

Inverse Fast Fourier Transform (IFFT)

IFFT can be obtained from some structures which are used to calculate FFT using DIF-FFT and DIF-FFT.

$$DFT[x(n)] = X(k) = \sum_{n=0}^{N-1} x(n) e^{\frac{j2\pi kn}{N}} ; k = 0 \text{ to } N-1$$

$$IDFT[x(k)] = x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k) e^{\frac{j2\pi kn}{N}} ; n = 0 \text{ to } N-1$$

$$N \cdot x(n) = \sum_{k=0}^{N-1} X(k) W_N^{-kn}$$

apply Conjugation on both sides.

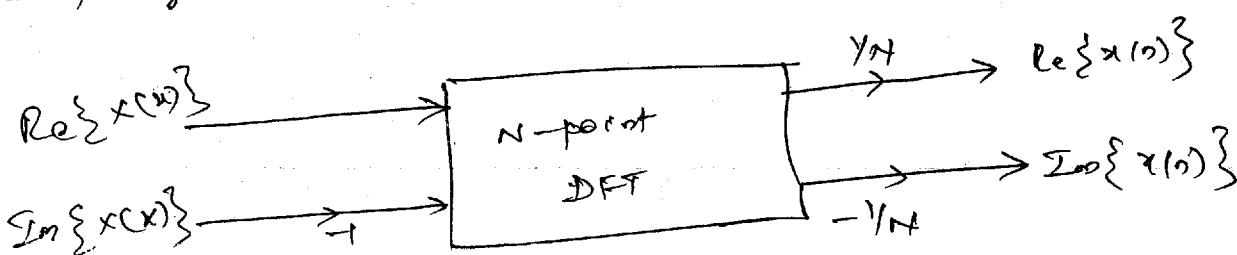
$$N \cdot x^*(n) = \sum_{k=0}^{N-1} X^*(k) W_N^{kn} ; n = 0 \text{ to } N-1$$

The above equation can be assumed as DFT of signal $X^*(k)$ and can be computed using any one of the FFT algorithms.

The desired $IDFT[x(n)]$ is then obtained as

$$x(n) = \frac{1}{N} \left\{ \sum_{k=0}^{N-1} X^*(k) W_N^{nk} \right\}^*$$

i.e. Given as N -point DFT $X(k)$, we first form its Complex Conjugate sequence $X^*(k)$, then compute N -point DFT of $X^*(k)$ and obtain Complex Conjugate of DFT computed, and finally divide each sample by N .



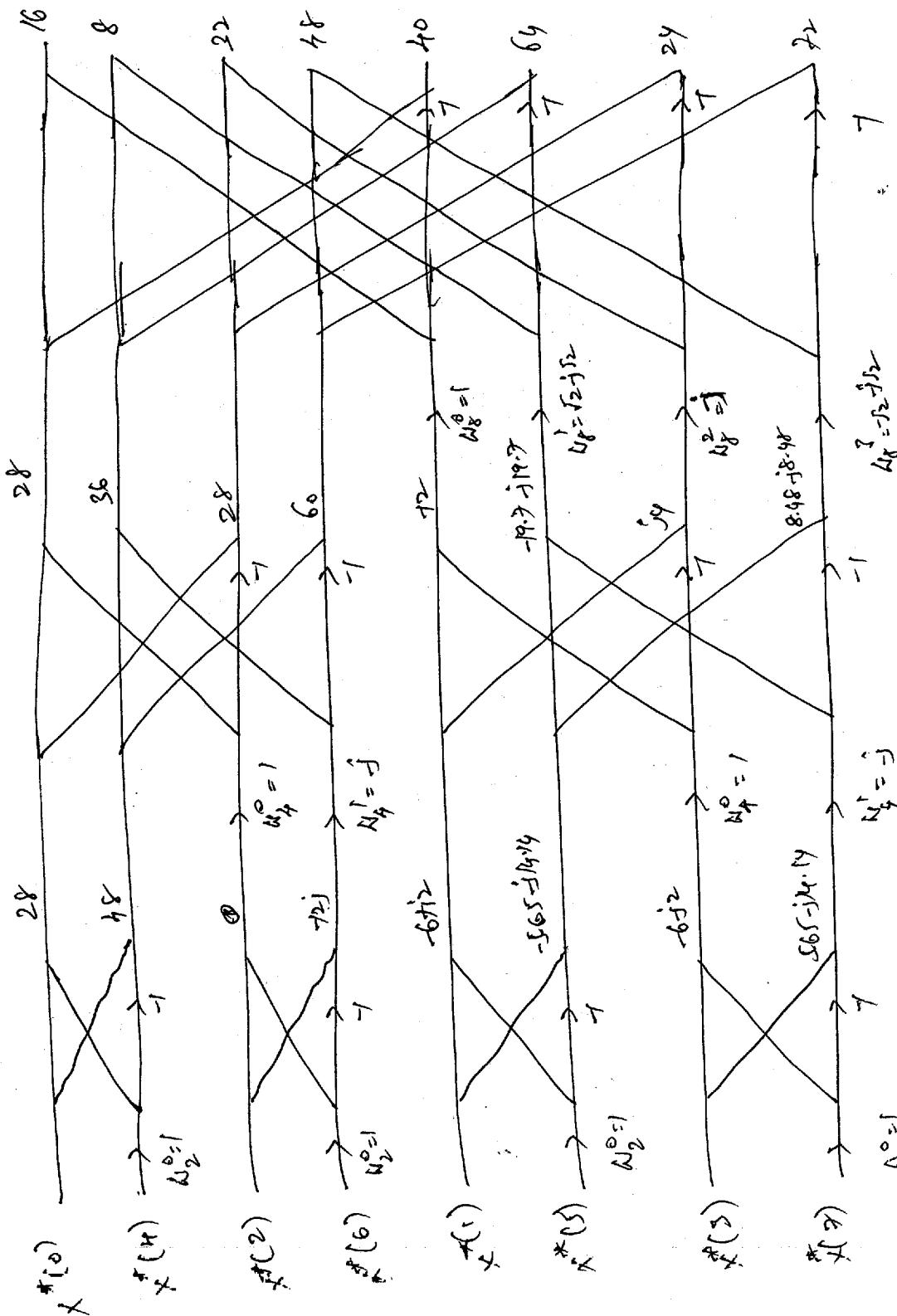
Prob 2: Calculate the IFFT for given α -coefficients

