

G.PULLAIAH COLLEGE OF ENGINEERING AND TECHNOLOGY, KURNOOL

Department of Electrical and Electronics Engineering

Digital Signal Processing

UNIT-V

Multi-rate signal processing:

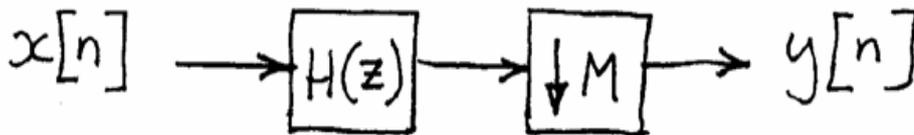
The process of converting a signal from a given rate to a different rate is called sampling rate conversion. Systems that employ multiple sampling rates in the processing of digital signals are called multi rate digital signal processing.

Down-sampling:

The process of reducing the sampling rate by an integer factor(D) is called decimation of the sampling rate. It is also called down sampling by factor(D).Decimator consists of decimation filter to band limit the signal and down sampler to decrease the sampling rate by an integer factor (D).

Decimation

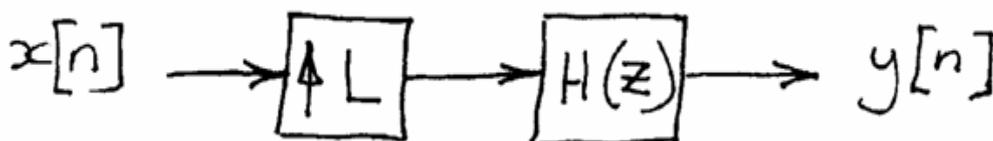
- Reduce the sampling rate of a discrete-time signal.
- Low sampling rate reduces storage and computation requirements.



up-sampling:

Increasing sampling rate of a signal by an integer factor I is known as Interpolation or up-sampling. An increase in the sampling rate by an integer factor I may be done by interpolating (I-1) new samples between successive values of the signals.

- Interpolation
- Increase the sampling rate of a discrete-time signal.
- Higher sampling rate preserves fidelity



Sampling Rate Conversion

Having discussed the special cases of decimation (down sampling by a factor D) and interpolation (upsampling by a factor I), we now consider the general case of sampling rate conversion by a rational factor I/D . Basically, we can achieve this sampling rate conversion by first performing interpolation by the factor I and then decimating the output of the interpolator by the factor D . In other words, a sampling rate conversion by the rational factor I/D is accomplished by cascading an interpolator with a decimator. We emphasize that the importance of performing the interpolation first and the decimation second is to preserve the desired spectral characteristics of $x(n)$.

Sample-rate conversion is the process of changing the sampling rate of a discrete signal to obtain a new discrete representation of the underlying continuous signal. Application areas include image scaling and audio/visual systems, where different sampling rates may be used for engineering, economic, or historical reasons.

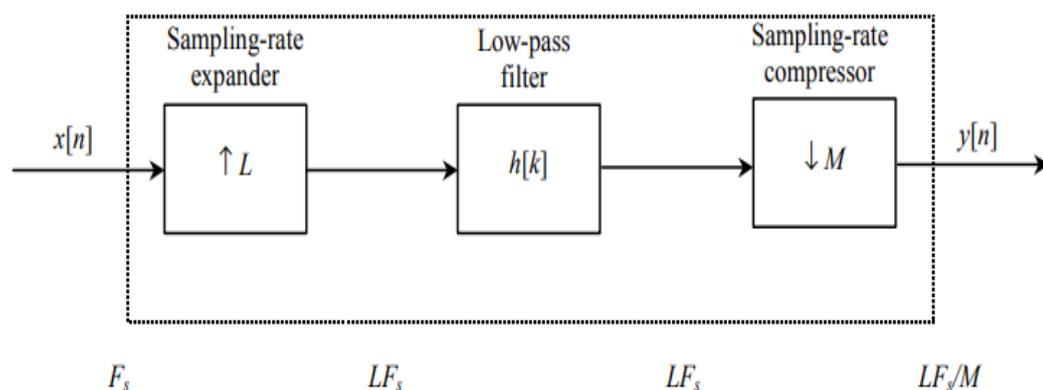


Fig: Sampling-rate conversion by expansion, filtering, and decimation

An example of sampling-rate conversion would take place when data from a CD is transferred onto a DAT. Here the sampling-rate is increased from 44.1 kHz to 48 kHz. To enable this process the non-integer factor has to be approximated by a rational number:

$$\frac{L}{M} = \frac{48}{44.1} = \frac{160}{147} = 1.08844$$

Hence, the sampling-rate conversion is achieved by interpolating by L i.e. from 44.1 kHz to $[44.1 \times 160] = 7056$ kHz.

Then decimating by M i.e. from 7056 kHz to $[7056/147] = 48$ kHz.

Multistage Approach

When the sampling-rate changes are large, it is often better to perform the operation in multiple stages, where $M_i(L_i)$, an integer, is the factor for the stage i .

$$M = M_1 M_2 \dots M_i \text{ or } L = L_1 L_2 \dots L_i$$

An example of the multistage approach for decimation is shown in Figure 9.8. The multistage approach allows a significant relaxation of the anti-alias and anti-imaging filters, with a consequent reduction in the filter complexity. The optimum number of stages is one that leads to the least computational effort in terms of either the multiplications per second (MPS), or the total storage requirement (TSR).

