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**GATE SOLVED PAPER
Mechanical Engineering
2003**

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GATE SOLVED PAPER - ME

2003

Q. 1 $\lim_{x \rightarrow 0} \frac{\sin^2 x}{x}$ is equal to

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ONE MARK

- (A) 0 (B) ∞
(C) 1 (D) -1

Sol. 1 Option (A) is correct

$$\begin{aligned} \text{Let, } f(x) &= \lim_{x \rightarrow 0} \frac{\sin^2 x}{x} = \lim_{x \rightarrow 0} \frac{\sin^2 x}{x} \times \frac{x}{x} \\ &= \lim_{x \rightarrow 0} \left(\frac{\sin x}{x} \right)^2 \times x \qquad \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1 \\ &= (1)^2 \times 0 = 0 \end{aligned}$$

Alternate:

$$\begin{aligned} \text{Let } f(x) &= \lim_{x \rightarrow 0} \frac{\sin^2 x}{x} \qquad \left[\frac{0}{0} \text{ form} \right] \\ f(x) &= \lim_{x \rightarrow 0} \frac{2 \sin x \cos x}{1} \quad \text{Apply L-Hospital rule} \\ &= \lim_{x \rightarrow 0} \frac{\sin 2x}{1} = \frac{\sin 0}{1} = 0 \end{aligned}$$

Q. 2 The accuracy of Simpson's rule quadrature for a step size h is

GATE ME 2003
ONE MARK

- (A) $O(h^2)$ (B) $O(h^3)$
(C) $O(h^4)$ (D) $O(h^5)$

Sol. 2 Option (D) is correct.

Accuracy of Simpson's rule quadrature is $O(h^5)$

Q. 3 For the matrix $\begin{bmatrix} 4 & 1 \\ 1 & 4 \end{bmatrix}$ the eigen values are

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ONE MARK

- (A) 3 and -3 (B) -3 and -5
(C) 3 and 5 (D) 5 and 0

Sol. 3 Option (C) is correct.

$$\text{Let, } A = \begin{bmatrix} 4 & 1 \\ 1 & 4 \end{bmatrix}$$

The characteristic equation for the eigen value is given by,

$$\begin{aligned} |A - \lambda I| &= 0 \qquad I = \text{Identity matrix } \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\ \left| \begin{bmatrix} 4 & 1 \\ 1 & 4 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right| &= 0 \\ \left| \begin{array}{cc} 4 - \lambda & 1 \\ 1 & 4 - \lambda \end{array} \right| &= 0 \\ (4 - \lambda)(4 - \lambda) - 1 &= 0 \\ (4 - \lambda)^2 - 1 &= 0 \end{aligned}$$

$$\lambda^2 - 8\lambda + 15 = 0$$

On solving above equation, we get

$$\lambda = 5, 3$$

Q. 4

GATE ME 2003
ONE MARK

The second moment of a circular area about the diameter is given by (D is the diameter).

- (A) $\frac{\pi D^4}{4}$
- (B) $\frac{\pi D^4}{16}$
- (C) $\frac{\pi D^4}{32}$
- (D) $\frac{\pi D^4}{64}$

Sol. 4

Option (D) is correct.

We know that, moment of inertia is defined as the second moment of a plane area about an axis perpendicular to the area.

Polar moment of inertia perpendicular to the plane of paper,

$$J \text{ or } I_P = \frac{\pi D^4}{32}$$

By the "perpendicular axis" theorem,

$$I_{XX} + I_{YY} = I_P \quad \text{For circular section } I_{XX} = I_{YY}$$

$$2I_{XX} = I_P$$

$$I_{XX} = \frac{I_P}{2} = \frac{\pi D^4}{64} = I_{YY}$$

Q. 5

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ONE MARK

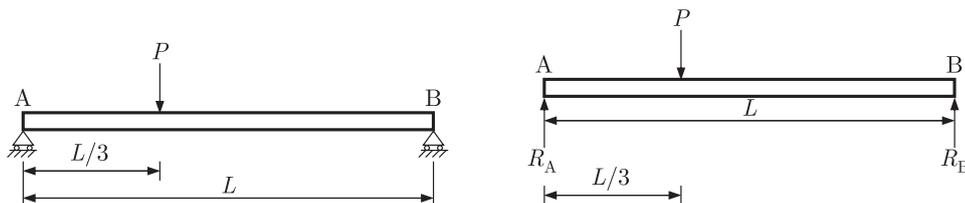
A concentrated load of P acts on a simply supported beam of span L at a distance $L/3$ from the left support. The bending moment at the point of application of the load is given by