$$X(k) = \{ 38, -5.82 + j6.07, 36, -0.172 + j8.07, -10, -0.172 - j8.07, \\ -j6, -5.82 - j6.07 \}$$

using DIT FFT structure

Soln: Given $X(n) = \{ 38, -5.82 + j6.07, 36, -0.172 + j8.07, -10, -0.172 - j8.07, \\ -j6, -5.82 - j6.07 \}$

$$X^*(n) = \{ 38, -5.82 - j6.07, -j6, -0.172 - j8.07, -10, -0.172 + j8.07, +j6, \\ -5.82 + j6.07 \}$$



$$x(n) = \frac{1}{8} \{ 16, 8, 48, 40, 69, 24, 72 \}$$

$$x(n) = \{ 16, 8, 48, 40, 69, 24, 72 \}$$

Repeat above problem using DIF-FFT

DSP-NOTES

~~AKASH SHORE~~
⇒ DFT for Composite N (or) Composite Radix FFT :-

A Composite or mixed radix is used when 'N' is not a power of 2 (or) when N is a composite number which has more than one prime factor. Because it is not always possible to work with sequences whose length is power of 2. Efficient DFT and DIF algorithms exists to compute DFT even if N is not a power of 2.

N can be written as $N = m_1 m_2 \dots m_r$

If $N = m_1 N_1$, where $N_1 = m_2 m_3 \dots m_r$, the sequence $x(n)$ can be separated into m_1 subsequences of N_1 elements each. Then the DFT can be written as

$$X(k) = \sum_{n=0}^{N_r-1} x(2m_r) W_N^{nm_r k} + \sum_{n=0}^{N_r-1} x(nm_r + 1) W_N^{(nm_r + 1)k}$$

$$+ \dots + \sum_{n=0}^{N_r-1} x(nm_r + m_r - 1) W_N^{(nm_r + m_r - 1)k}$$

prob :- Develop a radix-3 DIF FFT algorithm for evaluations of DFT for $N=9$.

Sol :- $N=9 = 3 \cdot 3$; $m_r = 3$; $n_r = 3$

$\therefore X(k)$ according to above equation.

$$X(k) = \sum_{n=0}^2 x(3n) W_9^{3nk} + \sum_{n=0}^2 x(3n+1) W_9^{(3n+1)k} + \sum_{n=0}^2 x(3n+2) W_9^{(3n+2)k}$$

$$= X_0(k) + W_9^k X_1(k) + W_9^{2k} X_2(k)$$

$$X_0(k) = \sum_{n=0}^2 x(3n) W_9^{3nk} = x(0) + x(3) W_9^{3k} + x(6) W_9^{6k}$$

$$X_1(k) = \sum_{n=0}^2 x(3n+1) W_9^{(3n+1)k} = x(1) + x(4) W_9^{3k} + x(7) W_9^{6k}$$

$$X_2(k) = \sum_{n=0}^2 x(3n+2) W_9^{3nk} = x(2) + x(5) W_9^{3k} + x(8) W_9^{6k}$$