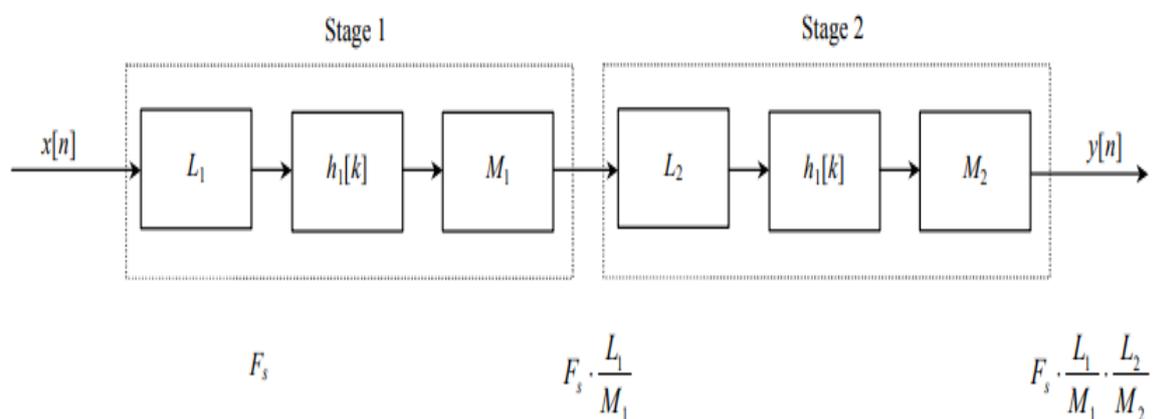


Fig: Multistage approach for the decimation process.

The meth to avoid aliasing:

- (i) Pre alias filter must be used to limit band of frequencies of the signal to f_m Hz.
- (ii) Sampling frequency ' f_s ' must be selected such that $f_s > 2 f_m$

The need for anti aliasing filter prior to down sampling:

Anti aliasing filter is used to avoid aliasing caused by down sampling the signal $x(n)$.

The need for anti imaging filter after up sampling a signal?

Anti imaging filter removes the unwanted images that are that are yielded by up sampling.

Applications of multi rate signal processing.

Multirate systems are used in a CD player when the music signal is converted from digital into analogue (DAC). Digital data (16-bit words) are read from the disk at a sampling rate of 44.1 kHz. If this data were converted directly into an analogue signal, image frequency bands

centred on multiples of the sampling-rate would occur, causing amplifier overload, and distortion in the music signal. To protect against this, a common technique called oversampling is often implemented nowadays in all CD players and in most digital processing systems of music signals. Fig.3 below illustrates a basic block diagram of a CD player and how oversampling is utilised. It is customary to oversample (or expand) the digital signal by a factor of x8, followed by an interpolation filter to remove the image frequencies. The sampling rate of the resulting signal is now increased up to 352.8 kHz. The digital signal is then converted into an analogue waveform by passing it through a 14-bit DAC. Then the output from this device is passed through an analogue low-pass filter before it is sent to the speakers.

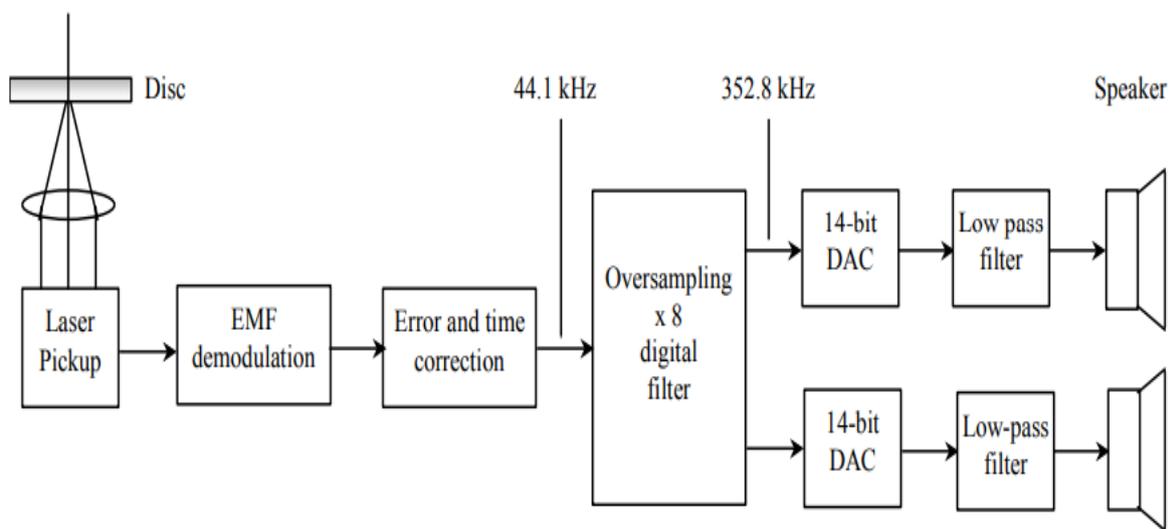


Fig. 3: Digital to analogue conversion for a CD player using x8 oversampling.

Fig. 4 illustrates the procedure of converting a digital waveform into an analogue signal in a CD player using x8 oversampling. As an example, Figure (a) illustrates a 20 kHz sinusoidal signal sampled at 44.1 kHz, denoted by $x[n]$. The six samples of the signal represent the waveform over two periods. If the signal $x[n]$ was converted directly into an analogue waveform, it would be very hard to exactly reconstruct the 20 kHz signal from this diagram. Now, Figure (b) shows $x[n]$ with an x8 interpolation, denoted by $y[n]$. Figure (c) shows the analogue signal $y(t)$, reconstructed from the digital signal $y[n]$ by passing it through a DAC. Finally, Figure (d) shows the waveform of $z(t)$, which is obtained by passing the signal $y(t)$ through an analogue low-pass filter.

