int_0^{\frac{1}{\sqrt{SNR}}} 2a\, da$$

$$= \cancel{2}\frac{a^2}{\cancel{2}}\Big|_0^{\frac{1}{\sqrt{SNR}}} = \frac{1}{SNR}$$

Probability of deep fade

- This is saying that the probability of a deep fade in a wireless system is 1/SNR
- As we already known BER=1/2SNR and
  Probability of deep fade =1/SNR

Comparing both eq's they are approximately equal .
Therefore BERis approximately equal toprobability of deep fade

- In wireless communication system BER is nothing but a deep fade event that the poor performance of wireless communication system not due to receiver noise but it is due to deep fade because of destructive interference in a wireless .

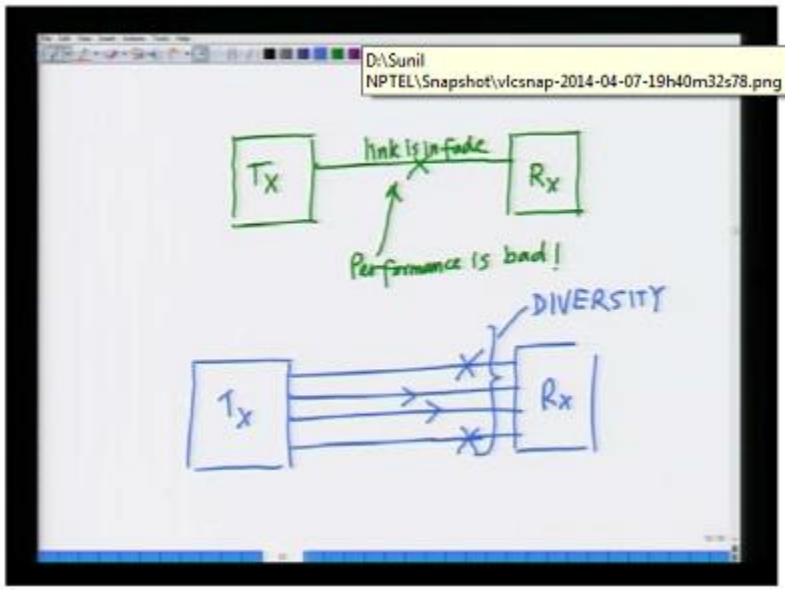
DIVERSITY:-

- Actually divesity is used to improve the performance of wireless system.

DEFINATION:

Diversity is a technique i.e., fundamentally important technique in a wireless communication system that can be employed to impose the performance of the wireless communication system.

- This is very important for the 3G and 4G wireless communication systems and analyzing their performance and improving their performance.
- We can improve reliability, so that we can communicate not just voice signals, but also which require a huge band width and we can transmit at very high data, rates over such wireless systems.
- The main reason for the poor performance of the wireless communication system is because of a deep fade, and diversity can be employed to improve the performance by combating the fading environmental (or) through controlling.
- One can employ diversity advantageously to remove (or) to avoid the harmful effects of the fading in wireless channels.
- Simple schematic representation of Diversity



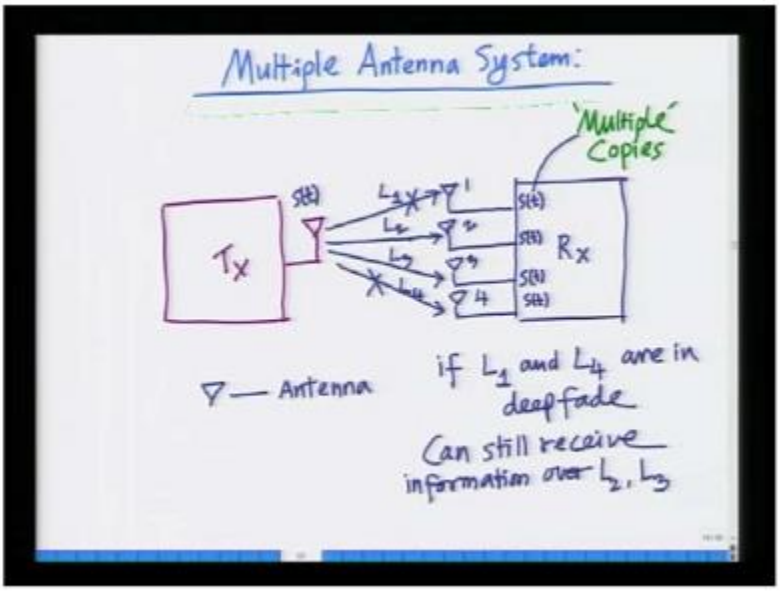
- In this above representation we can see the transmitter and receiver.

ADVANCED 3G AND 4G WIRELESS COMMUNICATIONS

- The $T_x$and R_x have one link it is not a wire. A link implies a port where we are transmitting data to the another past, we can tell this as an Antenna.
- If this link is deep fade because of multi part interference. The performance of system is bad.
- There is one possible method to avoid this problem. It
- Now in this multiple link system even if we see there are two links are in a deep fade and rest of the links can be employed to transmit information.

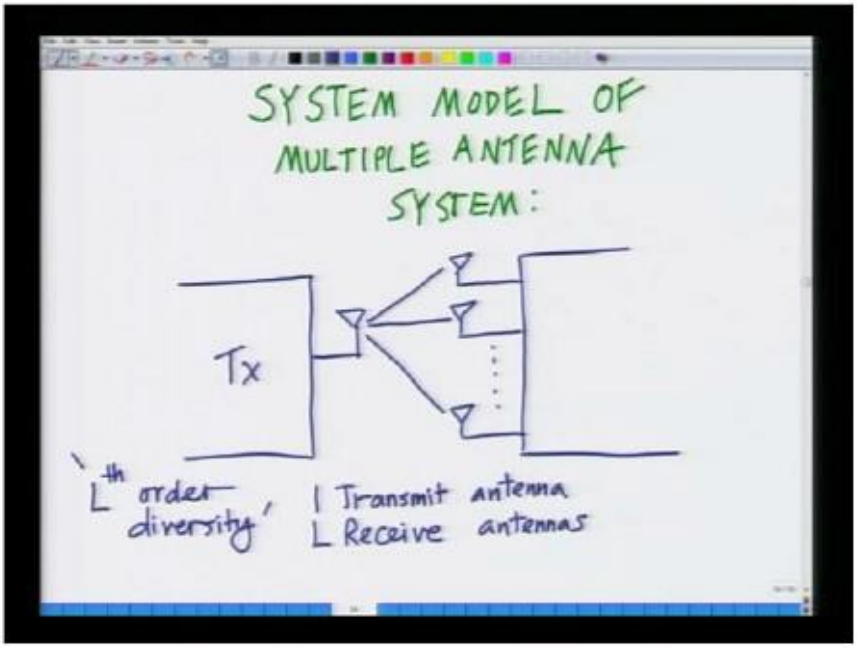
MULTIPLE ANTENNA SYSTEM:-

- This is a diversity system example.



- Let us consider a multiple antenna system which is having T_x and Receiver(R_x).
- Here the T_x have one transmitting antenna and R_x antenna have four receiving antennas.
- The path L1 and L4are in deep fade. Even though we can receive information through L2 and L3.
- can receive copies of the transmitted signal s(t) system model of multiple antenna system.
- In this system model for multiple antenna system contain transmitter which is having signal antenna and receiver which is having 'L' receiving antennas.
- This means we will receive 'L' signal copies hence this is known as an Lth order diversity.

