G.PULLAIAH COLLEGE OF ENGINEERING AND TECHNOLOGY, KURNOOL

Department of Electrical and Electronics Engineering

Digital Signal Processing

UNIT-1

BASIC TERMS AND DEFINITIONS:

Signal	A signal is a function of one or more independent variables which contain some information. Eg: Radio signal, TV signal, Telephone signal etc
System	A system is a set of elements or functional block that are connected together and produces an output in response to an input signal. Eg: An audio amplifier, attenuator, TV set etc.
Continuous time signals	Continuous time signals are defined for all values of time. It is also called as an analog signal and is represented by x(t).
Discrete time signals	Discrete time signals are defined at discrete instances of time. It is represented by x(n).
Unit step function	Unit step function is defined as U(t)= 1 for t >= 0 = 0 otherwise
ramp function	Unit ramp function is defined as r(t)= t for t>=0 = 0 for t<0
delta function	Unit delta function is defined as δ(t)= 1 for t=0 = 0 otherwise
UnitParabolic Function	The continuous-time unit parabolic function $p(t)$is defined as $p(t) = \begin{cases} \frac{t^2}{2} \; for\; t \geq 0 \\ 0 \; for\; t \; < 0 \end{cases}$
Deterministic signals	A deterministic signal is one which can be completely represented by Mathematical equation at any time. In a deterministic signal there is no uncertainty with respect to its value at any time.
Random signals	Random signal is a signal characterized by uncertainty about its

	occurrence. It cannot be represented by a mathematical equation. Eg: Noise generated in electronic components, transmission channels, cables etc.
Energy and power signals.	An energy signal is one whose total energy E = finite value and whose average power P = 0, whereas a power signal is the one whose average power P = finite value and total energy E = ∞.
Periodic and aperiodic signals.	continuous-time signal $x(t)$ is said to be periodic if it satisfies the condition $x(t) = x(t + T) for\ all\ t$ Whereas a continuous-time signal $x(t)$ is said to be aperiodic, if the above condition is not satisfied even for one value of t.
Odd and even signal	A DT signal x(n) is said to be an even signal if x(-n)=x(n) and an odd signal if x(-n)=-x(n). A CT signal is x(t) is said to be an even signal if x(t)=x(-t) and an odd signal if x(-t)=-x(t).
relation between impulse, step, ramp and parabolic signals	Step function is the integration of impulse function, ramp function is the integration of step function, and parabolic function is the integration of ramp function. Ramp function is the derivative function, step function is the derivative of ramp function and impulse function is the derivative of step function.
Causaland non-causal signals.	A continuous-time signal $x(t)$ is said to be causal, if $x(t)$ = 0 for t < 0. Otherwise the signal is non-causal. A discrete-time signal $x(n)$is said to be causal, if $x(n) = 0$ for n< 0, otherwise the signal is non-causal.
Continuous Time system	A Continuous – time system is a system which transforms continuous-time input signals into continuous-time output signals.
Discrete Time system	A Discrete – time system is a system which transforms discrete -time input signals into discrete -time output signals.
Static System	A Static or memory- less system is a system in which the response at any instant is due to present input alone. I.e. for a static or memory-less system, the output at any instant t (or n) depends only on the input applied at that instant t (or n) but not on the past or future values of input.

Dynamic System	A Dynamic or memory system is a system in which the response at any instant depends upon past or future inputs.
Bounded Input Bounded Output Stable System	A Bounded input-bounded output stable system is a system which produces a bounded input for every bounded output.
Unstable System	An unstable system is a system which produces an unbounded output for a bounded input.
Linear and non-linear System	A system is said to be linear if superposition theorem applies to that System. If it does not satisfy the superposition theorem, then it is said to be a nonlinear system.
Causal and non-Causal systems	A system is said to be a causal if its output at any time depends upon present and past inputs only. A system is said to be non-causal system if its output depends upon future inputs also.
time invariant and time varying systems	A system is time invariant if the time shift in the input signal results in corresponding time shift in the output. A system which does not satisfy the above condition is time variant system

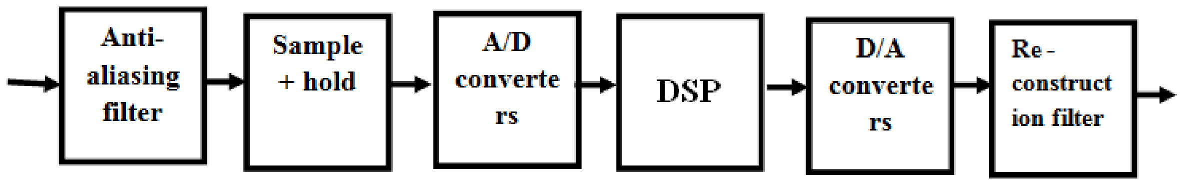
Concepts

Signal Processing: Signal processing is any operation that changes the characteristics of a signal. These characteristics include amplitude, frequency , phase and shape of the signal.