- (A) $\frac{PL}{3}$
- (B) $\frac{2PL}{3}$
- (C) $\frac{PL}{9}$
- (D) $\frac{2PL}{9}$

Sol. 5

Option (D) is correct.

We know that, the simplest form of the simply supported beams is the beam supported on rollers at ends. The simply supported beam and the *FBD* shown in the Figure.



Where, R_A & R_B are the reactions acting at the ends of the beam.

In equilibrium condition of forces,

$$P = R_A + R_B \quad \dots (i)$$

Taking the moment about point A,

$$R_B \times L = P \times \frac{L}{3}$$

$$R_B = \frac{P}{3}$$

From equation (i),

$$R_A = P - R_B = P - \frac{P}{3} = \frac{2P}{3}$$

Now bending moment at the point of application of the load

$$M = R_A \times \frac{L}{3} = \frac{2P}{3} \times \frac{L}{3} = \frac{2PL}{9}$$

Or,
$$M = R_B \times \frac{2L}{3} = \frac{2PL}{9}$$

Q. 6

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ONE MARK

Two identical circular rods of same diameter and same length are subjected to same magnitude of axial tensile force. One of the rod is made out of mild steel having the modulus of elasticity of 206 GPa. The other rod is made out of cast iron having the modulus of elasticity of 100 GPa. Assume both the materials to be homogeneous and isotropic and the axial force causes the same amount of uniform stress in both the rods. The stresses developed are within the proportional limit of the respective materials. Which of the following observations is correct ?

- (A) Both rods elongate by the same amount
- (B) Mild steel rod elongates more than the cast iron rod
- (C) Cast iron rod elongates more than the mild steel rods
- (D) As the stresses are equal strains are also equal in both the rods

Sol. 6

Option (C) is correct.

Given : $L_s = L_i$, $E_s = 206$ GPa, $E_i = 100$ GPa, $P_s = P_i$, $D_s = D_i$, $\Rightarrow A_s = A_i$

Where subscript s is for steel and i is for iron rod.

We know that elongation is given by,

$$\Delta L = \frac{PL}{AE}$$

Now, for steel or iron rod

$$\frac{\Delta L_s}{\Delta L_i} = \frac{P_s L_s}{A_s E_s} \times \frac{A_i E_i}{P_i L_i} = \frac{E_i}{E_s}$$

Substitute the values, we get

$$\frac{\Delta L_s}{\Delta L_i} = \frac{100}{206} = 0.485 < 1$$

Or,
$$\Delta L_s < \Delta L_i \Rightarrow \Delta L_i > \Delta L_s$$

So, cast iron rod elongates more than the mild steel rod.

Q. 7

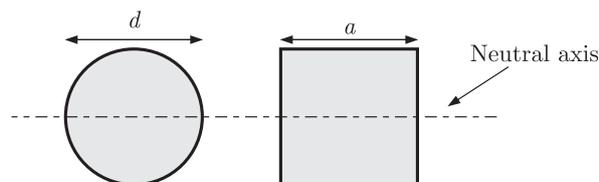
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ONE MARK

The beams, one having square cross section and another circular cross-section, are subjected to the same amount of bending moment. If the cross sectional area as well as the material of both the beams are same then

- (A) maximum bending stress developed in both the beams is same
- (B) the circular beam experience more bending stress than the square one
- (C) the square beam experience more bending stress than the circular one
- (D) as the material is same, both the beams will experience same deformation.

Sol. 7

Option (B) is correct.



Let, a = Side of square cross-section
 d = diameter of circular cross-section

Using subscripts for the square and c for the circular cross section.