ADVANCED 3G AND 4G WIRELESS COMMUNICATIONS



- Let the transmitted signal be 'x' for system model.
- As we already know that expression for the wireless communication system is

$$y=hx+n$$

x=transmitted signal

- Similarly the system model expression for the one TX antenna &RX antenna are

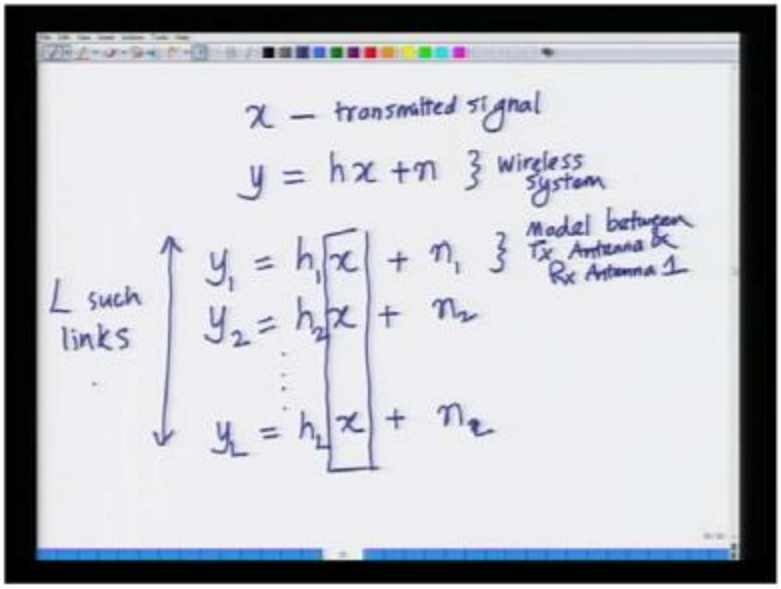
$$y_1=h_1x+n_1$$

$$y_2=h_2x+n_2$$

$$y_l=h_lx+n_l$$

- Where $y_1=h_1x+n_1$ is the system model expression between the Transmit antenna and Receive antenna-1.
- Here y_1 is the signal at RXing antenna1&h_1 is the fading coefficient between TXantenna
- This is repeated for 'L'such links.
- The exp can be represented as

ADVANCED 3G AND 4G WIRELESS COMMUNICATIONS



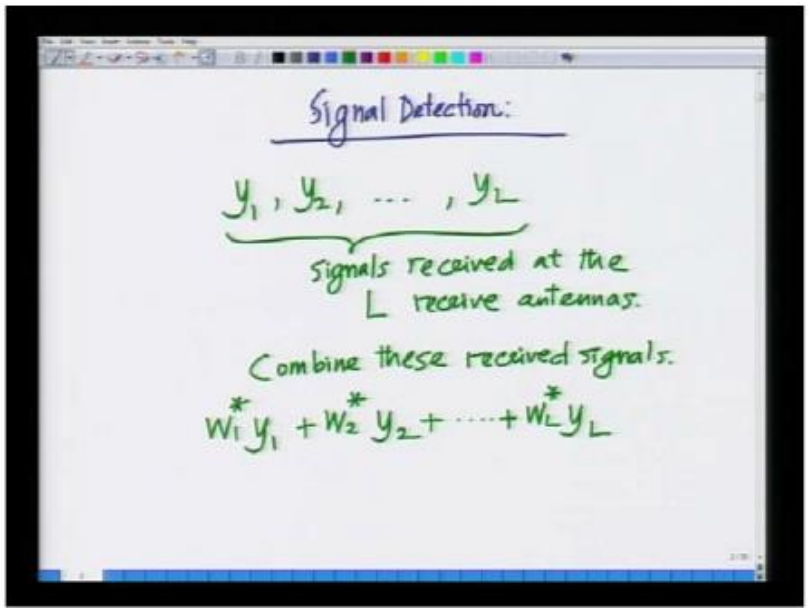
$\bar{y}$=velocity, $\bar{h}$=vector channel containing coefficient (h_1,h_2,h_l),$\bar{n}$=noise vector

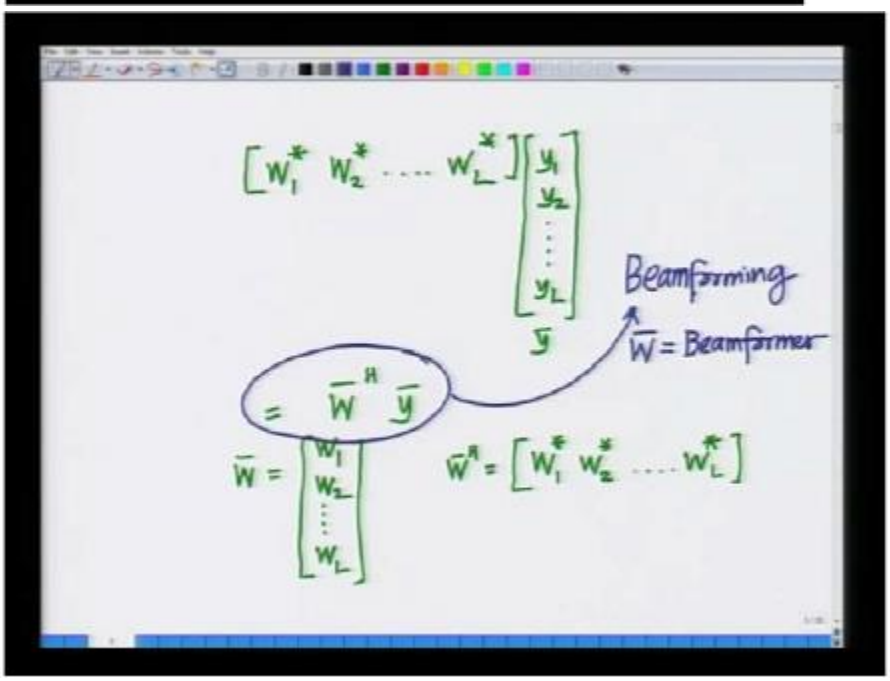
- vector notation is $\bar{y}= \bar{h}x+ \bar{n}$
- Lx1 Lx1Lx1
- In above vector notation we have given vectors with Lx1 where L is a number of Receive antennas.
- We will use this vector notation to represent the "Wireless diversity".
- wireless system in a compact fashion.

ANALYSIS OF RX ANTENNA DIVERSITY SYSTEM:-

- As we already known that system model is represented as is$\bar{y}=\bar{h}x+\bar{n}$.
- The expected value of noise i.e the noise varience,i.e.the power of the noise on each RX antenna is$\boldsymbol{\sigma}_n^2$.
- This means all the receive antennas are symmetric & the noise power of each RXantenna norm i.e.|ni(k)| is $\boldsymbol{\sigma}_n^2$.
- The analysis of RX antenna diversity system is given as the $E\{|ni(k)|2\} = \boldsymbol{\sigma}_n^2$.
- SIGNAL DETECTION:-
- To detect the signal in this system we have y_1,y_2.....yl.these are the signals received at L receive antennas.
- To detect the TX signal x we have to combine all the RX signals.
- We will combine these received signals as follows take y_1 weigh it by its 'w_1' conjugate.
- Now combine them linearly we can detect the transmitted signal x is present in each one of these signal copies.
- Combine these received signals as shown below is $w^*_1y_1+w^*_2y_2+.....+w^*_ly_l$

We represent vector eq as





- When we linearly combining the signals and add the output of receive antennas in a receive antenna diversity system or in a system with multiple antennas this has a name this is known as beam forming.
- beam forming is $\bar{w}^H\bar{y}$.
- beam former is$\bar{w}$
- As we already know that is $\bar{y}= \bar{h}x+ \bar{n}$
- Let we sub the $\bar{y}$ in the beam forming

$$\bar{w}^H\bar{y}=\bar{w}^H(\bar{h}x+ \bar{n})$$
$$=\bar{w}^H\bar{h}x+ \bar{w}^H \bar{n}$$