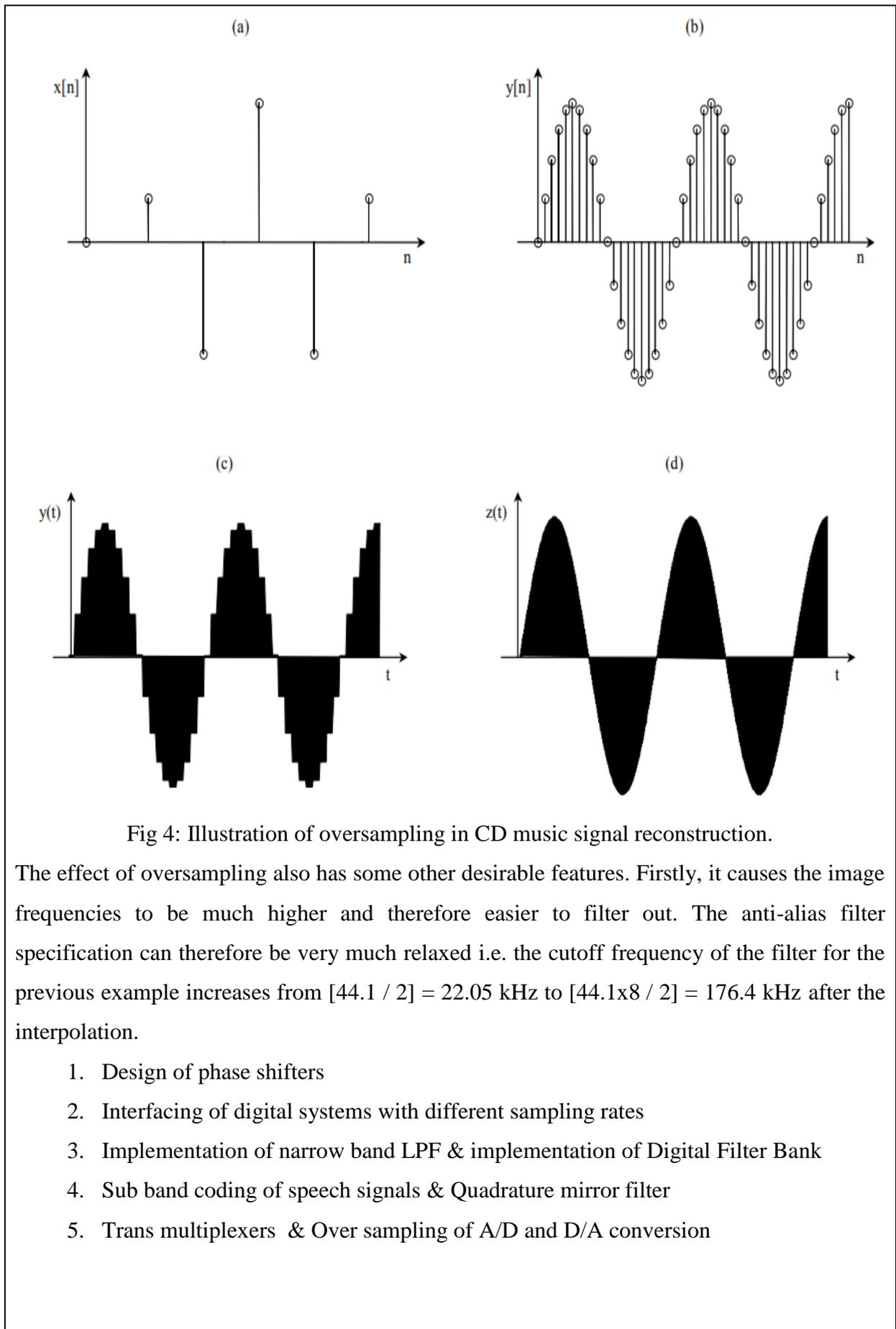


Fig 4: Illustration of oversampling in CD music signal reconstruction.

The effect of oversampling also has some other desirable features. Firstly, it causes the image frequencies to be much higher and therefore easier to filter out. The anti-alias filter specification can therefore be very much relaxed i.e. the cutoff frequency of the filter for the previous example increases from $[44.1 / 2] = 22.05$ kHz to $[44.1 \times 8 / 2] = 176.4$ kHz after the interpolation.

1. Design of phase shifters
2. Interfacing of digital systems with different sampling rates
3. Implementation of narrow band LPF & implementation of Digital Filter Bank
4. Sub band coding of speech signals & Quadrature mirror filter
5. Trans multiplexers & Over sampling of A/D and D/A conversion

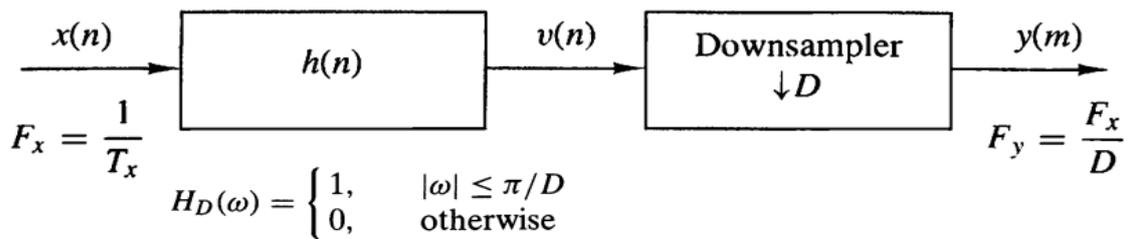
Mention the applications of speech coding.

Digital transmission like telephony, narrow band cellular radio, military communications and secrecy missions, voice mail sent on telephone networks, voice encryption, integrated voice and data transmission over packet networks.

Sub-band coding:

The speech signal is applied to an analysis filter bank consisting of a set of Q band pass filters. This digital filtration divides the speech signal into a non overlapping frequency bands. These filter banks are contiguous in frequency. Hence, by additive recombination of the set of sub band signals, one can approximately generate the original speech signal.