Given : $M_s = M_c$
 $A_c = A_s$

So, $\frac{\pi}{4}d^2 = a^2 \quad \dots (i)$

From the bending equation,

$$\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R} \Rightarrow \sigma = \frac{M}{I} \times y$$

Where, y = Distance from the neutral axis to the external fibre.
 σ = Bending stress

For square cross-section bending stress,

$$\sigma_s = \frac{M_s}{\frac{a^4}{12}} \times \frac{a}{2} = \frac{6M_s}{a^3} \quad \dots (ii)$$

And for circular cross-section,

$$\sigma_c = \frac{M_c}{\frac{\pi}{64}d^4} \times \frac{d}{2} = \frac{32M_c}{d^3} \quad \dots (iii)$$

On dividing equation (iii) by equation (ii), we get

$$\frac{\sigma_c}{\sigma_s} = \frac{32M_c}{d^3} \times \frac{a^3}{6M_s} = \frac{16}{3} \frac{a^3}{d^3} \quad M_c = M_s \quad \dots (iv)$$

From equation (i),

$$\left(\frac{\pi}{4}d^2\right)^{3/2} = (a^2)^{3/2} = a^3$$

$$\frac{a^3}{d^3} = \left(\frac{\pi}{4}\right)^{3/2} = 0.695$$

Substitute this value in equation (iv), we get

$$\frac{\sigma_c}{\sigma_s} = \frac{16}{3} \times 0.695 = 3.706$$

$$\frac{\sigma_c}{\sigma_s} > 1 \Rightarrow \sigma_c > \sigma_s$$

So, Circular beam experience more bending stress than the square section.

Q. 8

GATE ME 2003
ONE MARK

The mechanism used in a shaping machine is

- (A) a closed 4-bar chain having 4 revolute pairs
 (B) a closed 6-bar chain having 6 revolute pairs
 (C) a closed 4-bar chain having 2 revolute and 2 sliding pairs
 (D) an inversion of the single slider-crank chain

Sol. 8

Option (D) is correct.

A single slider crank chain is a modification of the basic four bar chain. It is found that four inversions of a single slider crank chain are possible. From these four inversions, crank and slotted lever quick return motion mechanism is used in shaping machines, slotting machines and in rotary internal combustion engines.

Q. 9

GATE ME 2003
ONE MARK

The lengths of the links of a 4-bar linkage with revolute pairs are $p, q, r,$ and s units. given that $p < q < r < s$. Which of these links should be the fixed one, for obtaining a "double crank" mechanism ?

- (A) link of length p (B) link of length q
 (C) link of length r (D) link of length s

Sol. 9

Option (A) is correct.

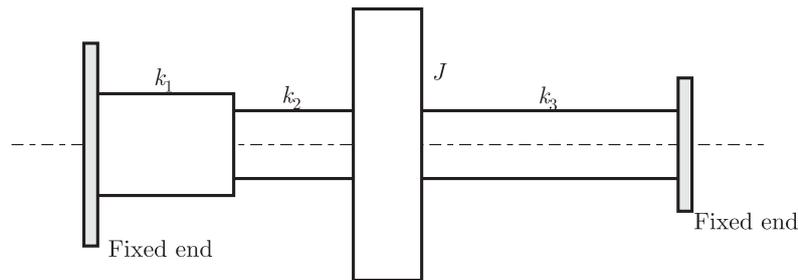
Given $p < q < r < s$

“Double crank” mechanism occurs, when the shortest link is fixed. From the given pairs p is the shortest link. So, link of length p should be fixed.

Q. 10

GATE ME 2003
ONE MARK

Consider the arrangement shown in the figure below where J is the combined polar mass moment of inertia of the disc and the shafts. k_1, k_2, k_3 are the torsional stiffness of the respective shafts. The natural frequency of torsional oscillation of the disc is given by



- (A) $\sqrt{\frac{k_1 + k_2 + k_3}{J}}$ (B) $\sqrt{\frac{k_1 k_2 + k_2 k_3 + k_3 k_1}{J(k_1 + k_2)}}$
 (C) $\sqrt{\frac{k_1 + k_2 + k_3}{J(k_1 k_2 + k_2 k_3 + k_3 k_1)}}$ (D) $\sqrt{\frac{k_1 k_2 + k_2 k_3 + k_3 k_1}{J(k_2 + k_3)}}$

Sol. 10

Option (B) is correct.