29

$$SNR = \frac{\text{Signal Power}}{\text{Noise power}}$$

$$\text{Signal} = \bar{w}^H \bar{h} x$$

$$\text{Signal power} = |\bar{w}^H \bar{h}|^2 \cdot P$$

Signal power is nothing but, the magnitude of $\bar{w}^H \bar{h}$ times the power

$$\text{Noise power} = \bar{w}^H \bar{n}$$

The output power is expected norm magnitude $(\bar{w}^H \bar{n})^2$ which can also be written as.

$$E\{|\bar{w}^H \bar{n}|^2\} = E\{(\bar{w}^H \bar{n})(\bar{w}^H \bar{n})^*\}$$

$$\bar{w}^H = [w_1^* \; w_2^* \; \cdots \; w_L^*]$$

$$\bar{n} = \begin{bmatrix} n_1 \\ n_2 \\ \vdots \\ n_L \end{bmatrix}$$

$$\bar{w}^H \bar{n} = [w_1^* \; w_2^* \; \cdots \; w_L^*] \begin{bmatrix} n_1 \\ n_2 \\ \vdots \\ n_L \end{bmatrix}$$

$$(\bar{w}^H \bar{n}) = [w_1^* n_1 + w_2^* n_2 + \cdots + w_L^* n_L]$$

$$(\bar{w}^H \bar{n})^* = [w_1 n_1^* + w_2 n_2^* + \cdots + w_L n_L^*]$$

$$E\{(\bar{w}^H \bar{n})(\bar{w}^H \bar{n})^*\} = [w_1^* n_1 + w_2^* n_2 + \cdots + w_L^* n_L] \times [w_1 n_1^* + w_2 n_2^* + \cdots + w_L$$

$$= E\left\{\sum_{i=1}^{L} |w_i|^2 |n_i|^2 + \sum_{i} \sum_{j} w_i w_j^* n_i^* n_j\right\}$$

$$i \neq j$$

$$= E\left\{\sum_{i}^{L} w_i w_i^* n_i n_i^* + \underbrace{\sum_{i} \sum_{j} w_i w_j^* n_i n_j^*}_{0}\right\}$$

$$i \neq j$$

$$(n_i n_j^*) \Rightarrow E(n_i) E(n_j^*)$$

$$\text{Noise power} = E\{\Sigma |w_i|^2 |n_i|^2\}$$
$$= \Sigma |w_i|^2 E\{n_i\}^2$$
$$= \sigma_n^2 \Sigma |w_i|^2$$
$$= \sigma_n^2 \, \|\bar{w}\|^2$$

$$\text{Noisepower} = \sigma_n^2 \, \bar{w}^H \bar{w}$$

This is the Noise power at the output of the beam former

* SNR at the output of the beamformer

$$SNR = \frac{\text{Signal Power}}{\text{Noise power}}$$

$$\boxed{SNR = \frac{|\bar{w}^H \bar{h}|^2 P}{\sigma_n^2 (\bar{w}^H \bar{w})}}$$

* ~~we~~ There is another technique to compute the power in the noise ~~because~~. This is also useful in many other contexts so we said noise vector $\bar{n}$ is

$$\bar{n} = \begin{bmatrix} n_1 \\ n_2 \\ \vdots \\ n_L \end{bmatrix}$$

* Now let we compute $\bar{n} \cdot \bar{n}^H$ i.e

$$\bar{n}\bar{n}^H = \begin{bmatrix} n_1 \\ n_2 \\ \vdots \\ n_L \end{bmatrix} [n_1^* \; n_2^* \cdots\cdots n_L^*]$$

$$\begin{bmatrix} \sigma_n^2 & 0 & 0 & \cdots & 0 \\ 0 & \sigma_n^2 & 0 & 0 & \cdots \\ \vdots & & \ddots & & \\ 0 & \cdots & & & \sigma_n^2 \end{bmatrix}$$

$$E\{\bar{n}\bar{n}^H\} = E\left\{\begin{bmatrix} |n_1|^2 & n_1 n_2^* & \cdots & \\ n_2 n_1^* & |n_2|^2 & \cdots & \\ \vdots & & \ddots & \\ & & & |n_L|^2 \end{bmatrix}\right\}$$

* If we take the Expectation of matrix, we can write specifically where all the diagonals are σ_n^2

$$E(\bar{n}\bar{n}^H) = \begin{bmatrix} \sigma_n^2 & 0 & 0 & 0 & \cdots \\ \vdots & \sigma_n^2 & \cdots & & \\ \vdots & \vdots & \ddots & & \\ & & & & \sigma_n^2 \end{bmatrix}$$

$$= \sigma_n^2 \begin{bmatrix} 1 & 0 & 0 & \cdots \\ 0 & 1 & 0 & \cdots \\ \vdots & \vdots & \ddots & \\ & & & 1 \end{bmatrix}$$

$$= \sigma_n^2 [I]$$

* Expected $\bar{n}\bar{n}^H$ is nothing but, a matrix which is σ_n^2 Identi
So this is just proportional to the identity matrix. This is simply σ_n^2 because any matrix identity itself, this is simply $\bar{w}^H\bar{w}$, which is $\sigma_n^2 \|\bar{w}\|^2$

i.e

Noise power $= \sigma_n^2 \|\bar{w}\|^2$ and

$$SNR = \frac{|\bar{w}^H\bar{h}|^2 P}{\sigma_n^2 \bar{w}^H\bar{w}}$$

choose $\bar{w}$ such that $\bar{w}$ maximizes the SNR

<u>Maximize the SNR:-</u>

$$\text{max SNR} = \frac{|\bar{w}^H\bar{h}|^2}{\bar{w}^H\bar{w}} \frac{P}{\sigma_n^2}$$

$$\boxed{\bar{w}^* = K\bar{w};\ \bar{w}^H = K\cdot\bar{w}^H}$$ ∵ Since it is scaled by 'K'

$$= \frac{K^2 |\bar{w}^H\bar{h}|^2}{K^2(\bar{w}^H\bar{w})} \cdot \frac{P}{\sigma_n^2}$$

$$= \frac{|\bar{w}^H\bar{h}|^2}{\bar{w}^H\bar{w}} \cdot \frac{P}{\sigma_n^2}$$

* Now let us say I scale that $\bar{w}$ by k. if I scale the vector $\bar{w}$ by k the SNR is still magnitude i.e

$$\text{maxSNR} = \frac{|\bar{w}^H\bar{h}|^2}{\bar{w}^H\bar{w}} \cdot \frac{P}{\sigma_n^2}$$

where k^2 is both Numerator & [illegible]

So this $\bar{w}$ hermitian is scale invariant that is we can maximize the SNR upto a scale factor k i.e that is we can maximize the SNR by choosing a vector $\bar{w}$

So, SNR simply becomes maximize magnitude $|\bar{w}^H \bar{h}|^2$ into some constant P over σ_n^2 such that $\|\bar{w}\|$ equal to 1

This is the problem of choosing the beamformer.

choose $\bar{w}$ such that

$$\|\bar{w}\|^2 = 1$$

$$\Rightarrow \bar{w}^H \bar{w} = 1$$

$$SNR = \underbrace{|\bar{w}^H \bar{h}|^2}_{\text{such that } \|\bar{w}\| = 1} \cdot P/\sigma_n^2$$

- we said the beam former SNR is invariant up to a scale that is unique upto a scale factor, so Now we are fixing the scale factor such that $\|w\| = 1$. Now problem is simplified to SNR equals to $|\bar{w}^H \bar{h}|^2 \; P/\sigma_n^2$

$$\max |w^H \bar{h}|^2 \frac{P}{\sigma_n^2}$$

$$\bar{w} = \overset{\text{constant}}{c}\bar{h}$$

$$\Rightarrow c^2 \|\bar{h}\|^2 = 1$$

$$c = \frac{1}{\|h\|}$$

$$\bar{a}^H \cdot \bar{b}$$

$$= |\bar{a}||\bar{b}| \underbrace{\cos\theta}_{\theta = 0}$$

$$= |\bar{a}||\bar{b}|$$

$$[\because c = \text{constant}]$$

- The receive beamforming vector, which maximizes the SNR is c times $\bar{h}$
- optimal Beamforming vector $\bar{w}$ the maximizes the receive SNR

$$\bar{w}_{opt} = \frac{\bar{h}}{\|\bar{h}\|}$$

- from SNR formulae i.e

$$SNR = |\bar{w}^H \bar{h}|^2 \frac{P}{\sigma_n^2}$$

$$SNR = \|\bar{h}\|^2 \frac{P}{\sigma_n^2}$$

* This combiner w_{opt} that maximizes the SNR has a name that is known as the maximal ratio combiner (or) MRC.