Decimation By A Factor D:



$$v(n) = \sum_{k=0}^{\infty} h(k)x(n - k)$$

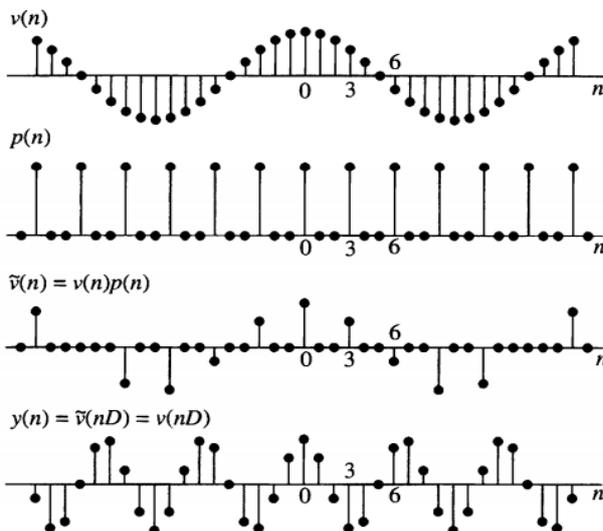
$$y(m) = v(mD) = \sum_{k=0}^{\infty} h(k)x(mD - k)$$

$$\tilde{v}(n) = \begin{cases} v(n), & n = 0, \pm D, \pm 2D, \dots \\ 0, & \text{otherwise} \end{cases}$$

$$\tilde{v}(n) = v(n)p(n)$$

$$p(n) = \frac{1}{D} \sum_{k=0}^{D-1} e^{j2\pi kn/D}$$

$$y(m) = \tilde{v}(mD) = v(mD)p(mD) = v(mD)$$



$$Y(z) = \sum_{m=-\infty}^{\infty} y(m)z^{-m} = \sum_{m=-\infty}^{\infty} \tilde{v}(mD)z^{-m} \quad \Rightarrow \quad Y(z) = \sum_{m=-\infty}^{\infty} \tilde{v}(m)z^{-m/D}$$

$$\tilde{v}(n) = v(n)p(n)$$

$$p(n) = \frac{1}{D} \sum_{k=0}^{D-1} e^{j2\pi kn/D}$$

$$Y(z) = \sum_{m=-\infty}^{\infty} v(m) \left[\frac{1}{D} \sum_{k=0}^{D-1} e^{j2\pi mk/D} \right] z^{-m/D}$$

$$= \frac{1}{D} \sum_{k=0}^{D-1} \sum_{m=-\infty}^{\infty} v(m) (e^{-j2\pi k/D} z^{1/D})^{-m}$$

$$= \frac{1}{D} \sum_{k=0}^{D-1} V(e^{-j2\pi k/D} z^{1/D})$$

$$v(n) = \sum_{k=0}^{\infty} h(k)x(n-k)$$

$$H_D(\omega) = \begin{cases} 1, & |\omega| \leq \pi/D \\ 0, & \text{otherwise} \end{cases}$$

$$= \frac{1}{D} \sum_{k=0}^{D-1} H_D(e^{-j2\pi k/D} z^{1/D}) X(e^{-j2\pi k/D} z^{1/D})$$

Evaluate the Z-transform on unit circle with frequency variable $\omega_y = \frac{2\pi F}{F_y} = 2\pi FT_y$

$$F_y = \frac{F_x}{D} \quad \omega_x = \frac{2\pi F}{F_x} = 2\pi FT_x \quad \Rightarrow \quad \omega_y = D\omega_x$$

Thus, $0 \leq |\omega_x| \leq \pi/D$ gets stretched to $0 \leq |\omega_y| \leq \pi$ by down-sampling

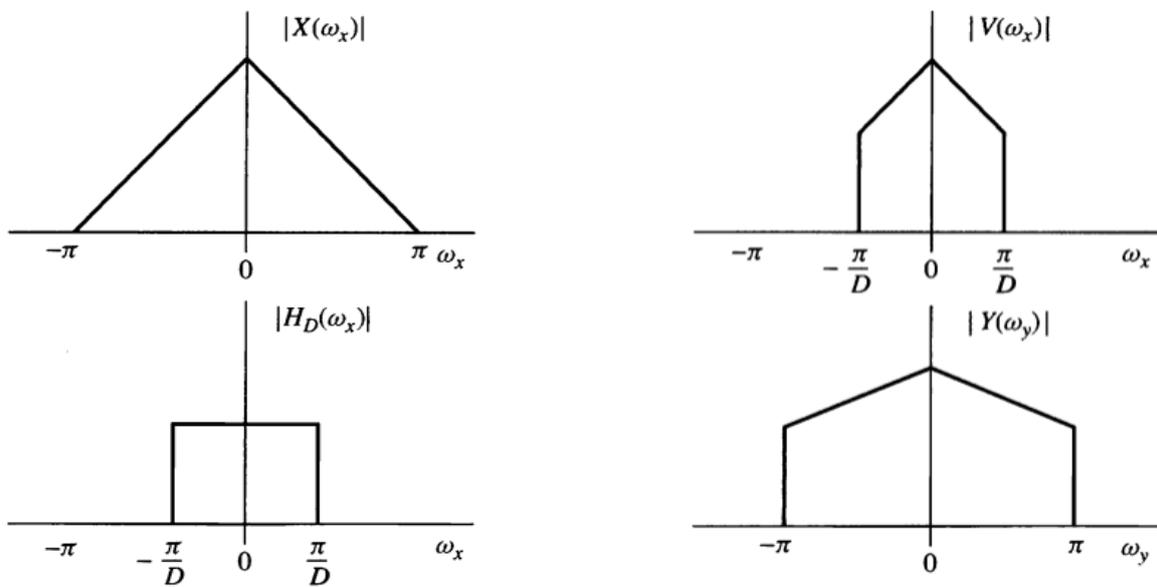
$$Y(z) = \frac{1}{D} \sum_{k=0}^{D-1} H_D(e^{-j2\pi k/D} z^{1/D}) X(e^{-j2\pi k/D} z^{1/D})$$



$$Y(\omega_y) = \frac{1}{D} \sum_{k=0}^{D-1} H_D\left(\frac{\omega_y - 2\pi k}{D}\right) X\left(\frac{\omega_y - 2\pi k}{D}\right)$$

If $H_D(\omega)$ is correctly designed, then aliasing is eliminated and

$$Y(\omega_y) = \frac{1}{D} H_D\left(\frac{\omega_y}{D}\right) X\left(\frac{\omega_y}{D}\right) = \frac{1}{D} X\left(\frac{\omega_y}{D}\right) \quad \text{for } 0 \leq |\omega_y| \leq \pi$$