$$\boxed{X_0(k+N_r) = X_0(k)}$$

$$X(0) = X_0(0) + W_9^0 X_1(0) + W_9^{2k} X_2(0)$$

$$X(1) = X_0(1) + W_9^1 X_1(1) + W_9^{2k} X_2(1)$$

$$X(2) = X_0(2) + W_9^2 X_1(2) + W_9^{4k} X_2(2)$$

$$X(3) = X_0(3) + W_9^3 X_1(3) + W_9^{6k} X_2(3)$$

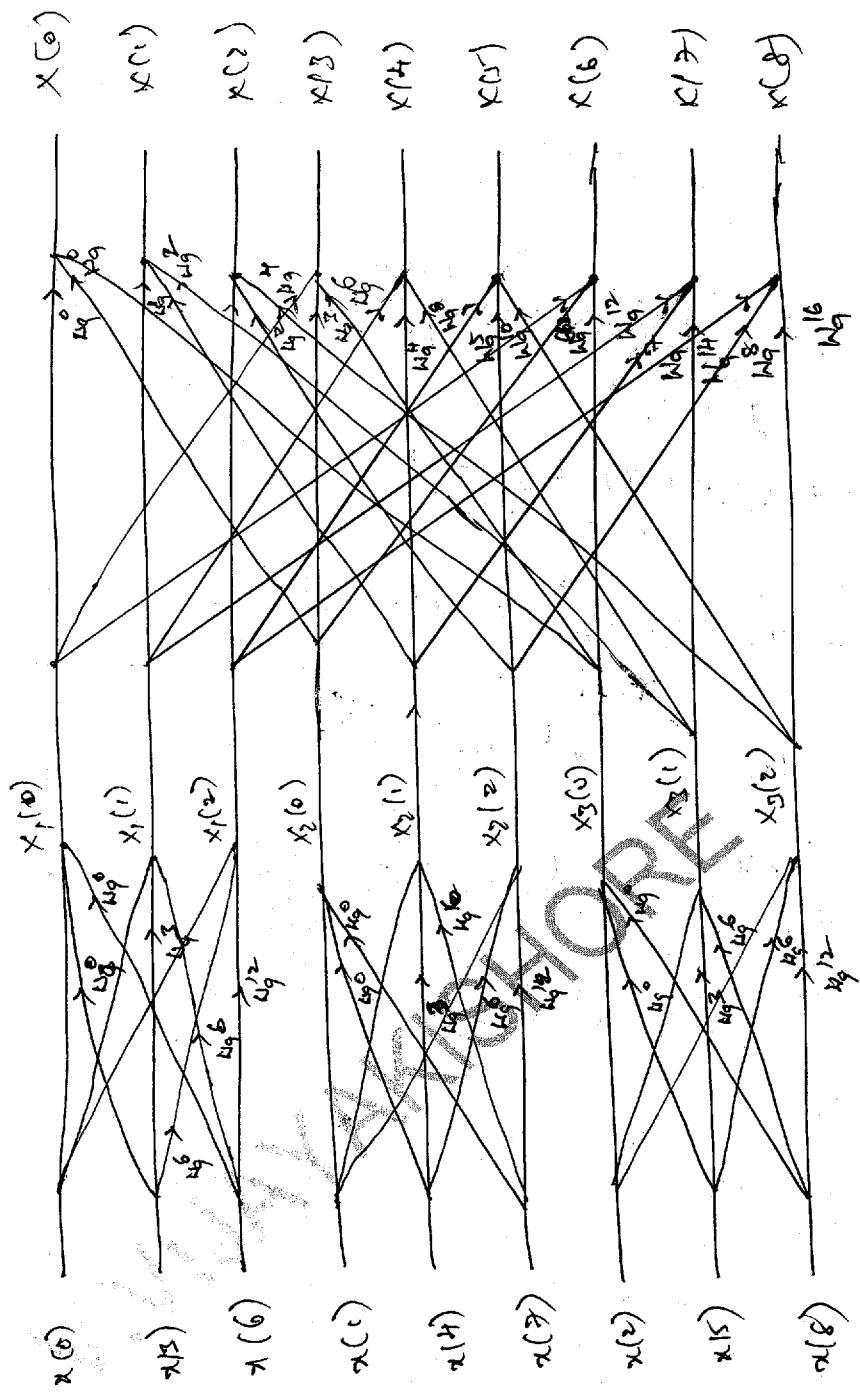
$$X(4) = X_0(4) + W_9^4 X_1(4) + W_9^{8k} X_2(4)$$

$$X(5) = X_0(5) + W_9^5 X_1(5) + W_9^{10k} X_2(5)$$

$$X(6) = X_0(6) + W_9^6 X_1(6) + W_9^{12k} X_2(6)$$

$$x(7) = x_r(7) + w_9^7 x_2(1) + w_9^{14} x_3(2)$$

$$x(8) = x_r(8) + w_9^8 x_2(2) + w_9^{16} x_3(2)$$



DSP-NOTES

* Develop a radix-3 DIF-FFT algorithm for evaluating
the DFT for $N=9$.

Given: $N=9=3 \cdot 3$

$$\begin{aligned}
 x(n) &= \sum_{n=0}^2 x(n) W_9^{nk} \\
 x(k) &= \sum_{n=0}^2 x(n) W_9^{nk} + \sum_{n=3}^5 x(n) W_9^{nk} + \sum_{n=6}^8 x(n) W_9^{nk} \\
 &= \sum_{n=0}^2 x(n) W_9^{nk} + \sum_{n=0}^2 x(n+3) W_9^{(n+3)k} + \sum_{n=0}^2 x(n+6) W_9^{(n+6)k} \\
 &= \sum_{n=0}^2 \left[x(n) + x(n+3) W_9^{3k} + x(n+6) W_9^{6k} \right] W_9^{nk} \\
 x(3k) &= \sum_{n=0}^2 \left(x(n) + x(n+3) W_9^{9k} + x(n+6) W_9^{18k} \right) W_9^{nk} \\
 &= \sum_{n=0}^2 (x(n) + x(n+3) + x(n+6)) W_9^{3nk} \quad (\because W_N = 1) \\
 &= \sum_{n=0}^2 f(n) W_9^{3nk} \\
 x(3k+1) &= \sum_{n=0}^2 \left(x(n) + x(n+3) W_9^3 + x(n+6) W_9^6 \right) W_9^{3nk} \cdot W_9^{3nk} \\
 &= \sum_{n=0}^2 g(n) W_9^n W_9^{3nk} \\
 x(3k+2) &= \sum_{n=0}^2 \left(x(n) + x(n+3) W_9^6 + x(n+6) W_9^3 \right) W_9^{2nk} \cdot W_9^{3nk} \\
 &= \sum_{n=0}^2 h(n) W_9^{2nk} W_9^{3nk} \\
 \text{Simplifying} \\
 f(0) &= x(0) + x(3) + x(6) \\
 f(1) &= x(1) + x(4) + x(7) \\
 f(2) &= x(2) + x(5) + x(8) \\
 \left| \begin{array}{l} g(0) = x(0) + x(3) W_9^3 + x(6) W_9^6 \\ g(1) = x(1) + x(4) W_9^3 + x(7) W_9^6 \\ g(2) = x(2) + x(5) W_9^3 + x(8) W_9^6 \end{array} \right.
 \end{aligned}$$