Here k_1 & k_2 are in series combination & k_3 is in parallel combination with this series combination.

$$\text{So, } k_{eq} = \frac{k_1 \times k_2}{k_1 + k_2} + k_3 = \frac{k_1 k_2 + k_2 k_3 + k_1 k_3}{k_1 + k_2}$$

Natural frequency of the torsional oscillation of the disc, $\omega_n = \sqrt{\frac{k_{eq}}{J}}$

Substitute the value of k_{eq} , we get

$$\omega_n = \sqrt{\frac{k_1 k_2 + k_2 k_3 + k_1 k_3}{J(k_1 + k_2)}}$$

Q. 11

GATE ME 2003
ONE MARK

Maximum shear stress developed on the surface of a solid circular shaft under pure torsion is 240 MPa. If the shaft diameter is doubled then the maximum shear stress developed corresponding to the same torque will be

- (A) 120 MPa (B) 60 MPa
(C) 30 MPa (D) 15 MPa

Sol. 11

Option (C) is correct.

Given : $\tau_1 = \tau_{\max} = 240 \text{ MPa}$ Let, diameter of solid shaft $d_1 = d$, And Final diameter $d_2 = 2d$ (Given)

From the Torsional Formula,

$$\frac{T}{J} = \frac{\tau}{r} \Rightarrow T = \frac{\tau}{r} \times J$$

Where, $J =$ polar moment of inertia

Given that torque is same,

$$\text{So, } \frac{\tau_1}{r_1} \times J_1 = \frac{\tau_2}{r_2} \times J_2$$

$$\frac{2\tau_1}{d_1} \times J_1 = \frac{2\tau_2}{d_2} \times J_2$$

$$J = \frac{\pi}{32} d^4$$

$$\frac{\tau_1}{d_1} \times \frac{\pi}{32} d_1^4 = \frac{\tau_2}{d_2} \times \frac{\pi}{32} d_2^4$$

$$\tau_1 \times d_1^3 = \tau_2 \times d_2^3 \Rightarrow \tau_2 = \tau_1 \times \frac{d_1^3}{d_2^3}$$

Substitute the values, we get

$$\tau_2 = 240 \times \left(\frac{d}{2d}\right)^3 = 240 \times \frac{1}{8} = 30 \text{ MPa}$$

Common Data For Q. Alternate method

From the Torsional Formula,

$$\tau = \frac{TR}{J} \quad r = \frac{d}{2} \text{ \& } J = \frac{\pi}{32} d^4$$

So, maximum shear stress,

$$\tau_{\max} = \frac{16T}{\pi d^3} = 240 \text{ MPa}$$

Given Torque is same & Shaft diameter is doubled then,

$$\tau'_{\max} = \frac{16T}{\pi (2d)^3} = \frac{16T}{8\pi d^3} = \frac{\tau_{\max}}{8} = \frac{240}{8} = 30 \text{ MPa}$$

Q. 12

GATE ME 2003
ONE MARK

A wire rope is designated as 6×19 standard hoisting. The numbers 6×19 represent

- (A) diameter in millimeter \times length in meter
- (B) diameter in centimeter \times length in meter
- (C) number of strands \times numbers of wires in each strand
- (D) number of wires in each strand \times number of strands

Sol. 12

Option (C) is correct.

