

B.Tech II Year II Semester (R15) Supplementary Examinations December 2018

MATHEMATICS – IV

(Common to EEE, ECE and EIE)

Time: 3 hours

Max. Marks: 70

PART – A

(Compulsory Question)

1 Answer the following: (10 X 02 = 20 Marks)

- Show that $\beta(m, n) = \beta(n, m)$.
- Compute $\int_0^{\infty} e^{-x} x^3 dx$.
- With usual notations prove that $J_n(-x) = (-1)^n J_n(x)$, where 'n' is a positive integer.
- Write the Rodrigue's formula.
- Define analytic function.
- Explain translation in bilinear transformation.
- Define removable singularity.
- Write Taylor's series expansion about the point $z=a$.
- Find the residue of $f(z) = \frac{2z+1}{z-2}$.
- Explain Laurent series expansion.

PART – B

(Answer all five units, 5 X 10 = 50 Marks)

UNIT – I

- Prove that $\int_0^{\pi/2} \sqrt{\sin \theta} d\theta \times \int_0^{\pi/2} \frac{d\theta}{\sqrt{\sin \theta}} = \pi$.
 - Compute $\int_0^{\infty} \frac{dx}{1+x^4}$ integrals by expressing in terms of gamma functions.

OR

- Solve $(x-1) \frac{d^2y}{dx^2} + x \frac{dy}{dx} + y = 0$ subject to the conditions $y(0) = 2$ and $y'(0) = -1$ by the method of series solution.

UNIT – II

- Compute the series solution of Bessel's differential equation $x^2 y'' + xy' + (x^2 - n^2)y = 0$ leading to $J_n(x)$.

OR

- Express $x^3 + 2x^2 - 4x + 5$ in terms of Legendre polynomials.
 - If $x^3 + 2x^2 - x + 1 = aP_0(x) + bP_1(x) + cP_2(x) + dP_3(x)$ compute the values of a, b, c, d.

UNIT – III

- Show that $V = 3x^2y + 6xy - y^3$ is harmonic. Also find harmonic conjugate. Hence find $f(z)$.

OR

- Find the Bilinear transformation which maps $z = 0, i, \infty$ into $w = 1, -i, -1$. Also find the invariant points of this transformation.

UNIT – IV

- Show that If C is the circle $|z - a| = 0$ then $\int_C (z - a)^n dz = \begin{cases} 0, & \text{if } n \neq -1 \\ 2\pi i, & \text{if } n = -1 \end{cases}$

OR

- State and prove Cauchy's integral formula. Also derive its general form.

UNIT – V

- Determine the poles and the residue for the functions $\frac{z+4}{(z-1)^2(z-2)^3}$.

OR

- State and prove Cauchy Residue theorem.

B.Tech II Year II Semester (R15) Supplementary Examinations December 2017

MATHEMATICS – IV

(Common to EEE, ECE and EIE)

Time: 3 hours

Max. Marks: 70

PART – A
(Compulsory Question)

1 Answer the following: (10 X 02 = 20 Marks)

- (a) Find $\beta(2.5, 1.5)$.
 (b) Compute $\Gamma(4.5)$.
 (c) Compute $J_1(1)$
 (d) $J_1(x) = \frac{1}{x}[xJ_1(x) - J_2(x)]$ use recurrence relation.
 (e) Find the fixed points of the bilinear transformation $w = (z - 1)/(z + 1)$.
 (f) Since the function $f(z) = r^2 \cos 2\theta + ir^2 \sin p\theta$ is analytic, the real and imaginary parts satisfies Cauchy-Riemann equations. Hence $p = 2$.
 (g) Find Laurent series for $f(z) = \frac{1}{1-z^2}$ about $z_0 = 1$.
 (h) Define removable singularity.
 (i) Evaluate $\oint_C e^{1/z^2} dz$ where C is $|z| = 2$ traversed counterclockwise.
 (j) Evaluate $\oint_C \frac{dz}{z^2(z+4)}$ where C is $|z| = 2$.

PART – B

(Answer all five units, 5 X 10 = 50 Marks)

UNIT – I

- 2 (a) Prove that $\beta\left(m, \frac{1}{2}\right) = 2^{2m-1}\beta(m, n)$.
 (b) Prove that $\Gamma(m)\Gamma\left(m + \frac{1}{2}\right) = \frac{\sqrt{\pi}}{2^{2m-1}}\Gamma(2m)$.