$$h(0) = \alpha(0) + \alpha(8) w_9^6 + \alpha(6) w_9^3$$

$$h(1) = \alpha(1) + \alpha(4) w_9^6 + \alpha(7) w_9^3$$

$$h(2) = \alpha(2) + \alpha(5) w_9^6 + \alpha(8) w_9^3$$

III

$$k(0) = \beta(0) + \beta(1) + \beta(2)$$

$$k(3) = \beta(0) + \beta(1) w_9^3 + \beta(2) w_9^6$$

$$k(6) = \beta(0) + \beta(1) w_9^6 + \beta(2) w_9^3$$

$x(3n+1)$

$$x(1) = g(0) + g(1) w_9^1 + g(2) w_9^2$$

$$x(4) = g(0) + g(1) w_9 w_9^3 + g(2) w_9^2 w_9 \cancel{w_9}$$

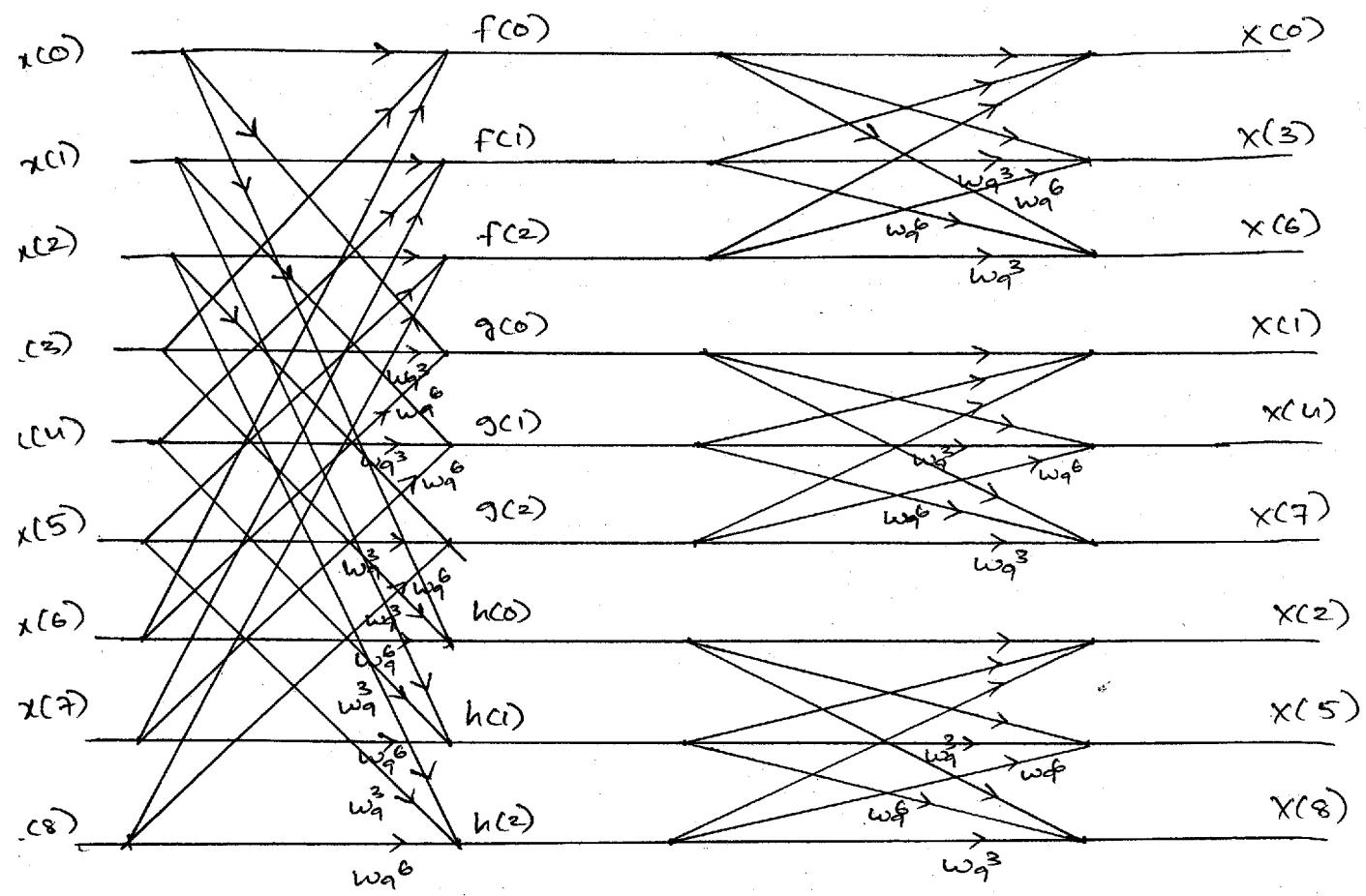
$$x(7) = g(0) + g(1) w_9 w_9^6 + g(2) w_9^2 w_9^{32}$$

$x(3n+2)$

$$x(2) = h(0) + h(1) w_9^2 + h(2) w_9^4$$

$$x(5) = h(0) + h(1) w_9^2 w_9^3 + h(2) w_9^4 w_9^6$$

$$x(8) = h(0) + h(1) w_9^2 w_9^6 + h(2) w_9^4 w_9^3$$



→ Develop DIT FFT algorithms for decomposing the DFT for $N=6$ and draw the flow diagrams for (a) $N=2 \cdot 3$ and (b) $N=3 \cdot 2$.

When $N=m_1 N_1$, the DFT can be written as

$$X(K) = \sum_{n=0}^{N_1-1} x(nm_1) W_N^{nm_1 K} + \sum_{n=0}^{N_1-1} x((nm_1+1)) W_N^{(nm_1+1)K} + \dots - \dots + \sum_{n=0}^{N_1-1} x((nm_1+m_1-1)) W_N^{(nm_1+m_1-1)K} \quad \rightarrow ①$$

(a) $N=6=2 \cdot 3$ where $m_1=2$ and $N_1=3$

then eq ① becomes

$$\begin{aligned} X(K) &= \sum_{n=0}^2 x(2n) W_6^{2nK} + \sum_{n=0}^2 x(2n+1) W_6^{(2n+1)K} \\ &= \sum_{n=0}^2 x(2n) W_6^{2nK} + W_6^K \sum_{n=0}^2 x(2n+1) W_6^{2nK} \quad \rightarrow ② \end{aligned}$$