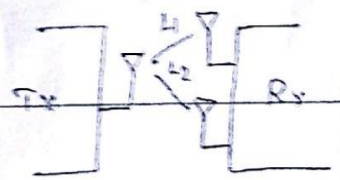
$$\bar{w}_{opt} = \frac{\bar{h}}{\|\bar{h}\|} \begin{array}{l} \rightarrow \text{MRC} \\ \rightarrow \text{spatial matching filter} \end{array}$$

* The temporal match filter, that is you match the filter characteristic across time, we are matching the beam former across the different receive antennas, that is across space hence this is known as 'spatial matched filter'

Example:

Given: $L = 2$ in Receiver i.e. this is taxi

$L = 2$ in Receiver means system is having 2 receiving antennas at the receiver



$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} x + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix}$$

$$h_1 = \left[\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} j\right] \Rightarrow |h_1| = \sqrt{\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2} = 1$$

if $a = (x + iy)$
$\left[\because |a| = \sqrt{x^2 + y^2}\right]$

$$h_2 = \left[\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} j\right] \Rightarrow |h_2| = \sqrt{\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2} = 1$$

$$\bar{h} = \begin{bmatrix} \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} j \\ \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} j \end{bmatrix}$$

$$\|\bar{h}\|^2 = |h_1|^2 + |h_2|^2$$

ADVANCED 3G AND 4G WIRELESS COMMUNICATIONS

$$\|h\|^2 = 2$$

$$\|h\| = \sqrt{2}$$

$$W_{MRC} = \frac{\bar{h}}{\|h\|} = \frac{1}{\sqrt{2}}\begin{bmatrix} \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}j \\ \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}j \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{2} + \frac{1}{2}j \\ \frac{1}{2} - \frac{1}{2}j \end{bmatrix}$$

Output of the Receive Beamformer

$$= \bar{w}^H \bar{y}$$

$$= \begin{bmatrix} \frac{1}{2} - \frac{1}{2}j & \frac{1}{2} + \frac{1}{2}j \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$$

$$\boxed{= \left(\frac{1}{2} - \frac{1}{2}j\right) y_1 + \left(\frac{1}{2} + \frac{1}{2}j\right) y_2}$$

$$h_1 = \left[\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}j\right] \quad |h_1| = \sqrt{\tfrac{1}{2} + \tfrac{1}{2}} = 1$$

$$h_2 = \left[\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}j\right] \quad |h_2| = 1$$

$$\bar{h} = \begin{bmatrix} \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}j \\ \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}j \end{bmatrix}$$

$$\|\bar{h}\|^2 = |h_1|^2 + |h_2|^2 = 1 + 1 = 2$$

ADVANCED 3G AND 4G WIRELESS COMMUNICATIONS

$$\|h\|^2 = 2$$
$$\|h\|^2 = 2$$

Rx SNR after MRC

$$= \frac{\|h\|^2 P}{\sigma_n^2} = \frac{2P}{\sigma_n^2}$$

Analysis of BER of Multiple Antenna System

$$R_x \text{ SNR} = \frac{\|h\|^2 P}{\sigma_n^2}$$
$$= \left(|h_1|^2 + |h_2|^2 + \cdots + |h_L|^2\right)\frac{P}{\sigma_n^2}$$
$$= g\frac{P}{\sigma_n^2}$$
$$g = |h_1|^2 + |h_2|^2 + \cdots + |h_L|^2$$

$g = \|h\|^2$ is a CHI-SQUARED Random variable with 2L degrees of freedom.

$$\boxed{f_G(g) = \frac{1}{(L-1)!} g^{L-1} e^{-g}}$$

compare equation ① & ②

$$\therefore \quad g = \|h\|^2 \quad \text{—— ③}$$

* equation ③ is also called CHI-SQUARED Random variable with 2L degree of freedom

$$F_G(g) = \frac{1}{(L-1)!} g^{L-1} e^{-g} \quad \text{—— ④}$$

* Here each fading coefficient is rely, magnitude is rely in nature that is each h_i has a fading coefficient whose real part and imaginary part are Guassian and also they are independent, uncorrelated of each other.

* Sub equation ③ in ①

$$R_x \; SNR = g \frac{P}{\sigma_n^2} \quad \text{—— ⑤}$$

* Instantaneous Bit Error Rate is

$$= Q\left[\sqrt{g \, SNR}\right]$$

* Average BER $= \int_0^\infty \underbrace{Q\sqrt{g\,SNR}}_{\text{Inst SNR}} \cdot \underbrace{F_G(g)}_{\text{distribution of } g} \, dg$

* BER with L Receive antennas after MRC combining

$$\boxed{BER = \left(\frac{1-\lambda}{2}\right)^L \sum_{l=0}^{L-1} \binom{L+l-1}{l} \left(\frac{1+\lambda}{2}\right)^l}$$

where $\lambda = \sqrt{\frac{SNR}{2+SNR}}$

W.K.T (we know that) $\quad {}^nC_k = \frac{n!}{k!(n-k)!}$

Put $l = 1$

$$= \left(\frac{1-\lambda}{2}\right) \sum_{l=0}^{0} {}^{l}C_{0} \left(\frac{1+\lambda}{2}\right)^{l}$$

$$= \left(\frac{1-\lambda}{2}\right) {}^{0}C_{0} \left(\frac{1+\lambda}{2}\right)^{0}$$

$$= \left(\frac{1-\lambda}{2}\right) (1)(1)$$

$$= \left(\frac{1-\lambda}{2}\right) \Rightarrow \frac{1}{2}(1-\lambda)$$

$$BER = \frac{1}{2}\left(1 - \sqrt{\frac{SNR}{2+SNR}}\right)$$

* This is the BER expression for the single Receiver Antenna

* BER for high SNR is

$$BER = \frac{1}{2}\left(1 - \sqrt{\frac{SNR}{2+SNR}}\right)$$

$$= \frac{1}{2}\left(1 - \left(\frac{1}{(1+2/SNR)^{1/2}}\right)\right)$$

$$= \frac{1}{2}\left(1 - \left(1 - \frac{1}{2}\left(\frac{2}{SNR}\right)\right)\right)$$

$$\boxed{BER = \frac{1}{2SNR}}$$

* If we compute the approximation for $\frac{1}{2}(1+\lambda)$ at high SNR then

$$BER = \frac{1}{2}(1+\lambda)$$

$$\approx \frac{1}{2}\left(1 + \sqrt{\frac{SNR}{2+SNR}}\right)$$

$$\approx \frac{1}{2}\left(1 + \left(1 - \frac{1}{2}\left(\frac{2}{SNR}\right)\right)\right)$$

$$\approx \frac{1}{2}\left(2 - \frac{1}{SNR}\right)$$

$$\approx \frac{1}{2}(2)$$

$$= 1$$

For high SNR:

$$\frac{1}{2}(1-\lambda)$$
$$= \frac{1}{2}\left(1 - \sqrt{\frac{SNR}{2+SNR}}\right)$$
$$= \frac{1}{2}\left(1 - \frac{1}{\left(1+\frac{2}{SNR}\right)^{1/2}}\right)$$
$$= \frac{1}{2}\left(1 - \left(1 - \frac{1}{2}\frac{2}{SNR}\right)\right)$$
$$= \frac{1}{2\,SNR}$$

$$\frac{1}{2}(1+\lambda)$$
$$\approx \frac{1}{2}\left(1+1-\frac{1}{SNR}\right)$$
$$= \frac{1}{2}\left(2-\frac{1}{SNR}\right)$$
$$\approx \frac{1}{2}\cdot 2 = 1$$