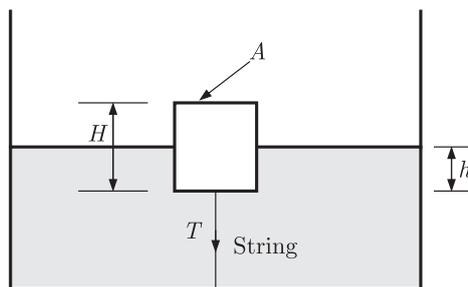
The wire ropes are designated by the number of strands multiplied by the number of wires in each strand. Therefore,

$$6 \times 19 = \text{Number of strands} \times \text{Number of wires in each strand.}$$

Q. 13

GATE ME 2003
ONE MARK

A cylindrical body of cross-sectional area A , height H and density ρ_s , is immersed to a depth h in a liquid of density ρ , and tied to the bottom with a string. The tension in the string is



- (A) ρghA
- (B) $(\rho_s - \rho) ghA$
- (C) $(\rho - \rho_s) ghA$
- (D) $(\rho h - \rho_s H) gA$

Sol. 13

Option (D) is correct.

Given :

Cross section area of body = A

Height of body = H

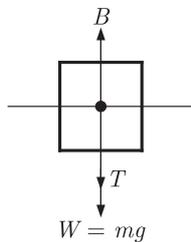
Density of body = ρ_s

Density of liquid = ρ

Tension in the string = T

We have to make the FBD of the block.

B = Buoyancy force



From the principal of buoyancy,

Downward force = Buoyancy force

$$m = \rho\nu$$

$$T + mg = \rho hAg$$

$$T + \rho_s \nu g = \rho hAg$$

$$\& \nu = A \times H$$

$$T + \rho_s AHg = \rho hAg$$

$$T = \rho hAg - \rho_s AHg = Ag(\rho h - \rho_s H)$$

Q. 14

GATE ME 2003
ONE MARK

A 2 kW, 40 liters water heater is switched on for 20 minutes. The heat capacity c_p for water is 4.2 kJ/kgK. Assuming all the electrical energy has gone into heating the water, increase of the water temperature in degree centigrade is

- (A) 2.7 (B) 4.0
(C) 14.3 (D) 25.25

Sol. 14

Option (C) is correct.

Given : $p = 2 \text{ kW} = 2 \times 10^3 \text{ W}$, $t = 20 \text{ minutes} = 20 \times 60 \text{ sec}$, $c_p = 4.2 \text{ kJ/kgK}$

Heat supplied, $Q = \text{Power} \times \text{Time}$

$$= 2 \times 10^3 \times 20 \times 60 = 24 \times 10^5 \text{ Joule}$$

And Specific heat at constant pressure,

$$Q = mc_p \Delta T$$



$$\Delta T = \frac{24 \times 10^5}{40 \times 4.2 \times 1000} = \frac{24 \times 100}{40 \times 4.2} = 14.3^\circ \text{C}$$

Q. 15

GATE ME 2003
ONE MARK

An industrial heat pump operates between the temperatures of 27°C and -13°C . The rates of heat addition and heat rejection are 750 W and 1000 W, respectively. The COP for the heat pump is

- (A) 7.5 (B) 6.5
(C) 4.0 (D) 3.0

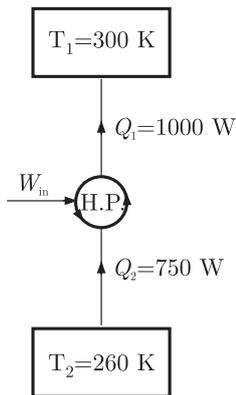
Sol. 15

Option (C) is correct.

Given : $T_1 = 27^\circ \text{C} = (27 + 273) \text{ K} = 300 \text{ K}$,

$T_2 = -13^\circ \text{C} = (-13 + 273) \text{ K} = 260 \text{ K}$

$Q_1 = 1000 \text{ W}$, $Q_2 = 750 \text{ W}$



So, $(COP)_{H.P.} = \frac{Q_1}{Q_1 - Q_2} = \frac{1000}{1000 - 750} = 4$

Common Data For Q. Alternate Method

From energy balance

$$W_{in} + Q_2 = Q_1$$

$$W_{in} = Q_1 - Q_2 = 1000 - 750 = 250 \text{ W}$$

And $(COP)_{H.P.} = \frac{\text{Desired effect}}{W_{in}} = \frac{Q_1}{W_{in}} = \frac{1000}{250} = 4$

