Code: 15A54402 **R15**

B.Tech II Year II Semester (R15) Supplementary Examinations December 2018

MATHEMATICS – IV

(Common to EEE, ECE and EIE)

Time: 3 hours Max. Marks: 70

PART – A

(Compulsory Question)

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1 Answer the following: (10 X 02 = 20 Marks)

(a) Show that $\beta(m,n) = \beta(n,m)$.

(b) Compute $\int_0^\infty e^{-x} x^3 dx$.

(c) With usual notations prove that $J_n(-x) = (-1)^n J_n(x)$, where 'n' is a positive integer.

(d) Write the Rodrigue's formula.

(e) Define analytic function.

(f) Explain translation in bilinear transformation.

(g) Define removable singularity.

(h) Write Taylor's series expansion about the point z=a.

(i) Find the residue of $f(z) = \frac{2z+1}{z-2}$.

(j) Explain Laurent series expansion.

PART – B

(Answer all five units, 5 X 10 = 50 Marks)

UNIT – I

2 (a) Prove that $\int_0^{\pi/2} \sqrt{\sin\theta}\, d\theta \ X \int_0^{\pi/2} \frac{d\theta}{\sqrt{sin\theta}} = \pi$.

(b) Compute $\int_0^\infty \frac{dx}{1+x^4}$ integrals by expressing in terms of gamma functions.

OR

3 Solve $(x-1)\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$ subject to the conditions y(0) = 2 and $y'0)$ = -1 by the method of series solution.

UNIT – II

4 Compute the series solution of Bessel's differential equation $x^2y'' + xy' + (x^2 - n^2)y = 0$ leading to $J_n(x)$.

OR

5 (a) Express $x^3 + 2x^2 - 4x + 5$ in terms of Legendre polynomials.

(b) If $x^3 + 2x^2 - x + 1 = aP_0(x) + bP_1(x) + cP_2(x) + dP_3(x)$ compute the values of a, b, c, d.

UNIT – III

6 Show that $V = 3x^2y + 6xy - y^3$ is harmonic. Also find harmonic conjugate. Hence find f(z).

OR

7 Find the Bilinear transformation which maps z = 0, i, ∞ into w = 1, -i, -1. Also find the invariant points of this transformation.

UNIT – IV

8 Show that If C is the circle $|z-a| = 0$ then $\int_c (z-a)^n dz = \begin{cases} 0, & if\ n \neq -1 \\ 2\pi i, & if\ n = -1 \end{cases}$.

OR

9 State and prove Cauchy's integral formula. Also derive its general form.

UNIT – V

10 Determine the poles and the residue for the functions $\frac{z+4}{(z-1)^2(z-2)^3}$.

OR

11 State and prove Cauchy Residue theorem.

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PART – A

(Compulsory Question)

*****

1 Answer the following: (10 X 02 = 20 Marks)

(a) Find $\beta(2.5, 1.5)$.

(b) Compute $\Gamma(4.5)$.

(c) Compute $J_1(1)$

(d) $J_1(x) = \frac{1}{x}[xJ_1(x) - J_2(x)]$ use recurrence relation.

(e) Find the fixed points of the bilinear transformation $w = (z-1)/(z+1)$.

(f) Since the function $f(z) = r^2 \cos 2\theta + ir^2 \sin p\theta$ is analytic, the real and imaginary parts satisfies Cauchy-Riemann equations. Hence p = 2.

(g) Find Laurent series for $f(z) = \frac{1}{1-z^2}$ about $z_0 = 1$.

(h) Define removable singularity.

(i) Evaluate $\oint_C e^{1/z^2}\, dz$ where C is $|z| = 2$ traversed counterclockwise.

(j) Evaluate $\oint_C \frac{dz}{z^2(z+4)} dz$ where C is $|z| = 2$.

PART – B

(Answer all five units, 5 X 10 = 50 Marks)

UNIT – I

2 (a) Prove that $\beta\left(m, \frac{1}{2}\right) = 2^{2m-1}\beta(m, n)$.

(b) Prove that $\Gamma(m)\Gamma\left(m + \frac{1}{2}\right) = \frac{\sqrt{\pi}}{2^{2m-1}}\Gamma(2m)$.

OR

3 (a) Find the value of $\Gamma\left(-\frac{1}{2}\right)$.