Interpolation By A Factor I:

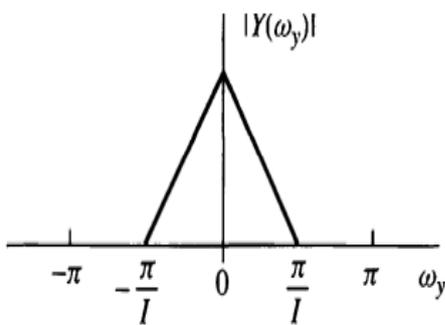
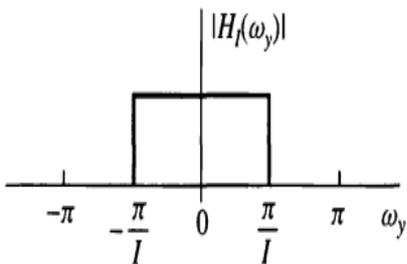
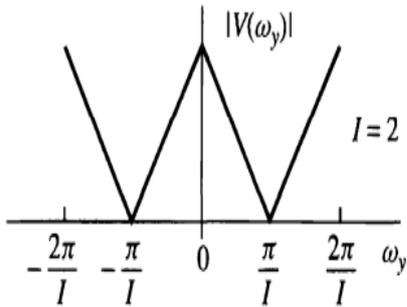
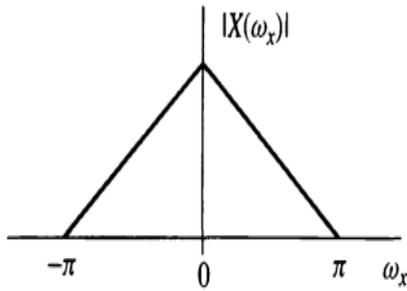
$$y(m) = x(m/I) \text{ for } m = 0, \pm I, \pm 2I, \dots$$

$$x(n) \xrightarrow{F_y = IF_x} v(m) = \begin{cases} x(m/I), & m = 0, \pm I, \pm 2I, \dots \\ 0, & \text{otherwise} \end{cases}$$

$$V(z) = \sum_{m=-\infty}^{\infty} v(m)z^{-m} = \sum_{m=-\infty}^{\infty} x(m)z^{-mI} = X(z^I)$$

$$\text{DTFT: } V(\omega_y) = X(\omega_y I)$$

$$\omega_y = 2\pi F/F_y, \quad F_y = IF_x \quad \Rightarrow \quad \omega_y = \frac{\omega_x}{I}$$



As the frequency component of $x(n)$ are unique in the range $0 \leq \omega_y \leq \pi/I$ Images beyond that in $v(n)$ should be rejected by low pass filtering

$$H_l(\omega_y) = \begin{cases} C, & 0 \leq |\omega_y| \leq \pi/I \\ 0, & \text{otherwise} \end{cases}$$

$$\Downarrow y(m) = \sum_{k=-\infty}^{\infty} h(m-k)v(k)$$

$$Y(\omega_y) = \begin{cases} CX(\omega_y I), & 0 \leq |\omega_y| \leq \pi/I \\ 0, & \text{otherwise} \end{cases}$$

$$y(m) = \sum_{k=-\infty}^{\infty} h(m-kI)x(k)$$

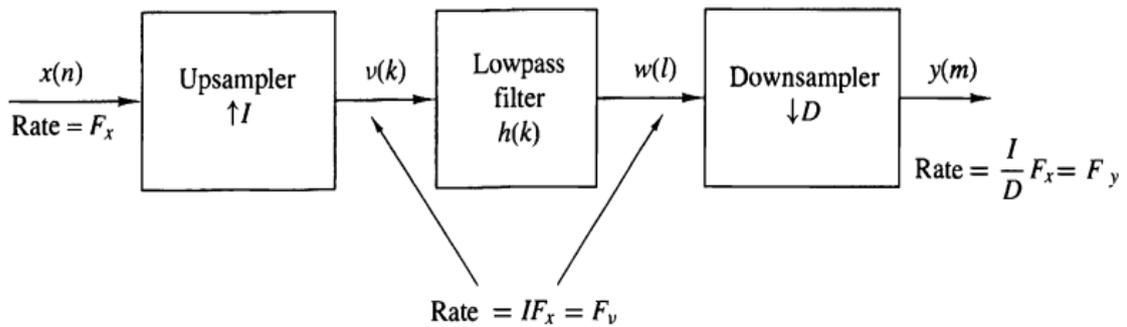
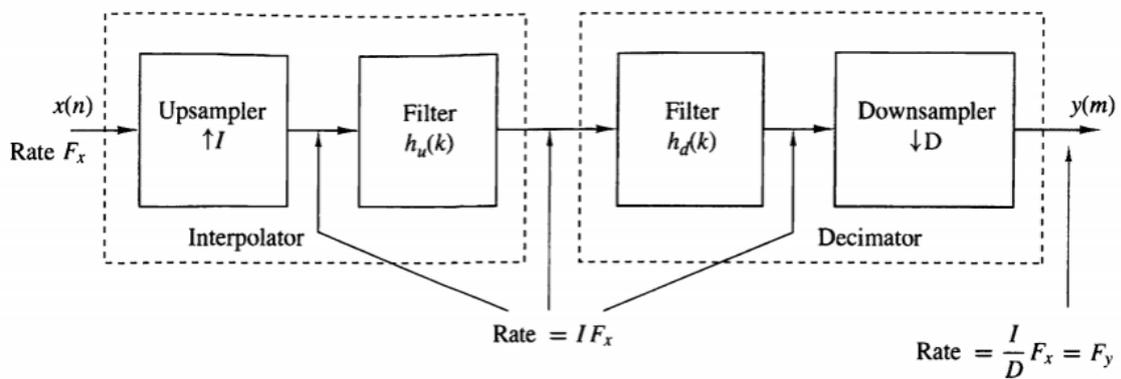
C = ?