Q. 16

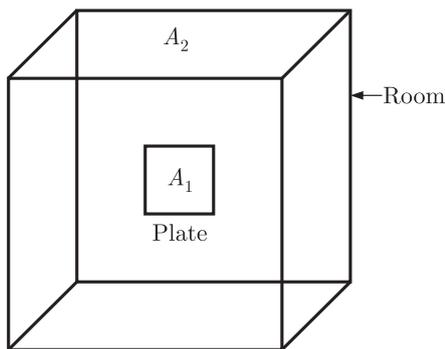
GATE ME 2003
ONE MARK

A plate having 10 cm^2 area each side is hanging in the middle of a room of 100 m^2 total surface area. The plate temperature and emissivity are respectively 800 K and 0.6 . The temperature and emissivity values for the surfaces of the room are 300 K and 0.3 respectively. Boltzmann's constant $\sigma = 5.67 \times 10^{-8} \text{ W/m}^2 \text{ K}^4$. The total heat loss from the two surfaces of the plate is

- (A) 13.66 W
- (B) 27.32 W
- (C) 27.87 W
- (D) 13.66 MW

Sol. 16

Option (B) is correct.



Given, for plate :

$$A_1 = 10 \text{ cm}^2 = 10 \times (10^{-2})^2 \text{ m}^2 = 10^{-3} \text{ m}^2, T_1 = 800 \text{ K}, \epsilon_1 = 0.6$$

For Room :

$$A_2 = 100 \text{ m}^2, T_2 = 300 \text{ K}, \epsilon_2 = 0.3$$

And $\sigma = 5.67 \times 10^{-8} \text{ W/m}^2 \text{ K}^4$

Total heat loss from one surface of the plate is given by,

$$(Q_{12}) = \frac{E_{b1} - E_{b2}}{\frac{(1 - \epsilon_1)}{A_1 \epsilon_1} + \frac{1}{A_1 F_{12}} + \frac{(1 - \epsilon_2)}{A_2 \epsilon_2}}$$

If small body is enclosed by a large enclosure, then $F_{12} = 1$ and from Stefan's Boltzman law $E_b = \sigma T^4$. So we get

$$\begin{aligned} (Q_{12}) &= \frac{\sigma (T_1^4 - T_2^4)}{\frac{1 - \varepsilon_1}{A_1 \varepsilon_1} + \frac{1}{A_1} + \frac{1 - \varepsilon_2}{A_2 \varepsilon_2}} \\ &= \frac{5.67 \times 10^{-8} [(800)^4 - (300)^4]}{\frac{1 - 0.6}{10^{-3} \times 0.6} + \frac{1}{10^{-3}} + \frac{1 - 0.3}{100 \times 0.3}} \\ &= \frac{22.765 \times 10^3}{666.66 + 1000 + 0.0233} = 13.66 \text{ W} \end{aligned}$$

Q_{12} is the heat loss by one surface of the plate. So, heat loss from the two surfaces is given by, $Q_{net} = 2 \times Q_{12} = 2 \times 13.66 = 27.32 \text{ W}$

Q. 17

GATE ME 2003
ONE MARK

For air with a relative humidity of 80%

- (A) the dry bulb temperature is less than the wet bulb temperature
- (B) the dew point temperature is less than wet bulb temperature
- (C) the dew point and wet bulb temperature are equal
- (D) the dry bulb and dew point temperature are equal

Sol. 17

Option (B) is correct.

We know that for saturated air, the relative humidity is 100% and the dry bulb temperature, wet bulb temperature and dew point temperature is same. But when air is not saturated, dew point temperature is always less than the wet bulb temperature.

$$\text{DPT} < \text{WBT}$$

Q. 18

GATE ME 2003
ONE MARK

For a spark ignition engine, the equivalence ratio (ϕ) of mixture entering the combustion chamber has values

- (A) $\phi < 1$ for idling and $\phi > 1$ for peak power conditions
- (B) $\phi > 1$ for both idling and peak power conditions
- (C) $\phi > 1$ for idling and $\phi < 1$ for peak power conditions
- (D) $\phi < 1$ for both idling and peak power conditions

Sol. 18

Option (B) is correct.