We know that $x_i(K+N_1) = x_i(K)$

Here $N_1=3$

$$\Rightarrow x_i(K+3) = x_i(K) \quad \rightarrow ③$$

$$\text{let } x_1(k) = \sum_{n=0}^2 x(2n) w_6^{2nK}$$

$$\text{Now, } = x(0) + x(2) w_6^{2K} + x(4) w_6^{4K}$$

$$x_1(0) = x(0) + x(2) + x(4)$$

$$x_1(1) = x(0) + x(2) w_6^2 + x(4) w_6^4$$

$$x_1(2) = x(0) + x(2) w_6^4 + x(4) w_6^8$$

$$= x(0) + x(2) w_6^4 + x(4) w_6^2$$

$$w_6^8 = w_6^6 \cdot w_6^2 = w_6^2$$

$$[w_6^6 = 1]$$

$$\text{let } x_2(k) = \sum_{n=0}^2 x(2n+1) w_6^{2nK}$$

$$= x(1) + x(3) w_6^{2K} + x(5) w_6^{4K}$$

$$\text{Now, } x_2(0) = x(1) + x(3) + x(5)$$

$$x_2(1) = x(1) + x(3) w_6^2 + x(5) w_6^4$$

$$x_2(2) = x(1) + x(3) w_6^4 + x(5) w_6^8$$

$$= x(1) + x(3) w_6^4 + x(5) w_6^2$$

eq(2) can be written as

$$x(k) = x_1(k) + w_6^k x_2(k)$$

$$x(0) = x_1(0) + x_2(0)$$

$$x(1) = x_1(1) + w_6 x_2(1)$$

$$x(2) = x_1(2) + w_6^2 x_2(2)$$

$$x(3) = x_1(3) + w_6^3 x_2(3) = x_1(0) + w_6^3 x_2(0)$$

$$x(4) = x_1(4) + w_6^4 x_2(4) = x_1(1) + w_6^4 x_2(1)$$

$$x(5) = x_1(5) + w_6^5 x_2(5) = x_1(2) + w_6^5 x_2(2)$$

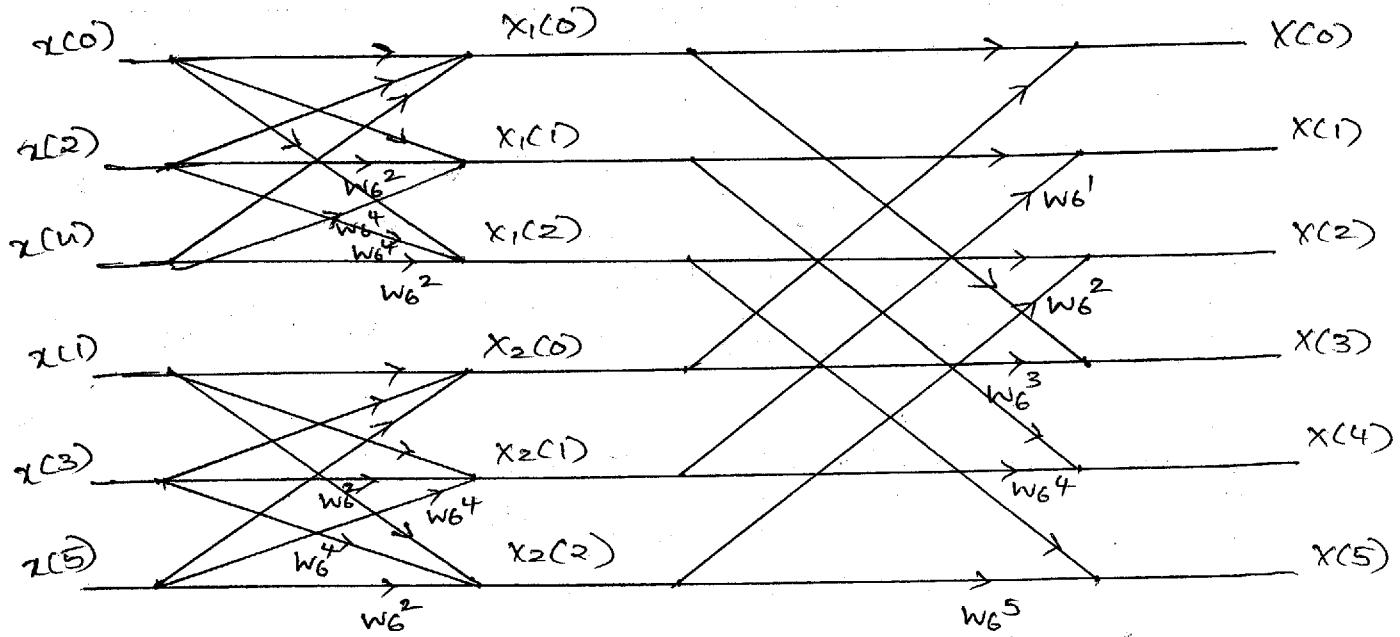
$$x(6) = x_1(6) + w_6^6 x_2(6) = x_1(0) + \cancel{w_6^3 x_2(0)}$$

$$x(7) = x_1(7) + w_6^7 x_2(7) = x_1(1) + \cancel{w_6^7}$$

$$x(8) = x_1(8) + w_6^8 x_2(8)$$

[from (3)]

flow diagram for $N=6 = 2 \cdot 3$



(b) $N=6 = 3 \cdot 2$ where $m_1=3$ and $N_1=2$

then eq① becomes

$$x(k) = \sum_{n=0}^1 x(3n)w_6^{3nK} + \sum_{n=0}^1 x(3n+1)w_6^{(3n+1)K} + \sum_{n=0}^1 x(3n+2)w_6^{(3n+2)K}$$

$$= \sum_{n=0}^1 x(3n)w_6^{3nK} + w_6^K \sum_{n=0}^1 x(3n+1)w_6^{3nK} + w_6^{2K} \sum_{n=0}^1 x(3n+2)w_6^{3nK} \quad \hookrightarrow ②$$