$$y(0) = \frac{1}{2\pi} \int_{-\pi}^{\pi} Y(\omega_y) d\omega_y = \frac{C}{2\pi} \int_{-\pi/I}^{\pi/I} X(\omega_y I) d\omega_y$$

$$\omega_y = \omega_x / I, \longrightarrow = \frac{C}{I} \frac{1}{2\pi} \int_{-\pi}^{\pi} X(\omega_x) d\omega_x = \frac{C}{I} x(0)$$

$C = I$ is the desired normalization factor

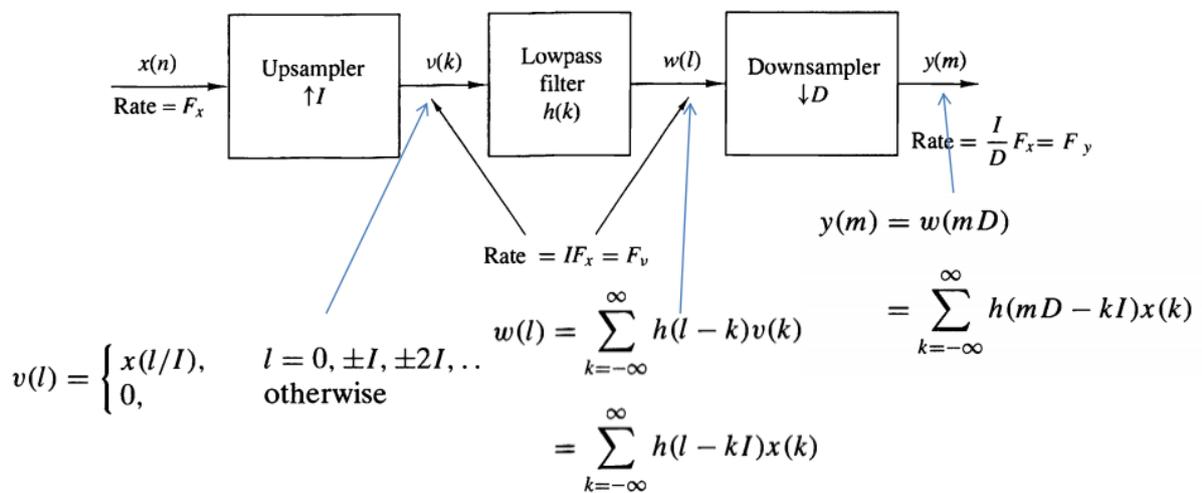
Sampling Rate Conversion By A Rational Factor I/D:



Frequency response of the combined filter

$$H(\omega_v) = \begin{cases} I, & 0 \leq |\omega_v| \leq \min(\pi/D, \pi/I) \\ 0, & \text{otherwise} \end{cases}$$

$$\omega_v = 2\pi F/F_v = 2\pi F/IF_x = \omega_x/I.$$



$$y(m) = w(mD)$$

$$= \sum_{k=-\infty}^{\infty} h(mD - kI)x(k)$$

Change of variable

$$k = \left\lfloor \frac{mD}{I} \right\rfloor - n$$



$$y(m) = \sum_{n=-\infty}^{\infty} h\left(mD - \left\lfloor \frac{mD}{I} \right\rfloor I + nI\right)x\left(\left\lfloor \frac{mD}{I} \right\rfloor - n\right)$$

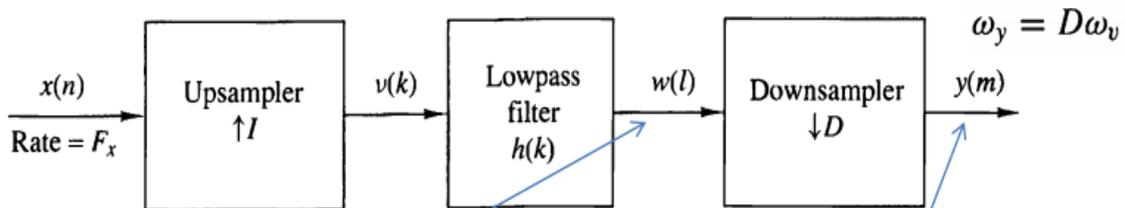
We know



$$mD - \left\lfloor \frac{mD}{I} \right\rfloor I = (mD)_I \Rightarrow y(m) = \sum_{n=-\infty}^{\infty} h(nI + (mD)_I)x\left(\left\lfloor \frac{mD}{I} \right\rfloor - n\right)$$

$$x(n) \xrightarrow[\substack{g(n, m) = h(nI + (mD)_I), \\ -\infty < m, n < \infty}]{\text{Time-varying filter}} y(m)$$