Equivalence Ratio or Fuel Air Ratio $\left(\frac{F}{A}\right)$

$$\begin{aligned} \phi &= \frac{\text{Actual Fuel - Air ratio}}{\text{stoichiometric Fuel air Ratio}} \\ &= \frac{\left(\frac{F}{A}\right)_{actual}}{\left(\frac{F}{A}\right)_{stoichiometric}} \end{aligned}$$

If $\phi = 1$, \Rightarrow stoichiometric (Chemically correct) Mixture.

If $\phi > 1$, \Rightarrow rich mixture.

If $\phi < 1$, \Rightarrow lean mixture.

Now, we can see from these three conditions that $\phi > 1$, for both idling & peak power conditions, so rich mixture is necessary.

Q. 19

GATE ME 2003
ONE MARK

- A diesel engine is usually more efficient than a spark ignition engine because
- (A) diesel being a heavier hydrocarbon, releases more heat per kg than gasoline
 - (B) the air standard efficiency of diesel cycle is higher than the Otto cycle, at a fixed compression ratio
 - (C) the compression ratio of a diesel engine is higher than that of an SI engine
 - (D) self ignition temperature of diesel is higher than that of gasoline

Sol. 19

Option (C) is correct.

The compression ratio of diesel engine ranges between 14 to 25 where as for S.I. engine between 6 to 12. Diesel Engine gives more power but efficiency of diesel engine is less than compare to the S.I. engine for same compression ratio.

Q. 20

GATE ME 2003
ONE MARK

- In Rankine cycle, regeneration results in higher efficiency because
- (A) pressure inside the boiler increases
 - (B) heat is added before steam enters the low pressure turbine
 - (C) average temperature of heat addition in the boiler increases
 - (D) total work delivered by the turbine increases

Sol. 20

Option (C) is correct.

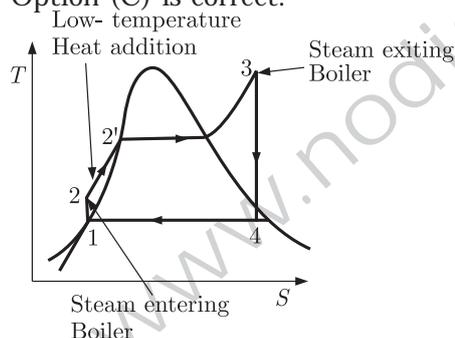


Fig : $T-s$ curve of simple Rankine cycle

From the observation of the $T-s$ diagram of the rankine cycle, it reveals that heat is transferred to the working fluid during process $2 - 2'$ at a relatively low temperature. This lowers the average heat addition temperature and thus the cycle efficiency.

To remove this remedy, we look for the ways to raise the temperature of the liquid leaving the pump (called the feed water) before it enters the boiler. One possibility is to transfer heat to the feed water from the expanding steam in a counter flow heat exchanger built into the turbine, that is, to use regeneration.

A practical regeneration process in steam power plant is accomplished by extracting steam from the turbine at various points. This steam is used to heat the feed water and the device where the feed water is heated by regeneration is called feed water heater. So, regeneration improves cycle efficiency by increasing the average temperature of heat addition in the boiler.