We know that $x_i(k+N_1) = x_i(k)$

Here $N_1=2$

$$x_i(k+2) = x_i(k) \rightarrow ③$$

$$\text{Let } x_i(k) = \sum_{n=0}^1 x(3n)w_6^{3nK}$$

$$= x(0) + x(3)w_6^{3K}$$

$$\text{Now, } x_i(0) = x(0) + x(3)$$

$$x_i(1) = x(0) + x(3)w_6^{3K}$$

$$\text{Let } x_i(k) = \sum_{n=0}^1 x(3n+1)w_6^{3nK} - x(1) + x(4)w_6^{3K}$$

$$x_i(0) = x(1) + x(4)$$

$$x_i(1) = x(1) + x(4)w_6^{3K}$$

$$\text{let } X_3(K) = \sum_{n=0}^1 x(3n+2) w_6^{3nK} = x(2) + x(5) w_6^{3K}$$

$$\text{Now, } X_3(0) = x(2) + x(5)$$

$$X_3(1) = x(2) + x(5) w_6^3$$

eq ② can be written as

$$x(k) = x_1(k) + w_6^k x_2(k) + w_6^{2k} x_3(k)$$

$$x(0) = x_1(0) + x_2(0) + x_3(0)$$

$$x(1) = x_1(1) + w_6^1 x_2(1) + w_6^2 x_3(1)$$

$$x(2) = x_1(2) + w_6^2 x_2(2) + w_6^4 x_3(2)$$

$$= x_1(0) + w_6^2 x_2(0) + w_6^4 x_3(0)$$

$$x(3) = x_1(3) + w_6^3 x_2(3) + w_6^6 x_3(3)$$

$$= x_1(1) + w_6^3 x_2(1) + w_6^6 x_3(1)$$

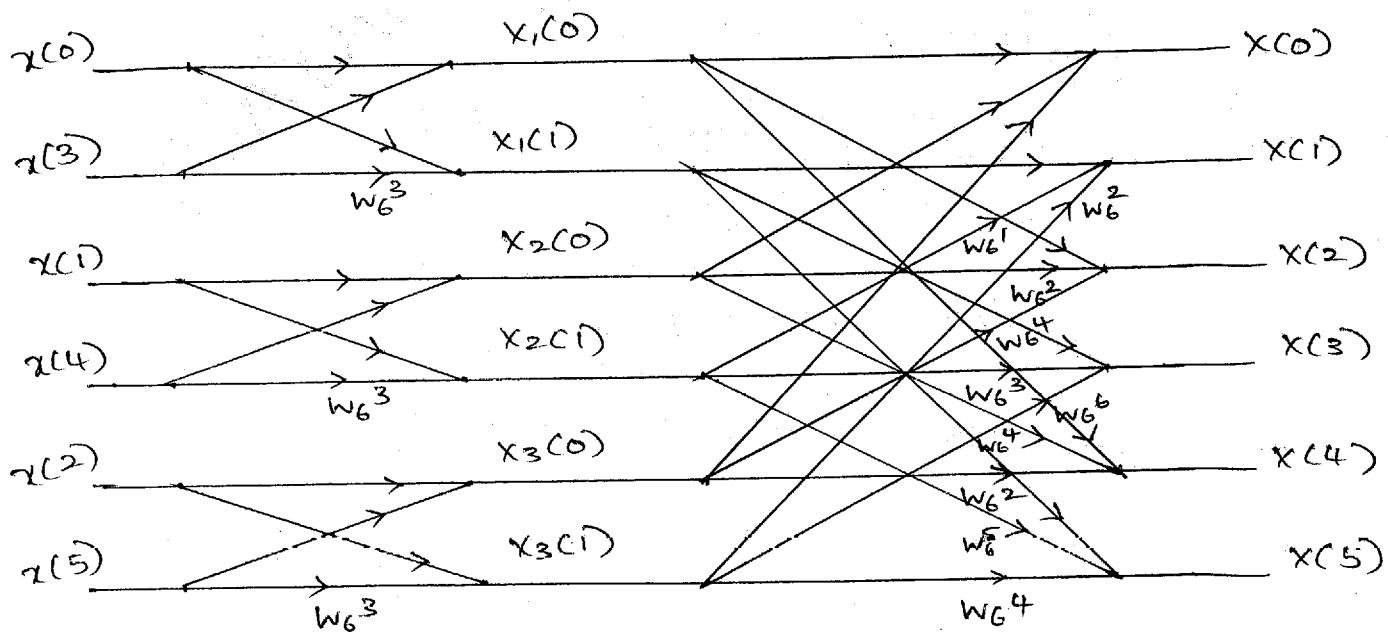
$$x(4) = x_1(4) + w_6^4 x_2(4) + w_6^8 x_3(4)$$

$$= x_1(0) + w_6^4 x_2(0) + w_6^2 x_3(0)$$

$$x(5) = x_1(5) + w_6^5 x_2(5) + w_6^{10} x_3(5)$$

$$= x_1(1) + w_6^5 x_2(1) + w_6^4 x_3(1)$$

flow diagram for $N = 6 = 3 \cdot 2$



→ Develop a DIF FFT algorithm for decomposing the DFT for $N=6$ and draw the flow diagrams for
a) $N=3 \cdot 2$ and b) $N=2 \cdot 3$.