Digital Signal Processing: Digital signal processing (DSP) is the process of analyzing and modifying a signal to optimize or improve its efficiency or performance. It involves applying various mathematical and computational algorithms to analog and digital signals to produce a signal that's of higher quality than the original signal.

Block diagram:

The digital signal processor consists of anti-aliasing filter, analog to digital converter (ADC), a digital filter represented by the transfer function H(z), a digital to analog converter and a reconstruction filter.



Anti-aliasing filter:

The input signal is applied to an anti-aliasingfilter. This is a low Pass filter used to remove the high frequency noise and to band limit the signal.

Sample + Hold:

The sample and hold device provides the input to the A/D converters and will be required if the inputs signal must remain relatively constant during the conversion of the analog signal to the digital format.

A/D converters:

The output of the sample and hold circuit serves as input to the ADC.The output of the ADC is an N-bit binary number depending on the value of analog signal at its input MCE converted into digital format,the signal can be processed using digital technique.

DSP:

The DSP may be large programmable digital computers (or) microprocessor performs the operating corresponding input signal.

D/A converter:

The DSP is applied to the input of a DAC. The o/p of DAC is continuous but not smooth,
The signal contain high frequency components that are unwanted.

Reconstruction filter:

To eliminate high frequency components the o/p of DAC is applied to reconstruction Filter. The o/p of reconstruction filter is smooth continuous signal.

Limitations of digital signal processing:

Bandwidth limitations: In case of DSP, if input signal is having wide bandwidth then it demands for high speed ADC. This is because to avoid aliasing effect, the sampling rate should be atleast twice the bandwidth. Thus such signals require fast digital signal processors. But always there is a practical limitation in the speed of processors and ADC.

System complexity : The digital signal processing system makes use of converters like ADC and DAC. This increases the system complexity compared to analog systems. Similarly in many applications the time required for this conversion is more.

Power Consumption: A typical digital signal processing chip contains more than 4 lakh transistors. Thus power dissipation is more in caps systems compared to analog systems.

Cost: DSP systems are expensive as compared to analog system.

Advantages of DSP:

1.Greater accuracy

2.Cheaper

3.Ease of data storage

4.Implementation of sophisticated algorithms

5.Flexibility in configuration

6.Applicability of Very Low Frequency signals

7.Time sharing

Applications of DSP:

1. Image processing like pattern recognition, animation, robotic vision, image enhancement.

2. Instrumentation and control like spectral analysis, noise reduction, data compression.

3. Speech/Audio like speech recognition, speech synthesis, equalisation.

4. Biomedical like scanners ECG analysis, patient monitoring

5. Telecommunication like in echo cancellation, spread spectrum and data communication.

6. Military like Sonar processing, radar processing, secure communication.

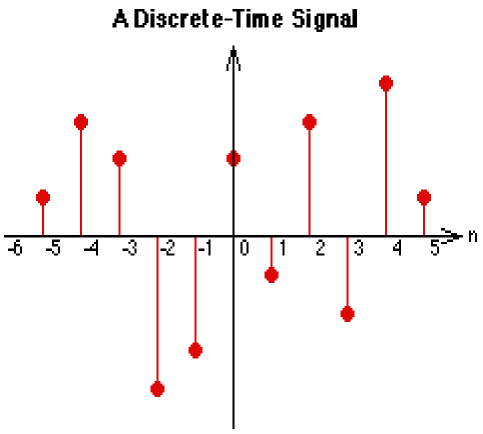
7. Consumer applications like digital audio and video, power like monitor.

8. Automotive applications like vibration analysis, voice commands and cellular telephones.

9. Industrial applications like robotics and CNC, power line monitors.

Discrete time signals:

The signals that are defined at discrete instants of time are known as discrete-time signals.The discrete –time signals are continuous in amplitude and discrete in time. They are denoted by x(n).