At high SNR $\begin{cases} (1-\lambda)/2 \approx 1/2SNR \\ (1+\lambda)/2 \approx 1 \end{cases}$

ADVANCED 3G AND 4G WIRELESS COMMUNICATIONS

at high SNR

Average BER

$$= \left(\frac{1-\lambda}{2}\right)^{L} \sum_{\ell=0}^{L-1} {}^{L+\ell-1}C_{\ell} \left(\frac{1+\lambda}{2}\right)^{\ell}$$

$$= \left(\frac{1}{2\,SNR}\right)^{L} \sum_{\ell=0}^{L-1} {}^{L+\ell-1}C_{\ell}$$

$$= {}^{2L-1}C_{L} \cdot \frac{1}{2^{L}} \cdot \left(\frac{1}{SNR}\right)^{L}$$

At High SNR $\begin{cases} \left(\frac{1-\lambda}{2}\right) \simeq \frac{1}{2SNR} \\ \left(\frac{1+\lambda}{2}\right) \simeq 1 \end{cases}$

Now, At high SNR the Average BER is

$$\text{Average BER} = \left(\frac{1-\lambda}{2}\right)^L \sum_{l=0}^{L-1} {}^{L+l-1}C_l \left(\frac{1+\lambda}{2}\right)^l$$

$$= \left(\frac{1}{2SNR}\right)^L \sum_{l=0}^{L-1} {}^{L+l-1}C_l$$

$$\text{Avg BER} = {}^{2L-1}C_L \cdot \frac{1}{2^L} \cdot \left(\frac{1}{SNR}\right)^L$$

This is the BER with 'L' R_x Antennas after maximal ratio combining at high SNR.

<u>Example</u>

$L=2$ Receive antennas what is the SNR required to acheive a BER of 10^{-6}?

Sol:- $\text{BER} = {}^{2L-1}C_L \frac{1}{2^L} \left(\frac{1}{SNR}\right)^L$

Given $L=2$

$\text{BER} = 10^{-6}$

$$10^{-6} = 2(2)^{-1}C_2 \frac{1}{2^2} \left(\frac{1}{SNR}\right)^2$$

$$= {}^3C_2 \frac{1}{4} \left(\frac{1}{SNR}\right)^2$$

$$= \frac{3!}{2!\,1!} \frac{1}{4} \left(\frac{1}{SNR}\right)^2$$

$$10^{-6} = \frac{3}{4} \left(\frac{1}{SNR}\right)^2$$

$$SNR = \frac{\sqrt{3}}{2} 10^3$$

SNR in dB is given as

$$(SNR)_{dB} = 10\log_{10}\left(\frac{\sqrt{3}}{8}\times 10^3\right)$$

$$= 29.37\ dB$$

The SNR required with one antenna was 57dB from previous lectures

$\therefore$ Reduction is $57 - 29$ dB

≈ 28 dB

Here power required with a single R_x-Antenna $= P_w^1$

Power required with a 2 R_x-Antennas $= P_w^2$

$$\therefore \quad 10\log_{10}\frac{P_w^1}{P_w^2} = 28 \approx 30\,dB$$

$$\frac{P_w^1}{P_w^2} = 10^3$$

$$P_w^2 = P_w^1/10^3$$

So add one more antenna i.e. receiver antenna as a result there will be improvement in the bit error rate at the receiver

ADVANCED 3G AND 4G WIRELESS COMMUNICATIONS

RECEIVER DIVERSITY:

-Receiver diversity is very important in 3G, 4G wireless communication system.

-Diversity is a key aspect, it is employed in technologies like

1. WCDMA
2. HSDPA
3. LTE
4. Wi MAX

-All these technologies are going to use some form of diversity because diversity results in a significant diversity in the SNR at receiver.

Why BER is decreasing with R_X-Antenna?

- We know that for 'L' Antenna BER is
  $BER=2L-1C_L\ (1/2^L)\ (1/SNR)^L$
  When L=1 then (1/2SNR) proportional to (1/SNR)

L=2 then $(3/4)\ (1/SNR)^2$ proportional to $(1/SNR^2)$

- At L=1 Receive Antenna , BER is decreasing as 1/SNR
- At L=2 Receive Antenna , BER is decreasing as $1/SNR^2$
- For L=3
  BER proportional to $(1/SNR^3)$
- As the number of R_X antenna is increasing, the BER decreasing at a much faster rate.

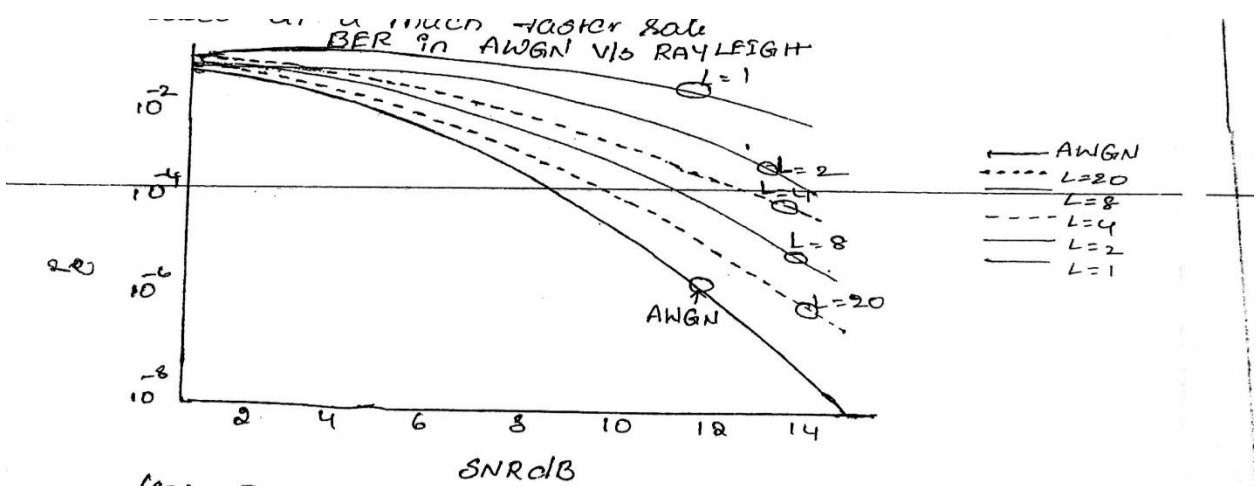


fig:- BER with Diversity

- From this we can observe as the number of receive antennas at a faster rate.
- So increase in antenna at receiver increase the performance of a system.