(b) Prove that $\int_0^{\frac{\pi}{2}} \cos^n x$.

UNIT – II

4 Show that $J_n(x) = \frac{1}{\pi}\int_0^\pi \cos(n\theta - x\sin\theta)d\theta$ where n being integer.

OR

5 Find the value of $J_{\frac{1}{2}}(x)$.

UNIT – III

6 Find the analytic function $f(z) = u(r,\theta) + i\, v(r,\theta)$, given that $v(r,\theta) = r^2\cos 2\theta - r\cos\theta + 2$.

OR

7 (a) Obtain the bilinear transformations which maps the points $z = \infty, i, 0$ into the points $w = 0, i, \infty$ respectively.

(b) Find the critical points of the transformation $w^2 = (z-a)(z-b)$.

UNIT – IV

8 Evaluate $\int_C \frac{e^{2z}}{(z-1)(z-2)} dz$ where C is the circle $|z| = 3$, using complex integration formula.

OR

9 If $0 < |z-1| < 2$ then express $(z) = \frac{z}{(z-1)(z-3)}$, in a series of positive and negative powers of $(z-1)$.

UNIT – V

10 Apply the calculus of residues evaluate $\int_0^{2\pi}\frac{d\theta}{(5-3\cos\theta)^2}$.

OR

11 (a) Evaluate $\int_C \frac{2z-3}{z^2+3z^2} dz$ where C is $|z| = 4$, traversed counterclockwise use residue theorem.

(b) Evaluate $\oint_C \frac{dz}{z^3(z+4)} dz$ where C is $|z+2| = 3$, traversed counterclockwise.

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B.Tech II Year II Semester (R15) Supplementary Examinations December 2017

MATHEMATICS – IV

(Common to EEE, ECE and EIE)

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PART – A

(Compulsory Question)

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(a) Find $\beta(2.5, 1.5)$.

(b) Compute $\Gamma(4.5)$.

(c) Compute $J_1(1)$

(d) $J_1(x) = \frac{1}{x}[xJ_1(x) - J_2(x)]$ use recurrence relation.

(e) Find the fixed points of the bilinear transformation $w = (z-1)/(z+1)$.

(f) Since the function $f(z) = r^2 \cos 2\theta + ir^2 \sin p\theta$ is analytic, the real and imaginary parts satisfies Cauchy-Riemann equations. Hence p = 2.

(g) Find Laurent series for $f(z) = \frac{1}{1-z^2}$ about $z_0 = 1$.

(h) Define removable singularity.

(i) Evaluate $\oint_C e^{1/z^2}\, dz$ where C is $|z| = 2$ traversed counterclockwise.

(j) Evaluate $\oint_C \frac{dz}{z^2(z+4)} dz$ where C is $|z| = 2$.

PART – B

(Answer all five units, 5 X 10 = 50 Marks)

UNIT – I

2 (a) Prove that $\beta\left(m, \frac{1}{2}\right) = 2^{2m-1}\beta(m, n)$.

(b) Prove that $\Gamma(m)\Gamma\left(m + \frac{1}{2}\right) = \frac{\sqrt{\pi}}{2^{2m-1}}\Gamma(2m)$.

OR

3 (a) Find the value of $\Gamma\left(-\frac{1}{2}\right)$.

(b) Prove that $\int_0^{\frac{\pi}{2}} \cos^{\mathrm{n}}\, x$.

UNIT – II

4 Show that $J_n(x) = \frac{1}{\pi}\int_0^{\pi} \cos(n\theta - x\sin\theta)d\theta$ where n being integer.

OR

5 Find the value of $J_{\frac{1}{2}}(x)$.

UNIT – III

6 Find the analytic function $f(z) = u(r,\theta) + i\, v(r,\theta)$, given that $v(r,\theta) = r^2\cos 2\theta - r\cos\theta + 2$.

OR

7 (a) Obtain the bilinear transformations which maps the points $z = \infty,\ i,\ 0$ into the points $w = 0,\ i,\ \infty$ respectively.

(b) Find the critical points of the transformation $w^2 = (z-a)(z-b)$.