Elementary discrete time signals are

1. Unit impulse sequence
2. Unit step sequence
3. Unit ramp sequence

Classification of discrete time signals:

1.Energy and power signals

2.Periodic and Aperiodic signals

3.Even and odd signals

4. Casual and Non-casual signals.

Classification of Discrete time systems:

1.Static and dynamic system

2.casual and Non-casual systems

3.Linear and Non-linear systems

4.Time-varaian and Time-invariant systems

5.FIR and IIR systems

6.Stable and Unsatable systems

Static and Dynamic systems:-

A discrete time system is called static or memory less if its output at any instant 'n' depends on the input samples at the same time ,but not past or future samples of input .In any other case , the system is said to be dynamic or to have memory.

- The system is described by the following equation

y(n)=a*x(n)

y(n)=a*x^2(n) are static

- The system described by (n)=x(n-1)+x(n-2) & y(n)=x(n-1)+x(n) are called the Dynamic systems.

Casual & non causal system:-

A system is said to be casual if the output of the system at any time "n" depends at "present and past inputs", but does not depends on future inputs. This can be represented as

Y(n) = F[x(n),x(n – 1), x(n - 2), - - - - - -]

If the output of a system depends on future inputs then that system is said to be Non – Causal or Anticipatory system.

Y(n) = F[x(n),x(n + 1), x(n + 2), - - - - - -]

Example:

Y(n) = x(n)+x(n – 1)+ x(n - 2) --- Causal

Y(n) = x(n)+x(n + 1)+ x(n + 2) --- Non – Causal

Linear and nonlinear system:-

A system that satisfies the superposition principle is said to be linear system. It states that the response of the system to a weighted sum of signals be equal to the corresponding weighted sum of the system to each of the individual input signal.

A system is said to be take linear if and only if

$$T[a_1 x_1(n) + a_2 x_2(n)] = a_1 T[x_1(n)] + a_2 T[x_2(n)]$$

For any obituary constants a_1 & a_2.

A system does not satisfy the superposition principle then that system is called non – linear system.

Time variant and Time invariant System:-

A system is said to be time-invariant (or) Shift variant if the characteristics of the system does not change the time. For time invariant system, if Y(n) is the response of the system for the input X(n), then the response the system to the input X(n-k) is Y(n-k).

To test if any system is time-invariant first apply an orbitrary sequence X(n) and find Y(n) . Now delay the input sequence by k samples and find output sequence, denote it as Y(n,k)=T[x(n-k)].

Delay the output sequence by k samples, denote it as Y(n,k).If Y(n,k)=Y(n-k) for all possible value of k, the system is time-invariant system on the other hand if the output.

$$Y(n,k) \neq Y(n-k)$$

Even for one value of k, the system is time-invariant

NOTE:

A Linear time-invariant discrete time system safety both the linearity and the time invariant properties.

FIR AND IIR SYSTEMS:-

Linear time invariant system can be classified according to the type of impulse response.

They are

(i)FIR system (ii) IIR system

(i)FIR system:-

If the impulse response of the system is of finite duration of the system is called Finite impulse response of the system.

An example of FIR system is

H(n) = 1 for n = -1,2

2 for n = 1

3 for n = 0, 3

0 otherwise

(ii)IIR system:-

An infinite impulse response of the system has an impulse response for finite duration.

An example for IIR system is

$$H(n) = a^n . u(n)$$

Stable and Unstable system:-

Linear time-invariant system is stable if it produces a bounded output sequence for every bounded input sequence. Otherwise the system is called unstable system.

Let X (n) be a bounded input sequence, h (n) be the impulse response of the system and Y (n) be the output sequence taking the magnitude of the output.
The condition will be satisfied when the system is said to stable is

$$\sum_{k=-\infty}^{\infty} |h(k)| < \infty$$

TIME RESPONSE ANALYSIS OF DISCRETE -TIME SYSTEMS:

Time domain behaviour of linear time invariant discrete –time systems for different standard input signals. There are two basic methods for analysing the response of a linear system to given input signal.

1) The response y(n) for any input signal x(n) using convolution sum if the impulse response h(n) is known.

2) The second method is based on the direct solution of the difference equation representing the system. The general form of difference equation of an Nth order linear time invariant discrete time system is

$$Y(n) = - \sum \mathbf{a_k}\, y(n - \mathbf{k}) + \sum_{\mathbf{k=0}}^{\mathbf{M}} \mathbf{b_k}\, x(n - \mathbf{k}) \qquad (1.1)$$

Where ak and b_k are constants. The response of any discrete time system can be decomposed as

Total response= Zero state response+ Zero input response

Zero state response or forced response:

The zero state response of the system is the response of the system due to input alone when the initial state of the system is zero. That is ,the system is initially relaxed at n=o. It depends on the nature of the input signal.