Multiple Antenna system:-

* As we already know that the system model can be expressed as

$$\bar{y} = \bar{h}x + \bar{n}$$

* Signal power after MRC is $\|h\|^2 \frac{P}{\sigma_n^2} = g\frac{P}{\sigma_n^2}$
* In the system with multiple receive antennas, the received signal to Noise power ratio after MRC (or) at the output of the MRC is Nothing but $\|h\|^2 P/\sigma_n^2$
* Signal to Noise Ratio SNR is

$$SNR = \|h\|^2 P/\sigma_n^2$$

where $\|h\|^2 P \rightarrow$ signal power

$\sigma_n^2 \rightarrow$ Noise power

$$\|h\|^2 P < \sigma_n^2$$

$$gP < \sigma_n^2 \qquad [\|h\|^2 = g]$$

$$g < \left(\frac{1}{SNR}\right) \rightarrow \left[SNR = \frac{P}{\sigma_n^2}\right)$$

$$g < \left(\frac{1}{SNR}\right) \rightarrow \text{deep fade}$$

* Probability distribution of the g is

$$F_G(g) = \frac{g^{L-1}}{(L-1)!} e^{-g}$$

$$P\left(g < \frac{1}{SNR}\right) = \int_0^{1/SNR} \frac{g^{L-1}}{(L-1)!} \underbrace{e^{-g}}_{1} dg$$

$$\int_0^{1/SNR} \frac{g^{L-1}}{(L-1)!} dg$$

$$= \left[g^L \left(\frac{1}{L!} \right) \right]_0^{1/SNR}$$

$$= \left[\frac{g^L}{L!} \right]_0^{1/SNR}$$

$$= \frac{1}{L!} \left[g^L \right]_0^{1/SNR}$$

$$= \frac{1}{L!} \left[\left(\frac{1}{SNR} \right)^L - 0 \right]$$

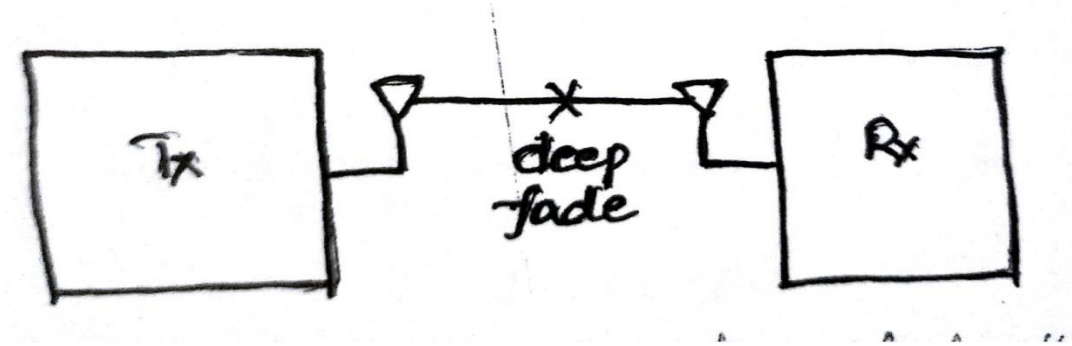
$$= \frac{1}{L!} \left[\frac{1}{SNR} \right]^L$$

Probability of deep fade $\propto \left(\frac{1}{SNR} \right)^L$

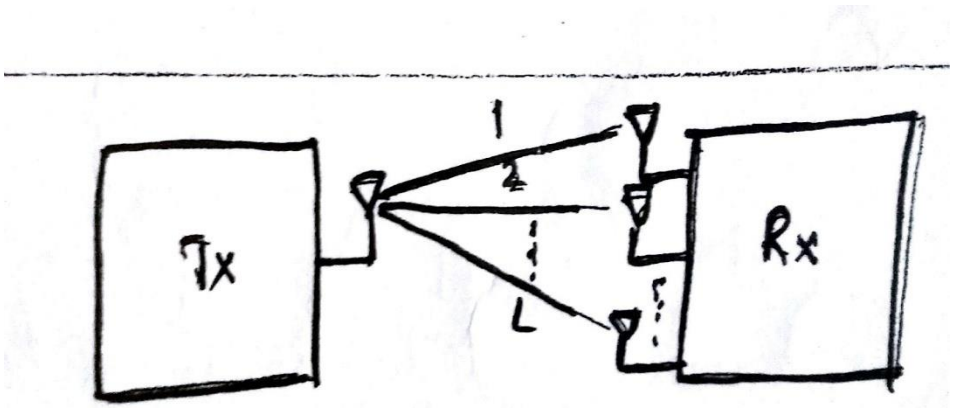
- The probability of deep fade decreases significantly with more receive antennas.

INTUITION:

- Consider a system with one transmitter and one receiver , this system will have only one link between Tx and Rx.



- If the link was in a deep fade then the bit error rate (BER) is high.
- The probability with which this single link is in a deep fade is 1/SNR hence, the BER is 1/SNR.
  If we consider a system with multiple receive antennas and there are L links between transmitter and receiver.



- If E_I is the event that link-I (or) i^{th} link is in a deep fade.
- Now we want deep fade event , Net deep fade event is P i.e. represented as

$$P(E1 \cap E2 \cap \ldots\ldots EL)$$

  Here all the links are independent
- P(E1) is the probability that links one is in a deep fade that is 1/SNR
- P(E2) is the probability that link two is in a deep fade 1/SNR
- P[E1NE2…….El]=p(E1) p(E2)……p(El)
- i.e = p(E1) p(E2)…….p(El)
- = (1/SNR)(1/SNR)……..(1/SNR)=$(1/SNR)^L$
- This is the deep facts of probability,which is decreasing the deep for a 1/SNR

SPATIAL DIVERSITY

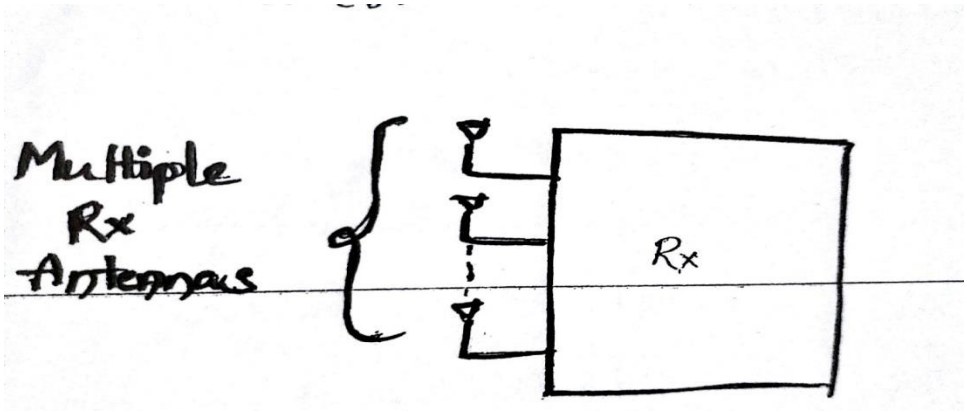
-Multiple antenna systems are key to technology in 3G and 4G wireless communication

ADVANCED 3G AND 4G WIRELESS COMMUNICATIONS

Systems .they are a part of WCDMA,HSDPA,4G systems LTE and WIMAX ,so it is important in

understanding of multiple antenna systems and especially of diversity the system

-we consider multiple receive antennas in the system, these antennas are placed apart from each other



-so unlike time or in frequency, these antennas are placed in different spatial locations. Hence,this multiple antennas diversity is also known as spatial diversity, so this multiple antenna diversity is also known as spatial diversity , so this multiple antenna diversity is also known as spatial diversity

-let us assume one critical assumption that each of these 'L' channels are independent. we assume E1,E2,…….El which is events across the 'L' receive antennas are independent

-we have'L' links corresponding to the 'L'receive antennas and each of these antennas are independent.We had the probability of intersection as the product of the probability, so the key assumption is the independence of these channels for these channels to be independent, the antennas have to be placed sufficiently for apart

-we have 'L' receive antennas, the receive antenna are close to one another than the signal received. These different antennas are highly correlated, that is the reason the independent assumption does not hold

ADVANCED 3G AND 4G WIRELESS COMMUNICATIONS

-to hold this assumption, the antennas are to be placed sufficiently for apart so that the received signals across these 'L' receive antennas are independent

HOW FAR APART?