OR

- 3 (a) Find the value of $\Gamma\left(-\frac{1}{2}\right)$.
 (b) Prove that $\int_0^{\pi/2} \cos^n x$.

UNIT – II

- 4 Show that $J_n(x) = \frac{1}{\pi} \int_0^\pi \cos(n\theta - x \sin \theta) d\theta$ where n being integer.

OR

- 5 Find the value of $J_{\frac{1}{2}}(x)$.

UNIT – III

- 6 Find the analytic function $f(z) = u(r, \theta) + i v(r, \theta)$, given that $v(r, \theta) = r^2 \cos 2\theta - r \cos \theta + 2$.

OR

- 7 (a) Obtain the bilinear transformations which maps the points $z = \infty, i, 0$ into the points $w = 0, i, \infty$ respectively.
 (b) Find the critical points of the transformation $w^2 = (z - a)(z - b)$.

UNIT – IV

- 8 Evaluate $\int_C \frac{e^{2z}}{(z-1)(z-2)} dz$ where C is the circle $|z| = 3$, using complex integration formula.

OR

- 9 If $0 < |z - 1| < 2$ then express $(z) = \frac{z}{(z-1)(z-3)}$, in a series of positive and negative powers of $(z - 1)$.

UNIT – V

- 10 Apply the calculus of residues evaluate $\int_0^{2\pi} \frac{d\theta}{(5-3 \cos \theta)^2}$.

OR

- 11 (a) Evaluate $\int_C \frac{2z-3}{z^2+3z^2} dz$ where C is $|z| = 4$, traversed counterclockwise use residue theorem.
 (b) Evaluate $\oint_C \frac{dz}{z^3(z+4)}$ where C is $|z + 2| = 3$, traversed counterclockwise.

B.Tech II Year II Semester (R15) Supplementary Examinations December 2017

MATHEMATICS – IV

(Common to EEE, ECE and EIE)

Time: 3 hours

Max. Marks: 70

PART – A
(Compulsory Question)

1 Answer the following: (10 X 02 = 20 Marks)

- (a) Find $\beta(2.5, 1.5)$.
 (b) Compute $\Gamma(4.5)$.
 (c) Compute $J_1(1)$
 (d) $J_1(x) = \frac{1}{x}[xJ_1(x) - J_2(x)]$ use recurrence relation.
 (e) Find the fixed points of the bilinear transformation $w = (z - 1)/(z + 1)$.
 (f) Since the function $f(z) = r^2 \cos 2\theta + ir^2 \sin p\theta$ is analytic, the real and imaginary parts satisfies Cauchy-Riemann equations. Hence $p = 2$.
 (g) Find Laurent series for $f(z) = \frac{1}{1-z^2}$ about $z_0 = 1$.
 (h) Define removable singularity.
 (i) Evaluate $\oint_C e^{1/z^2} dz$ where C is $|z| = 2$ traversed counterclockwise.
 (j) Evaluate $\oint_C \frac{dz}{z^2(z+4)}$ where C is $|z| = 2$.

PART – B

(Answer all five units, 5 X 10 = 50 Marks)

UNIT – I

- 2 (a) Prove that $\beta\left(m, \frac{1}{2}\right) = 2^{2m-1}\beta(m, n)$.
 (b) Prove that $\Gamma(m)\Gamma\left(m + \frac{1}{2}\right) = \frac{\sqrt{\pi}}{2^{2m-1}}\Gamma(2m)$.

OR

- 3 (a) Find the value of $\Gamma\left(-\frac{1}{2}\right)$.
 (b) Prove that $\int_0^{\frac{\pi}{2}} \cos^n x$.

UNIT – II

- 4 Show that $J_n(x) = \frac{1}{\pi} \int_0^\pi \cos(n\theta - x \sin \theta) d\theta$ where n being integer.

OR

- 5 Find the value of $J_{\frac{1}{2}}(x)$.

UNIT – III

- 6 Find the analytic function $f(z) = u(r, \theta) + i v(r, \theta)$, given that $v(r, \theta) = r^2 \cos 2\theta - r \cos \theta + 2$.