It consists of two parts , homogeneous solution and particular solution.

Zero input response or natural response:

The zero input response is obtained by letting the input signal to zero. It depends on the nature of the system and initial conditions.

The natural response $y_n(n)$ is the solution of $\sum_{k=0}^{N} \text{aky}(\mathrm{n}-\mathrm{k})= \sum_{k=0}^{M} b_k$ x(n-k) with

X(n)=0.Therefore for a discrete –time systemthe natural response is the solution of homogeneous equation $\sum_{k=0}^{N} \text{aky}(\mathrm{n}-\mathrm{k}) = 0$

Discrete Fourier Transform

The Discrete Fourier Transform (DFT) is purely discrete: discrete-time data sets are converted into a discrete-frequency representation.

The Discrete Fourier Transform is a numerical variant of the Fourier Transform. Specifically, given a vector of *n* input amplitudes such as $\{f_0, f_1, f_2, \ldots, f_{n-2}, f_{n-1}\}$, the Discrete Fourier Transform yields a set of *n* frequency magnitudes.

The DFT is defined as such

$$x(k) = \sum_{n=0}^{N-1} x(n).e^{\frac{-j2\pi kn}{N}} \tag{1.2}$$

here, k is used to denote the frequency domain ordinal, and n is used to represent the time-domain ordinal. The big "N" is the length of the sequence to be transformed.

The Inverse DFT (IDFT) is given by the following equation:

$$x(n) = \frac{1}{N}\sum_{k=0}^{N-1} X(k)\cdot e^{j\left(\frac{2\pi}{N}\right)nk} \quad (n = 0,1,\ldots,N-1) \tag{1.3}$$

Properties of Discrete Fourier Transform

As a special case of general Fourier transform, the discrete time transform shares all properties (and their proofs) of the Fourier transform discussed above, except now some of these properties may take different forms. In the following, we always assume $\mathcal{F}[x[m]] = X(e^{j\omega})$ and $\mathcal{F}[y[m]] = Y(e^{j\omega})$.

- Linearity

$$\mathcal{F}[ax[m] + by[m]] = aX(e^{j\omega}) + bY(e^{j\omega}) \tag{1.4}$$

- Time Shifting

$$\mathcal{F}[x[m - m_0]] = e^{-jm_0\omega}X(e^{j\omega}) \tag{1.5}$$

- Time Reversal

$$\mathcal{F}[x[-m]] = X(e^{-j\omega}) \tag{1.6}$$

- Frequency Shifting

$$\mathcal{F}[x[m]e^{j\omega_0 m}] = X(e^{j(\omega-\omega_0)}) \tag{1.7}$$

- Differencing

  Differencing is the discrete-time counterpart of differentiation.

$$\mathcal{F}[x[m] - x[m-1]] = (1 - e^{-j\omega})X(e^{j\omega}) \tag{1.8}$$

- Differentiation in frequency

$$\mathcal{F}^{-1}[j\frac{d}{d\omega}X(e^{j\omega})] = m\ x[m] \tag{1.9}$$

- Convolution Theorems

The convolution theorem states that convolution in time domain corresponds to multiplication in frequency domain and vice versa:

$$\mathcal{F}[x[n] * y[n]] = X(e^{j\omega})\, Y(e^{j\omega}) \tag{1.10}$$

$$\mathcal{F}[x[n]\; y[n]] = X(e^{j\omega}) * Y(e^{j\omega}) \tag{1.11}$$

- Parseval's Relation

$$\sum_{n=-\infty}^{\infty} |x[n]|^2 = \frac{1}{2\pi} \int_0^{2\pi} |X(e^{j\omega})|^2 d\omega \tag{1.12}$$

Linear filtering methods based on DFT or Filtering long duration sequences

Suppose input sequence x(n) of long duration is to be processed with a system having impulse response of finite duration by convolving two sequences. The length of input sequence is long it would not be practical to store it all before performing linear convolution. Therefore the input sequence must be divided into blocks.

The successive blocks are processed separately one at a time and the results are combined later to get desired output sequence which is identical to the sequence obtained by linear convolution.

Two methods that are commonly used for filtering section data and combining the results are

1) Overlap-Add method 2) Overlap-Save method

Frequency analysis of discrete time signal:-

Any continuous – time periodic signal X(t) with period(t) can be expressed as a weighted sum of harmonically related sinusoidal or complex exponentials.