$$g(n, m + kI) = h(nI + (mD + kDI)_I) = h(nI + (mD)_I) = g(n, m)$$

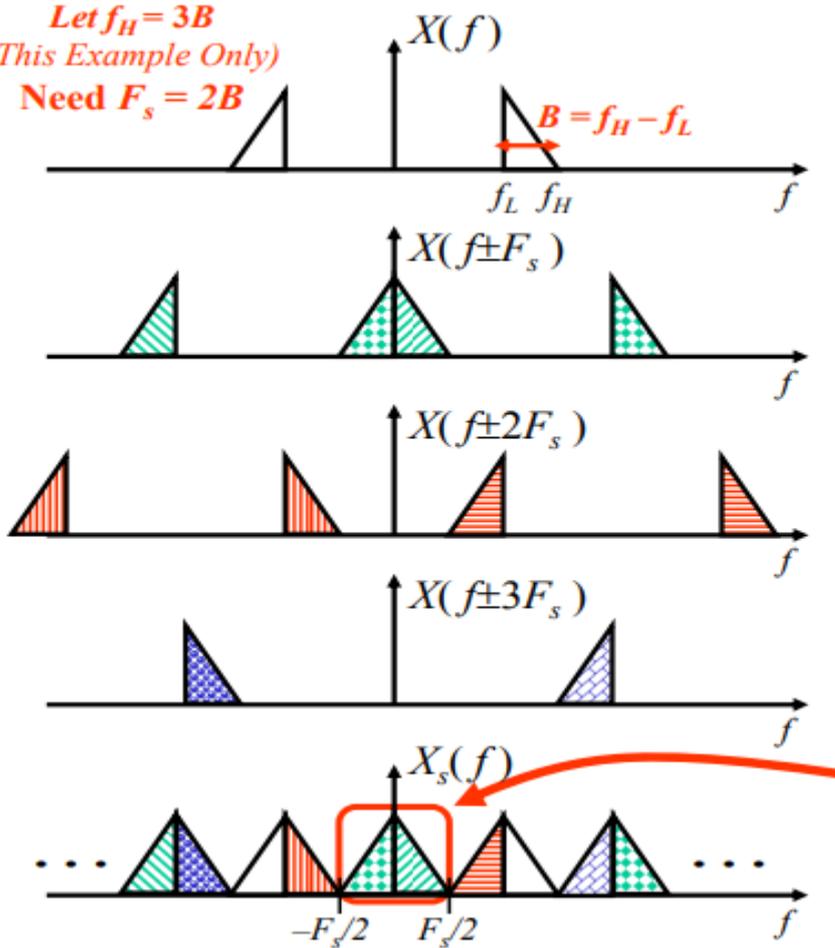


$$H(\omega_v)X(\omega_v I) = \begin{cases} IX(\omega_v I), & 0 \leq |\omega_v| \leq \min(\pi/D, \pi/I) \\ 0, & \text{otherwise} \end{cases}$$

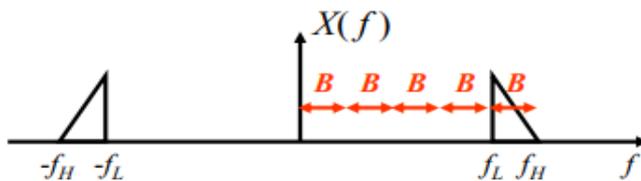
$$Y(\omega_y) = \begin{cases} \frac{I}{D}X\left(\frac{\omega_y}{D}\right), & 0 \leq |\omega_y| \leq \min\left(\pi, \frac{\pi D}{I}\right) \\ 0, & \text{otherwise} \end{cases}$$

Sampling rate conversion of band pass signals:

Let $f_H = 3B$
 (This Example Only)
 Need $F_s = 2B$

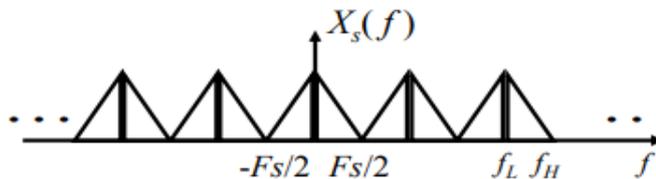


Consider the case where $f_H = kB$ (k an Odd Integer)



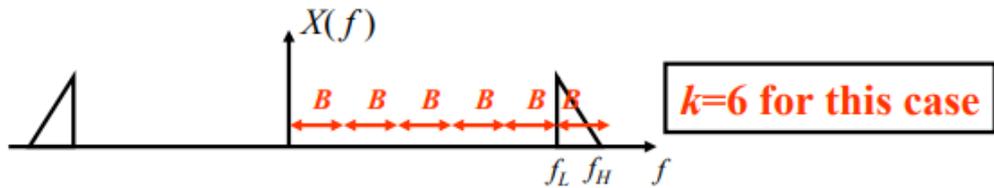
$k=5$ for this case

Whenever $f_2 = LB$, we can choose $F_s = 2B$ to perfectly “interweave” the shifted spectral replicas

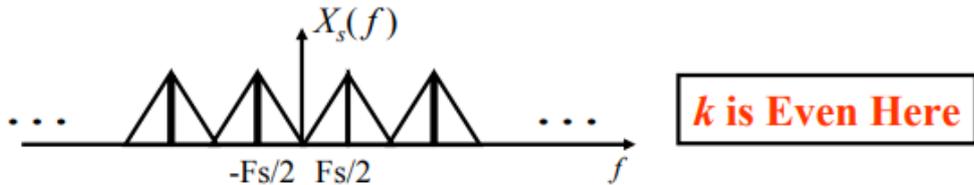


k is Odd Here

Consider the case where $f_H = LB$ (k an Even Integer)



Whenever $f_H = LB$, we can choose $F_s = 2B$ to perfectly “interweave” the shifted spectral replicas



Note: If k is EVEN the spectrum in the 0 to $F_s/2$ range is flipped. This is not usually a problem since the next step after BP sampling is usually to create the lowpass equivalent signal, which can be done in a way that gives either spectral orientation.