OR

- 7 (a) Obtain the bilinear transformations which maps the points $z = \infty, i, 0$ into the points $w = 0, i, \infty$ respectively.
 (b) Find the critical points of the transformation $w^2 = (z - a)(z - b)$.

UNIT – IV

- 8 Evaluate $\int_C \frac{e^{2z}}{(z-1)(z-2)} dz$ where C is the circle $|z| = 3$, using complex integration formula.

OR

- 9 If $0 < |z - 1| < 2$ then express $(z) = \frac{z}{(z-1)(z-3)}$, in a series of positive and negative powers of $(z - 1)$.

UNIT – V

- 10 Apply the calculus of residues evaluate $\int_0^{2\pi} \frac{d\theta}{(5-3 \cos \theta)^2}$.

OR

- 11 (a) Evaluate $\int_C \frac{2z-3}{z^2+3z^2} dz$ where C is $|z| = 4$, traversed counterclockwise use residue theorem.
 (b) Evaluate $\oint_C \frac{dz}{z^3(z+4)}$ where C is $|z + 2| = 3$, traversed counterclockwise.

B.Tech II Year II Semester (R15) Regular Examinations May/June 2017

MATHEMATICS – IV

(Common to EEE, ECE and EIE)

Time: 3 hours

Max. Marks: 70

PART – A
(Compulsory Question)

- 1 Answer the following: (10 X 02 = 20 Marks)
- Compute $\Gamma(3.5)$.
 - $\beta\left(\frac{9}{2}, \frac{7}{2}\right)$.
 - Compute $J_0(2)$
 - $J_0(x) = \frac{9}{2}[J_2(x) - J_0(x)]$ use recurrence relation.
 - Find the critical points of the transformation $w^2 = (z - a)(z - b)$.
 - Compute $V(r, \theta)$ when $f(z) = u(r, \theta) + iv(r, \theta)$. Here $u(r, \theta) = \left(r + \frac{1}{r}\right) \cos \theta$.
 - Expand Taylor's series $\cos z$ about the point $z = \pi/2$.
 - Write the formula of pole of order n at $z = z_0$.
 - Evaluate $\oint_C e^{1/z^2} dz$ where C is $|z| = 2$ traversed counterclockwise.
 - Evaluate $\oint_C \frac{dz}{z^2(z+4)}$ where C is $|z| = 2$.

PART – B

(Answer all five units, 5 X 10 = 50 Marks)

UNIT – I

- 2 State and prove relation between Beta and Gamma function.
- OR
- 3 (a) Find the value of $\Gamma\left(\frac{1}{2}\right)$.
- (b) Derive $\int_0^{\pi} \sin^n \theta d\theta$.

UNIT – II

- 4 Show that $J_0(x) = \frac{1}{\pi} \int_0^{\pi} \cos(x \cos \phi) d\phi$.
- OR
- 5 Find the value of $J_{\frac{1}{2}}(x)$.

UNIT – III

- 6 Find the analytical function of the complex potential for an electric field $w = \phi + i\Psi$, given that $\Psi = x^2 - y^2 + \frac{x}{x^2 + y^2}$. Use Milne Thomson method.
- OR
- 7 (a) Find the bilinear transformation that maps the points $z_1 = -i, z_2 = 0, z_3 = i$ into the points $w_1 = 0, w_2 = -1$ and $w_3 = \infty$.
- (b) Find the fixed points of the bilinear transformation $w = (z - 1)/(z + 1)$.

UNIT – IV

- 8 Evaluate $\int_C \frac{e^{2z}}{(z+1)^4} dz$ where C is the circle $|z| = 2$, using complex integration formula.
- OR
- 9 Represent the function $f(z) = \frac{4z+3}{z(z-3)(z-2)}$ as Laurent series:
- (i) Within $|z| = 1$. (ii) In the annulus region $|z| = 2$ and $|z| = 3$. (iii) Exterior to $|z| = 3$.

UNIT – V

- 10 Apply the calculus of residues to evaluate $\int_0^{2\pi} \frac{d\theta}{2 - \sin \theta}$.
- OR
- 11 (a) Evaluate $\int_{|z|=\frac{1}{2}} \frac{dz}{(z-1)(z+2)^2} = 0$ using Residue theorem.
- (b) Evaluate $\oint_C \frac{z^2}{z^2 - jz + 2} dz$ where C is $|z| = 3/2$, traversed counterclockwise.