(a) To develop DIF FFT algorithm for $N=3 \cdot 2$

$N=3 \cdot 2 \Rightarrow 2$ sequences of 3 elements each.

$$X(K) = \sum_{n=0}^5 x(n) W_6^{nk} \quad \left[X(K) = \sum_{n=0}^{N-1} x(n) W_N^{nk} \right]$$

$$= \sum_{n=0}^2 x(n) W_6^{nk} + \sum_{n=3}^5 x(n) W_6^{nk}$$

$$= \sum_{n=0}^2 x(n) W_6^{nk} + \sum_{n=0}^2 x(n+3) W_6^{(n+3)k}$$

$$= \sum_{n=0}^2 x(n) W_6^{nk} + W_6^{3k} \sum_{n=0}^2 x(n+3) W_6^{nk}$$

$$= \sum_{n=0}^2 [x(n) + x(n+3) W_6^{3k}] W_6^{nk}$$

$$X(2k) = \sum_{n=0}^2 [x(n) + x(n+3) W_6^{6k}] W_6^{2nk}$$

$$= \sum_{n=0}^2 [x(n) + x(n+3)] W_6^{2nk} \quad \left[W_6^{6k} = (W_6^6)^k = 1^k = 1 \right]$$

$$X(2k+1) = \sum_{n=0}^2 [x(n) + x(n+3) W_6^{3(2k+1)}] W_6^{(2k+1)n}$$

$$= \sum_{n=0}^2 [x(n) + x(n+3) W_6^{6k} W_6^3] W_6^{(2k+1)n}$$

$$W_N = e^{-j2\pi/N}$$

$$W_6^3 = [e^{-j2\pi/6}]^3 = e^{-j2\pi/2} = \cos \frac{\pi}{2}$$

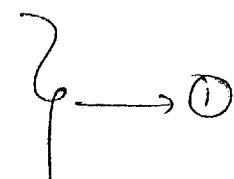
$$= e^{-j\pi} = \cos \pi - j \sin \pi = -1$$

$$\therefore X(2k+1) = \sum_{n=0}^2 [x(n) - x(n+3)] W_6^n W_6^{2kn}$$

$$\text{let } g(n) = x(n) + x(n+3), h(n) = x(n) - x(n+3)$$

$$\text{Now, } X(2k) = \sum_{n=0}^2 g(n) W_6^{2nk}$$

$$X(2k+1) = \sum_{n=0}^2 h(n) W_6^n \cdot W_6^{2nk}$$



$$g(0) = x(0) + x(3)$$

$$g(1) = x(1) + x(4)$$

$$g(2) = x(2) + x(5)$$

$$h(0) = x(0) - x(3)$$

$$h(1) = x(1) - x(4)$$

$$h(2) = x(2) - x(5)$$

Now,

$$x(0) = \sum_{n=0}^2 g(n) w_6^n = g(0) + g(1) + g(2)$$

$$x(2) = \sum_{n=0}^2 g(n) w_6^{2n} = g(0) + g(1) w_6^2 + g(2) w_6^4$$

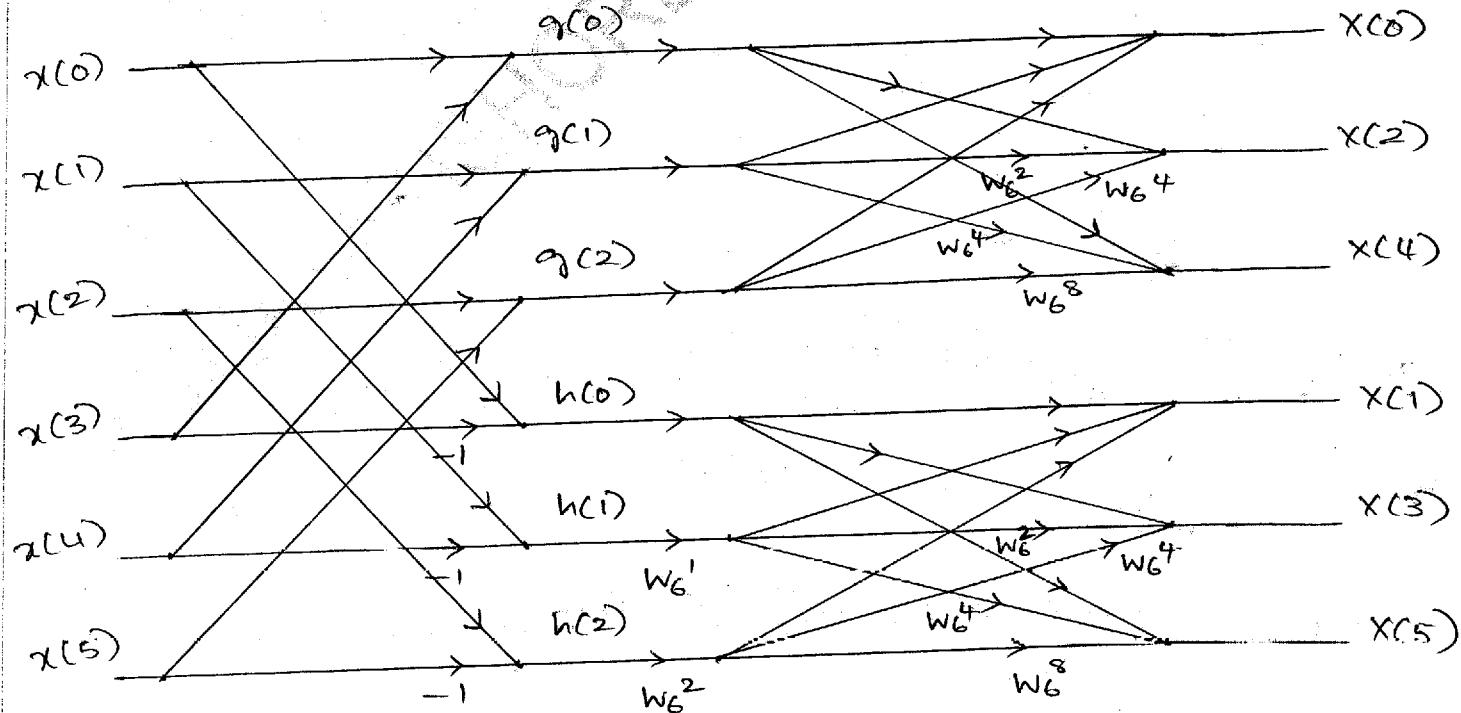
$$x(4) = \sum_{n=0}^2 g(n) w_6^{4n} = g(0) + g(1) w_6^4 + g(2) w_6^8$$