UNIT – IV

8 Evaluate $\int_C \frac{e^{2z}}{(z-1)(z-2)} dz$ where C is the circle $|z| = 3$, using complex integration formula.

OR

9 If $0 < |z-1| < 2$ then express $(z) = \frac{z}{(z-1)(z-3)}$, in a series of positive and negative powers of $(z-1)$.

UNIT – V

10 Apply the calculus of residues evaluate $\int_0^{2\pi} \frac{d\theta}{(5-3\cos\theta)^2}$.

OR

11 (a) Evaluate $\int_C \frac{2z-3}{z^2+3z^2} dz$ where C is $|z| = 4$, traversed counterclockwise use residue theorem.

(b) Evaluate $\oint_C \frac{dz}{z^3(z+4)} dz$ where C is $|z+2| = 3$, traversed counterclockwise.

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Code: 15A54402 **R15**

B.Tech II Year II Semester (R15) Regular Examinations May/June 2017

MATHEMATICS – IV

(Common to EEE, ECE and EIE)

Time: 3 hours Max. Marks: 70

PART – A

(Compulsory Question)

*****

1 Answer the following: (10 X 02 = 20 Marks)

(a) Compute $\Gamma(3.5)$.

(b) $\beta\left(\frac{9}{2},\frac{7}{2}\right)$.

(c) Compute $J_0(2)$

(d) $J_0(x)=\frac{9}{2}[J_2(x)-J_0(x)]$ use recurrence relation.

(e) Find the critical points of the transformation $w^2=(z-a)(z-b)$.

(f) Compute $\mathrm{V}(\mathrm{r},\theta)$ when $\mathrm{f(z)}=\mathrm{u}(\mathrm{r},\theta)+\mathrm{iv}(\mathrm{r},\theta)$. Here $\mathrm{u}(\mathrm{r},\theta)=\left(\mathrm{r}+\frac{1}{\mathrm{r}}\right)\cos\theta$.

(g) Expand Taylor's series $\cos z$ about the point $z=\pi/2$.

(h) Write the formula of pole of order n at $z=z_0$.

(i) Evaluate $\oint_C \ e^{1/z^2}\,dz$ where C is $|z|=2$ traversed counterclockwise.

(j) Evaluate $\oint_C \ \frac{dz}{z^2(z+4)}dz$ where C is $|z|=2$.

PART – B

(Answer all five units, 5 X 10 = 50 Marks)

UNIT – I

2 State and prove relation between Beta and Gamma function.

OR

3 (a) Find the value of $\Gamma\left(\frac{1}{2}\right)$.

(b) Derive $\int_0^{\frac{\pi}{2}}\sin^{n\theta}d\theta$.

UNIT – II

4 Show that $J_0(x)=\frac{1}{\pi}\int_0^{\pi}\cos(x\cos\phi)d\phi$.

OR

5 Find the value of $J_{\frac{1}{2}}(x)$.

UNIT – III

6 Find the analytical function of the complex potential for an electric field $\mathrm{w}=\varphi+\mathrm{i}\Psi$, given that $\Psi=x^2-y^2+\frac{x}{x^2+y^2}$. Use Milne Thomson method.

OR

7 (a) Find the bilinear transformation that maps the points $z_1=-i$, $z_2=0, z_3=i$ into the points $w_1=0$, $w_2=-1\ and\ w_3=\infty$.

(b) Find the fixed points of the bilinear transformation $w=(z-1)/(z+1)$.

UNIT – IV

8 Evaluate $\int_C \ \frac{e^{2z}}{(z+1)^4}\,dz$ where C is the circle $|z|=2$, using complex integration formula.

OR

9 Represent the function $f(z)=\frac{4z+3}{z(z-3)(z-2)}$ as Laurent series:

(i) With in $|z|=1$. (ii) In the annulus region $|z|=2\ and\ |z|=3$. (iii) Exterior to $|z|=3$.

UNIT – V

10 Apply the calculus of residues to evaluate $\int_0^{2\pi}\frac{d\theta}{2-\sin\theta}$.

OR

11 (a) Evaluate $\int_{|z|=\frac{1}{2}} \ \frac{dz}{(z-1)(z+2)^2}=0$ using Residue theorem.

(b) Evaluate $\oint_C \ \frac{z^2}{z^2-jz+2}dz$ where C is $|z|=3/2$, traversed counterclockwise.

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