-For independent channels across the receive antennas the spacing is required or the minimum spacing required equals lambda/2 i.e.

The spacing required = lambda/2

Example:-

Consider a GSM system, $f_c = 900 MHz$.

$$\lambda_{GSM} = \frac{c}{f_c}$$

$$= \frac{3 \times 10^8 \, m/s}{900 \times 10^6 \, /s}$$

$$= \frac{\cancel{3} \times \cancel{10^8}}{\cancel{9} \times \cancel{10^8}}$$

$$= \frac{1}{3} m = 0.333 m = 0.333 \times 10^2 cm$$

$$\lambda_{GSM} = 33.33 \, cm$$

Hence, spacing between antennas

$$= \frac{\lambda}{2} = \frac{33.33}{2}$$

$$= 16.66 \, cm$$

-Hence the spacing between the antennas is in aGSM system for independence to hold id 16.66

-if we look at a normal cell phone , it is around 6 to 7cms in dimension so ,on a phone in a GSM mobile phone it is not possible to plan multiple antennas well at least not possible to place multiple antennas for independent channels.

Therefore this is not possible to place multiple antennas

This is greater than the dimension because h, the spacing is greater than that dimension

EXAMPLE:

What is the minimum spacing required for diversity or independence?

Let us consider a 3G, 4Gsystem. the spacing can be either at least this or greater the spacing between the antennas but the restriction is that the device itself is fairly small.

The system is 3G, 4G system. The carrier frequency for a 3G system is given by

The system is a 3G/4G system. The carrier frequency for a ~~3G system is given by~~

$$f_c = 2.3\,GHz$$

$$\therefore \lambda = \frac{c}{f_c} = \frac{3\times10^8}{2.3\times10^9}$$

$$= \frac{3\times10^8\ m/s}{23\times10^8\ 1/s}$$

$$= 0.1304\ m$$

$$= 0.1304\times10^2\ cm$$

$$\lambda = 13.04\,cm, \quad \frac{\lambda}{2} = 6.52\,cm$$

minimum spacing

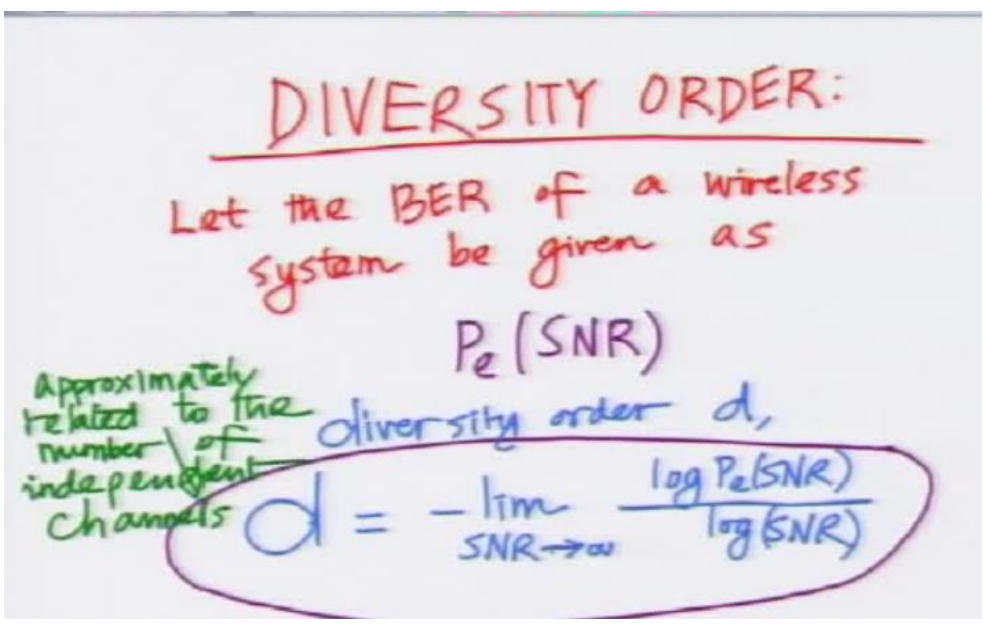
ADVANCED 3G AND 4G WIRELESS COMMUNICATIONS

The minimum spacing required for diversity or independence is 6.52cms and this is comparable to the dimensions of mobile phone i.e., the mobile phone can have around 6 to 7 cm

Therefore here, it is possible to place multiple antennas on the mobile phone in a third generation or fourth generation mobile phone The reason is that as the carrier frequency increases the required spacing decreases , the wavelength decreases and hence the antenna spacing decreases
<u>i.e lamda proportional to 1/fc</u>

DIVERSITY ORDER

- By reducing the bit error rate in a 3G,4G wireless system , there is a possibility of placing multiple antennas on the phone and also possible to employ multiple receive antennas or to the receive diversity therefor , extracting receive diversity and drastically reducing the bit error rate
- Let the bit error rate of wireless system given as a probability as a function of SNR i.e. Pe (SNR)
- So, the bit error rate decreases with respect to SNR



This is approximately related to the numbers of independent channels
This definition of diversity order gives approximately .if we know the bit error rate function as function of SNR. It gives the approximate number of independent

channels in the system or it also tells what rate the bit error rate is decreasing with respect to SNR.

WIRELESS SYSTEM-DIVERSITY OF WITH ONE RECEIVE ANTENNA

- Let us take an example of considering the wireless one receive antenna
- Now let us consider a wireless system i.e , two kinds of wireless systems .
- One is wireless system which has only one transmit antenna and one receive antenna
- The other is a wireless system which has one transmit antenna and multiple receive antennas
- The original wireless systems which has only receive antenna which means L = 1

The bit error rate or $P_e(SNR) = \frac{1}{2SNR}$.

diversity order, $d = -\lim_{SNR \to \infty} \frac{\log(1/2SNR)}{\log SNR}$.

$$= -\lim_{SNR \to \infty} \frac{\log 1 - (\log SNR + \log 2)}{\log SNR}$$

$$= -\lim_{SNR \to \infty} \frac{\log 1 - [\log 2SNR]}{\log SNR}$$

$$= -\lim_{SNR \to \infty} \frac{0 - [\log 2 + \log SNR]}{\log SNR}$$

$$= -\lim_{SNR \to \infty} \frac{-\log 2 - \log SNR}{\log SNR}$$

$$= \lim_{SNR \to \infty} \frac{\log SNR + \log 2}{\log SNR}$$

$$= -\lim_{SNR\to\infty} \frac{\log\left[{}^{2L-1}C_L \cdot \frac{1}{2^L}\right] - \log(SNR)^L}{\log SNR}$$

$$= \lim_{SNR\to\infty} \frac{\log(SNR)^L - \log\left[{}^{2L-1}C_L \cdot \frac{1}{2^L}\right]}{\log SNR}$$

$$= \lim_{SNR\to\infty} \frac{L\log(SNR) - \log\left[{}^{2L-1}C_L \cdot \frac{1}{2^L}\right]}{\log SNR}$$

$$= \lim_{SNR\to\infty} \frac{L\,\cancel{\log(SNR)}}{\cancel{\log SNR}} - \frac{\log\left[{}^{2L-1}C_L \cdot \frac{1}{2^L}\right]}{\log SNR}$$

$$= \lim_{SNR\to\infty} \frac{L - \log\left[{}^{2L-1}C_L \frac{1}{2^L}\right]}{\log SNR}$$

$$= \lim_{SNR\to\infty} L - \frac{\log\left[{}^{2L-1}C_L \cdot \frac{1}{2^L}\right]}{\log SNR}.$$