$$x(1) = \sum_{n=0}^2 h(n) w_6^n = h(0) + h(1) w_6^1 + h(2) w_6^2$$

$$x(3) = \sum_{n=0}^2 h(n) w_6^n w_6^{2n} = h(0) + h(1) w_6^1 w_6^2 + h(2) w_6^2 w_6^4$$

$$x(5) = \sum_{n=0}^2 h(n) w_6^n w_6^{4n} = h(0) + h(1) w_6^1 w_6^4 + h(2) w_6^2 w_6^8$$

Flow diagram :-



b) To develop DIF FFT algorithm for $N=2, 3$

$N=2, 3 \Rightarrow 3$ sequences of 2 elements each.

$$X(k) = \sum_{n=0}^{N-1} x(n) w_N^{nk}$$

$$\Rightarrow X(k) = \sum_{n=0}^5 x(n) w_6^{nk} \quad [\because N=6]$$

$$= \sum_{n=0}^1 x(n) w_6^{nk} + \sum_{n=2}^3 x(n) w_6^{nk} + \sum_{n=4}^5 x(n) w_6^{nk}$$

$$= \sum_{n=0}^1 x(n) w_6^{nk} + \sum_{n=0}^1 x(n+2) w_6^{(n+2)k} + \sum_{n=0}^1 x(n+4) w_6^{(n+4)k}$$

$$= \sum_{n=0}^1 [x(n) + x(n+2) w_6^{2k} + x(n+4) w_6^{4k}] w_6^{nk}$$

$$X(3k) = \sum_{n=0}^1 [x(n) + x(n+2) w_6^{6k} + x(n+4) w_6^{12k}] w_6^{nk}$$

$$= \sum_{n=0}^1 [x(n) + x(n+2) + x(n+4)] w_6^{nk} \quad [w_6^{6k}=1; w_6^{12k}=1]$$

$$X(3k+1) = \sum_{n=0}^1 [x(n) + x(n+2) w_6^{2(3k+1)} + x(n+4) w_6^{4(3k+1)}] w_6^{n(3k+1)}$$

$$= \sum_{n=0}^1 [x(n) + x(n+2) w_6^{6k} w_6^2 + x(n+4) w_6^{12k} w_6^4] w_6^{n(3k+1)}$$

$$= \sum_{n=0}^1 [x(n) + x(n+2) w_6^2 + x(n+4) w_6^4] w_6^n w_6^{3nk}$$

$$X(3k+2) = \sum_{n=0}^1 [x(n) + x(n+2) w_6^{2(3k+2)} + x(n+4) w_6^{4(3k+2)}] w_6^{n(3k+2)}$$

$$= \sum_{n=0}^1 [x(n) + x(n+2) w_6^{6k} w_6^4 + x(n+4) w_6^{12k} w_6^8] w_6^{n(3k+2)}$$

$$= \sum_{n=0}^1 [x(n) + x(n+2) w_6^4 + x(n+4) w_6^8] w_6^{2n} w_6^{3nk}$$

$$\text{let } f(n) = x(n) + x(n+2) + x(n+4)$$

$$g(n) = x(n) + x(n+2) w_6^2 + x(n+4) w_6^4$$

$$h(n) = x(n) + x(n+2) w_6^4 + x(n+4) w_6^8$$

$$h(n) = x(0) + x(2)w_6^4 + x(4)w_6^2 \quad [w_6^8 = w_6^2]$$

$$\text{Now, } f(0) = x(0) + x(2) + x(4)$$

$$f(1) = x(1) + x(3) + x(5)$$

$$g(0) = x(0) + x(2)w_6^2 + x(4)w_6^4$$

$$g(1) = x(1) + x(3)w_6^2 + x(5)w_6^4$$

$$\therefore h(0) = x(0) + x(2)w_6^4 + x(4)w_6^2$$

$$h(1) = x(1) + x(3)w_6^4 + x(5)w_6^2$$

$x(3k)$, $x(3k+1)$ & $x(3k+2)$ can be written as

$$x(3k) = \sum_{n=0}^1 f(n) w_6^{3nk}$$

$$x(3k+1) = \sum_{n=0}^1 g(n) w_6^n w_6^{3nk}$$

$$x(3k+2) = \sum_{n=0}^1 h(n) w_6^{2n} w_6^{3nk}$$

Now,

$$x(0) = \sum_{n=0}^1 f(n) w_6^0 = f(0) + f(1)$$

$$x(1) = \sum_{n=0}^1 g(n) w_6^n w_6^0 = g(0) + g(1)w_6^1$$

$$x(2) = \sum_{n=0}^1 h(n) w_6^{2n} w_6^0 = h(0) + h(1)w_6^2$$

$$x(3) = \sum_{n=0}^1 f(n) w_6^{3n} = f(0) + f(1)w_6^3$$

$$x(4) = \sum_{n=0}^1 g(n) w_6^n w_6^{3n} = g(0) + g(1)w_6^1 w_6^3$$

$$x(5) = \sum_{n=0}^1 h(n) w_6^{2n} w_6^{3n} = h(0) + h(1)w_6^2 w_6^3$$

Flow diagram