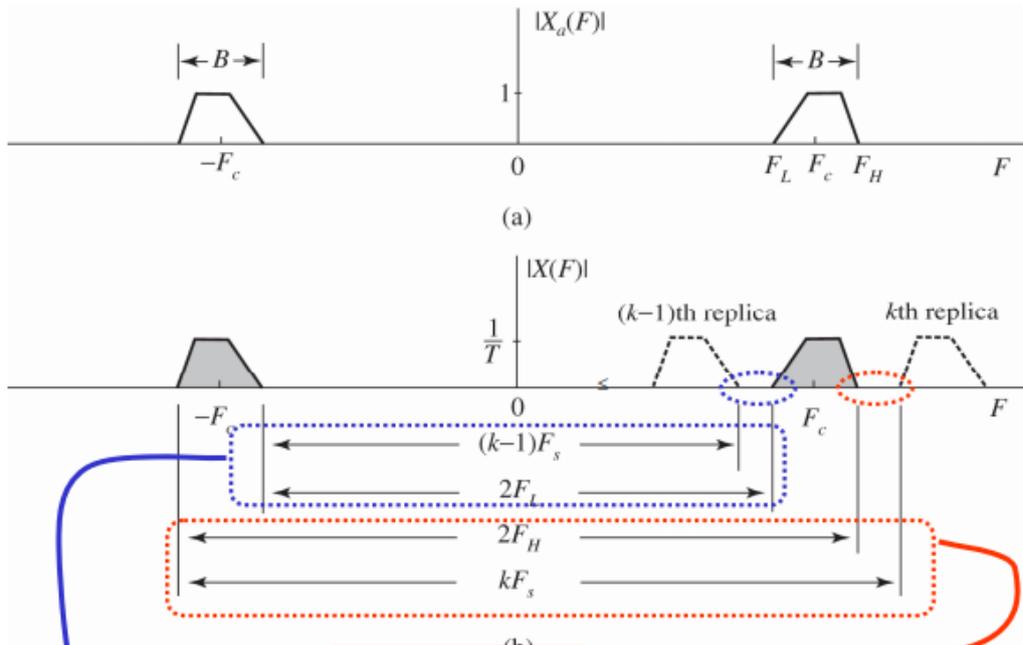


Figure 6.4.2

$$2f_H \leq kF_s$$

$$(k-1)F_s \leq 2f_L$$

$$\frac{2f_H}{k} \leq F_s \leq \frac{2f_L}{k-1}$$

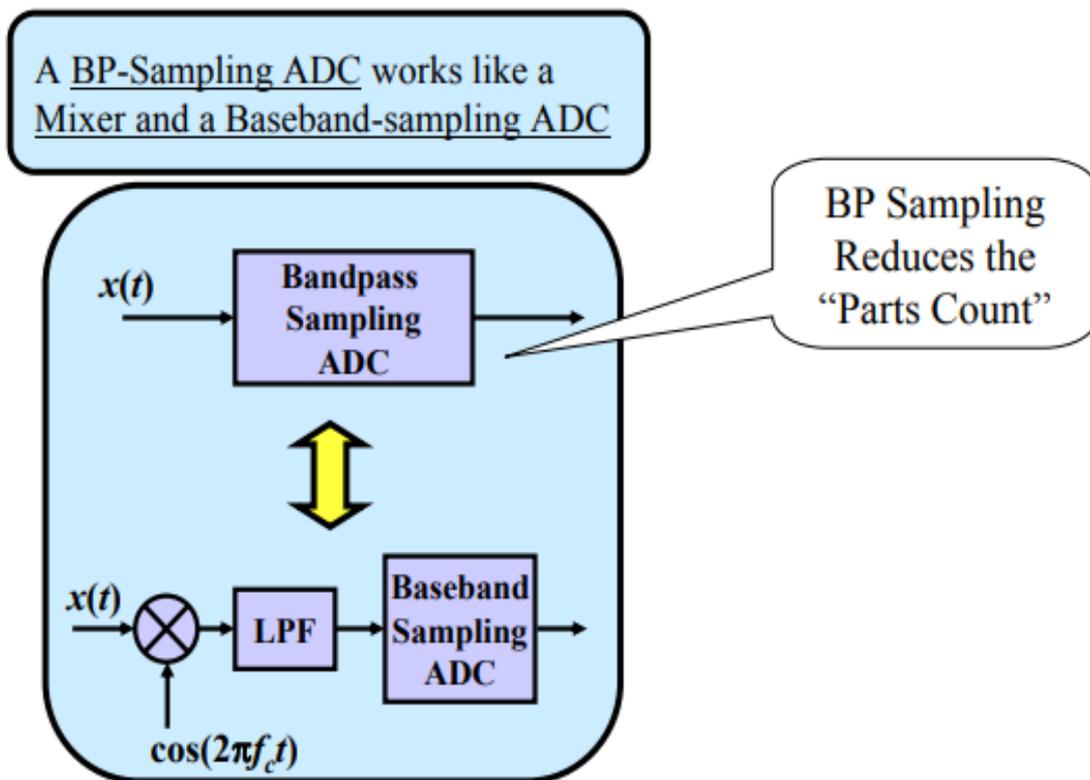
To find the required value of k ... re-write as:

$$\begin{array}{ccc} \boxed{2f_H \leq kF_s} & \longrightarrow & \boxed{\frac{1}{F_s} \leq \frac{k}{2f_H}} \\ \boxed{(k-1)F_s \leq 2f_L} & & \boxed{(k-1)F_s \leq 2(f_H - B)} \end{array}$$

Now... solving these for k gives:

$$\boxed{k \leq \frac{f_H}{B}} \longrightarrow \boxed{k_{\max} \leq \left\lfloor \frac{f_H}{B} \right\rfloor}$$

Advantages of BP Sampling:



Previous Papers Questions

1. Discuss in detail the errors resulting from rounding and truncation?
2. (i) Explain the limit cycle oscillations due to product round off and overflow errors? (ii) Explain how reduction of product round-off error is achieved in digital filters?
3. (i) Explain the effects of co-efficient quantization in FIR filters? (ii) Distinguish between fixed point and floating point arithmetic
4. With respect to finite word length effects in digital filters, with examples discuss about
 - (i) Over flow limit cycle oscillation
 - (ii) Signal scaling
5. What is called quantization noise? Derive the expression for quantization noise power.
6. (i) Compare the truncation and rounding errors using fixed point and floating point representation. (ii) Represent the following numbers in floating point format with five bits for mantissa and three bits for exponent.
 - (a) 710
 - (b) 0.2510
 - (c) -710
 - (d) -0.2510
7. Determine the dead band of the system $y(n) = 0.2y(n-1) + 0.5y(n-2) + x(n)$ Assume 8 bits are used for signal representation.
8. (a) (i) Explain the characteristics of limit cycle oscillation with respect to the system described by the difference equation $y(n) = 0.95y(n-1) + x(n)$; $x(n) = 0$ and $y(n-1) = 13$. Determine the dead range of the system.
(ii) Explain the effects of coefficient quantization in FIR filters.
9. (i) Derive the signal to quantization noise ratio of A/D converter.
(ii) Compare the truncation and rounding errors using fixed point and floating point representation. ‘