At high SNR i.e., $SNR \to \infty$ $\quad \frac{\log\left[{}^{2L-1}C_L \cdot \frac{1}{2^L}\right]}{\log SNR} = 0.$

$$\therefore d = \lim_{SNR\to\infty} L - 0$$

$$= \lim_{SNR\to\infty} L$$

$$\therefore d = L$$

(40)

Diversity Order of a Wired or Wireline Communication System:-

→ The bit error rate of a wireline communication system as

$$Q(\sqrt{SNR}).$$

The Probability is given by

$$P_e(SNR) = Q(\sqrt{SNR})$$

$$\simeq e^{-SNR/2}$$

$$\therefore \text{Diversity Order}, d = -\lim_{SNR\to\infty} \frac{\log e^{-SNR/2}}{\log SNR}$$

$$= -\lim_{SNR\to\infty} \frac{-\frac{SNR}{2}\log e}{\log SNR}$$

$$= -\lim_{SNR\to\infty} \frac{-\frac{SNR}{2}(1)}{\log SNR}$$

$$= \frac{1}{2}\lim_{SNR\to\infty} \frac{SNR}{\log SNR}$$

By using log Pitho's rule, we get

$$= \frac{1}{2}\lim_{SNR\to\infty} \frac{SNR}{\log SNR}$$

Differentiating the numerator with SNR we get 1 and differentiating the denominator with SNR i.e., $\frac{\log SNR}{SNR} = 1/SNR$

$$\therefore d = \frac{1}{2}\lim_{SNR\to\infty} \frac{1}{(1/SNR)} = \frac{1}{2}\lim_{SNR\to\infty} (SNR).$$

$$d = \infty$$

ADVANCED 3G AND 4G WIRELESS COMMUNICATIONS

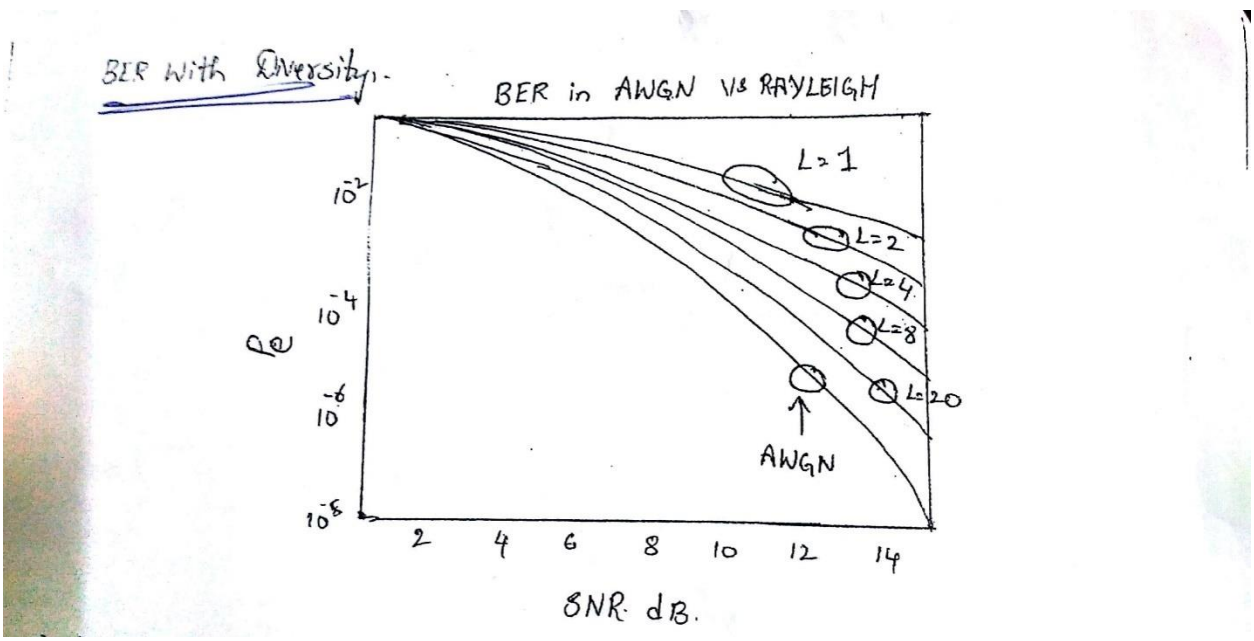
INTUTION

Wireless channel has diversity order, d=∞

The main institution similar similar to a wireless, wired or wire line communication system having multiple receive antennas that is L receive antennas where L tends to infinity.

That is the region why the wire line communication system has such a superior performance because it can be thought of as a system having infinite number of independent links which means a very small bit of error rate which decreases exponentially

- We saw that , for diversity order 1 bit error rate is high
- For order 2 , slightly lower
- For order 4 , low
- For order 8, lower
- For order 20 ,even lower



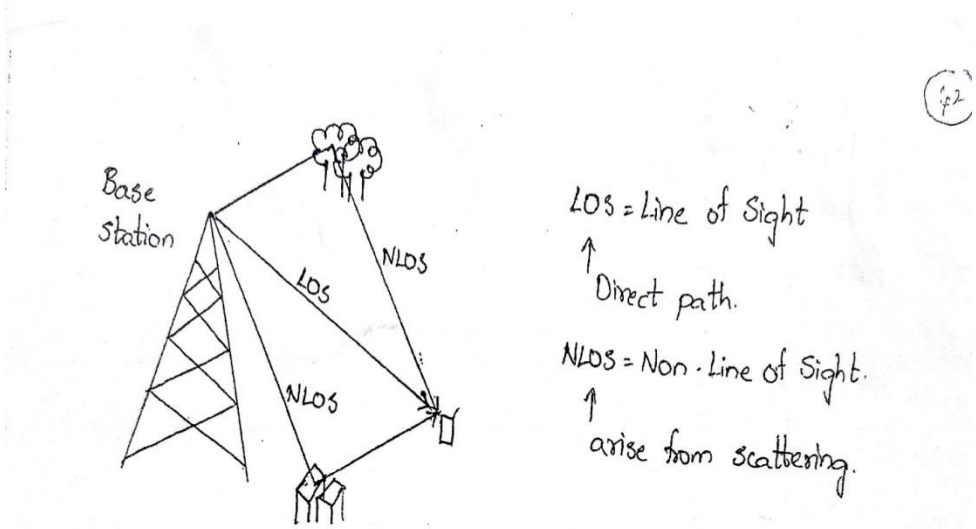
- We approach the limit of the digital communication channel as L as the diversity order progressively keeps increasing in the limit.
- We approach the AWGN channel because we can think of a wired or a wire line communication channel as having infinite diversity order. That the

wireline channel can thought of as comprising of an infinite number of independent links that is the reason its bit error rate is significantly smaller.

WIRELESS CHANNEL

How does the radio wireless channel affect 3G/4G wireless mobile communications?

- We have a base station that is transmitting , to a mobile phone and mobile station



- There is a direct line of sight path , but there are also indirect non line of sigth paths . trees act as scatter the signal and building act as another set of scatters. They also scatter the received signal so this is line of sight paths (NLSO PATH) which are arising due to scatter
- The line of sight which is direct path and there are several non line of sight which arise due to scattering and scatters that are present in the environment
- The net multipath channel or multipath wireless channel can be modeled as a delay an attenuation corresponding to each of these paths.

The i th path can be modeled as an attenuation a_i and a delay which is given as a impulse $\Delta(t - Ti)$.

$$\mathrm{h(t)} = \sum_{i=0}^{L-1} \mathrm{ai}\Delta(\mathrm{t} - \mathrm{Ti})$$