It can be written as

$$x(t) = a_0 + \sum_{n=1}^{\infty} \left[a_n \cos(n\Omega_0 t) + b_n \sin(n\Omega_0 t) \right]$$

Similarly X(t) is represented by weighted sum of harmonically complex exponentially, then the series is known as exponential Fourier series which can be expressed as

$$x(t) = \sum_{n=-\infty}^{\infty} C_n \cdot e^{jn\Omega_0 t}, \quad \Omega_0 = \frac{2\pi}{T}$$

Hence C_n is known as exponential fourier series co-efficient.

$$C_n = |C_n| \, e^{j\theta_n}$$

$$x(n) = \sum_{k=0}^{N-1} C_k \, e^{j\left(\frac{2\pi}{N}\right)kn}$$

Solving above, It can also be expressed as

$$\boxed{C_k = \frac{1}{N} \sum_{k=0}^{N-1} x(n) \cdot e^{-\frac{j2\pi nk}{N}}} \quad k = 0, 1, 2, \ldots N-1$$

$$\boxed{x(n) = \sum_{k=0}^{N-1} C_k \cdot e^{j\left(\frac{2\pi}{N}\right)nk}} \quad n = 0, 1, \ldots N-1$$

Discrete Fourier Transform:-

If $x(n)$ is the finite duration sequence then the N-point DFT of $x(n)$ is given by

$$DFT[x(n)] = X(K) = \sum_{n=0}^{N-1} x(n) . e^{\frac{-j2\pi Kn}{N}} , \quad 0 \le K \le N-1$$

where $K = 0,1,2,\ldots,N-1$

The DFT transforms N-samples of a discrete time signal to the same number of discrete frequency samples. The IDFT can be represented by

$$x(n) = IDFT[x(K)] = \frac{1}{N} \sum_{K=0}^{N-1} X(K) . e^{\frac{j2\pi Kn}{N}} , \quad 0 \le n \le N-1$$

Let us define the term $W_N = e^{\frac{-j2\pi}{N}} = \left[\cos\frac{2\pi}{N} - j\sin\frac{2\pi}{N}\right]$ which is known as twiddle factor then the DFT pair can be simplified as

$$DFT[x(n)] = x(K) = \sum_{n=0}^{N-1} x(n).\, W_N^{nK}, \quad 0 \le K \le N-1$$

where $K = 0, 1, 2 \ldots\ldots, N-1$

$$x(n) = IDFT[X(K)] = \frac{1}{N} \sum_{K=0}^{N-1} x(K).\, W_N^{nK}, \quad 0 \le K \le N-1$$

problems:-

Find the DFT of a sequence $x(n) = \{1,1,0,0\}$ and IDFT of $y(n) = \{1,0,1,0\}$

sol:- let us assume $N = L = 4$

We have $X(K) = \sum_{n=0}^{N-1} x(n).\, e^{\frac{-j2\pi Kn}{N}}$, $K = 0,1,2, \ldots\ldots, N-1$

$$X(0) = \sum_{n=0}^{3} x(n) = x(0) + x(1) + x(2) + x(3)$$

$$= 1 + 1 + 0 + 0 = 2$$

$$X(1) = \sum_{n=0}^{3} x(n).\, e^{\frac{-j2\pi K}{N}} = x(0) + x(1)e^{-j\frac{2\pi}{2}} + x(2)e^{-j\pi} + x(3).\, e^{-j3\pi/2}$$

$$= 1 + \cos\frac{\pi}{2} - j\sin\pi/2 = 1 - j$$

$$X(2) = \sum_{n=0}^{3} x(n).\, e^{-j\pi n} = x(0) + x(1)e^{-j\pi} + x(2).\, e^{-j2\pi} + x(3)e^{-j3\pi}$$

$$= 1 + \cos\pi - j\sin\pi = 1 - 1 = 0$$

$$X(3) = \sum_{n=0}^{3} x(n).\, e^{\frac{-j3\pi n}{2}} = x(0) + x(1).\, e^{-j\frac{3\pi}{2}} + x_2(2)e^{-j3\pi} + x(3)\, e^{-j9\pi/2}$$

$$= 1 + \cos 3\pi/2 - j\sin 3\pi/2 = 1 + j$$

Previous Questions

1. Explain in detail about the classification of discrete time systems.

2. (a) Describe the different types of discrete time signal representation.

(b) Define energy and power signals. Determine whether a discrete time unit step signal $x(n)=u(n)$is an energy signal or a power signal.

3.(a)Givethevariousrepresentationofthegivendiscretetimesignal$x(n)=\{-1,2,1,-2,3\}$in Graphical, Tabular, Sequence, Functional and Shifted functional.

(b)Give the classification of signals and explain it.

4. (a) Drawand explain the following sequences:

i) Unit sample sequence ii) Unit step sequence iii)Unit ramp sequence

iv) Sinusoidal sequenceandv)Real exponential sequence

(b) Determine if the system described by the following equations are causal or non causal

i)$y(n) =x(n)+ (1/ (x(n-1))$

ii)$y(n) =x(n^2)$

5. Determine the values of power and energy of the following signals. Find whether the signals are power, energy or neither energy nor power signals.

i) $x(n)= (1/3)n\ u(n)$

ii) $x(n)=e^{j((\pi/2)n}+ (\pi/4)$

iii) $x(n)=\sin (\pi/4)n$

iv) $x(n)=e^{2n}u(n)$

6. (a) Determine if the following systems are time-invariant or time-variant

i) $y(n) =x(n)+x(n-1)$

ii)$y(n) =x(-n)$

(b)Determineifthesystemdescribedbythefollowinginput-outputequationsarelinearor non-linear.i)$y(n) =x(n)+ (1 /(x(n-1))$

ii)$y(n)=x^2(n)$

iii)$y(n)=nx(n)$

7. Test if the following system are stable or not. i)$y(n) = \cos x(n)$

ii)$y(n)= ax(n)$

iii)$y(n)=x(n)$ en

iv)$y(n) = ax(n)$

8. (a) Explain the principle of operation of analog to digital conversion with a neat diagram.

(b) Explain the significance of Nyquist rate and aliasing during the sampling of continuous time signals.

9. (a)List the merits and demerits of Digital signal processing.
(b)Write short notes about the applications of DSP.

10. (a)Find the convolution of the following sequences i) $x(n)=u(n)h(n)=u(n-3)$

ii) $x(n)=\{1,2,-1,1\}$ $h(n)=\{1,0,1,1\}$

(b)Determinethe response ofthe causal system $y(n)-y(n-1)=x(n)+x(n-1)$ to inputs $x(n)=u(n)$andx$(n)=2^{-n}u(n)$.

11. (a)Determinethe solution ofthe differenceequation

$$y(n) =5/6y(n-1) – 1/6y(n-2)+x(n) \text{ for} x(n)=2nu(n)$$

(b)Determinetheresponsey(n),n>=0ofthesystemdescribedby thesecondorderdifference equation$y(n)–4y(n-1)+4y(n-2)=x(n)–x(n-1)$whentheinputis$x(n)=(-1)^n u(n)$andthe initial condition are $y(-1)=y(-2)=1$.

12. State and prove anytwo properties ofz-transform.

13. Find thez-transformand ROCof the causal sequence. $X(n)=\{1,0,3,-1,2\}$

14. Find thez-transformand ROCof theanticausal sequence$X(n)=\{-3,-2,-1,0,1\}$

13.(a)Determinethez-transformandROCofthesignal

i)$x(n)=anu(n)$and

ii)$x(n)=-bnu(-n)$

(b) Find thestabilityof thesystem whoseimpulse response $h(n) =(2)nu(n)$

14. (a) Determinethe z-transform of $x(n)=\cos\omega nu(n)$

(b)Stateandprovethefollowingpropertiesofz-transform.

i)Timeshifting

ii)Timereversal

iii) Differentiation

iv) Scalingin z-domain

15. Determinethe inversez-transform of x(z) =(1+3z-1)/ (1+3z-1+2z-2)forz>2

16. Find theinversez-transformofx(z) = (z2+z) /(z-1)(z-3), ROC: z> 3. Using(i) Partialfraction method, (ii) Residuemethod and (iii) Convolutionmethod

17. Determinetheunitstep responseofthesystemwhosedifferenceequationis

y(n)-0.7y(n-1)+0.12y(n-2) = x(n-1)+x(n-2) ify(-1) =y(-2) = 1.

18. Determinetheconvolution sumoftwo sequencesx(n) = {3,2,1,2}, h(n) ={1,2,1,2}

19.Findtheconvolution of the signalsx(n)=1n=-2,0,1,2n=-1=0elsewhere

h(n)=δ(n)-δ(n-1)+ δ(n-2)- δ(n-3)