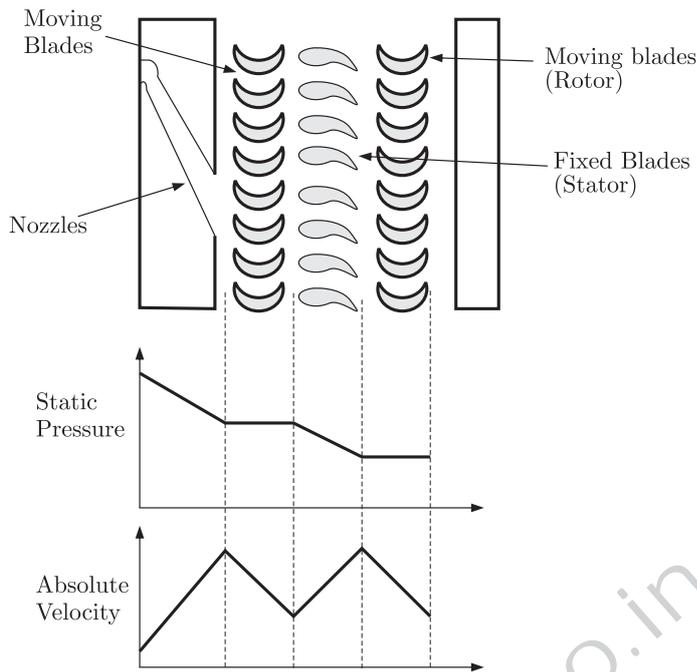
Q. 21

GATE ME 2003
ONE MARK

- Considering the variation of static pressure and absolute velocity in an impulse steam turbine, across one row of moving blades
- (A) both pressure and velocity decreases
 - (B) pressure decreases but velocity increases
 - (C) pressure remains constant, while velocity increases
 - (D) pressure remains constant, while velocity decreases

Sol. 21

Option (D) is correct.



Easily shows that the diagram that static pressure remains constant, while velocity decreases.

Q. 22

GATE ME 2003
ONE MARK

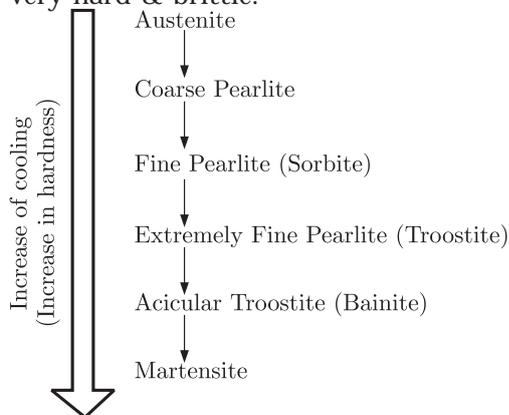
During heat treatment of steel, the hardness of various structures in increasing order is

- (A) martensite, fine pearlite, coarse pearlite, spherodite
- (B) fine pearlite, Martensite, spherodite, coarse pearlite
- (C) martensite, coarse pearlite, fine pearlite, spherodite
- (D) spherodite, coarse pearlite, fine pearlite, martensite

Sol. 22

Option (D) is correct.

Steel can be cooled from the high temperature region at a rate so high that the austenite does not have sufficient time to decompose into sorbite or troostite. In this case the austenite is transformed into martensite. Martensite is ferromagnetic, very hard & brittle.



So hardness is increasing in the order,
Spherodite → Coarse Pearlite → Fine Pearlite → Martensite

- Q. 23 Hardness of green sand mould increases with
GATE ME 2003 (A) increase in moisture content beyond 6 percent
ONE MARK (B) increase in permeability
(C) decrease in permeability
(D) increase in both moisture content and permeability

Sol. 23 Option (C) is correct.
Permeability or porosity of the moulding sand is the measure of its ability to permit air to flow through it.
So, hardness of green sand mould increases by restricted the air permitted in the sand i.e. decrease its permeability.

- Q. 24 In Oxyacetylene gas welding, temperature at the inner cone of the flame is around
GATE ME 2003 (A) 3500° C (B) 3200° C
ONE MARK (C) 2900° C (D) 2550° C

Sol. 24 Option (B) is correct.
In OAW, Acetylene (C_2H_2) produces higher temperature (in the range of 3200° C) than other gases, (which produce a flame temperature in the range of 2500° C) because it contains more available carbon and releases heat when its components (C & H) dissociate to combine with O_2 and burn.

- Q. 25 Cold working of steel is defined as working
GATE ME 2003 (A) at its recrystallisation temperature
ONE MARK (B) above its recrystallisation temperature
(C) below its recrystallisation temperature
(D) at two thirds of the melting temperature of the metal

Sol. 25 Option (C) is correct.
Cold forming or cold working can be defined as the plastic deforming of metals and alloys under conditions of temperature and strain rate.
Theoretically, the working temperature for cold working is below the recrystallization temperature of the metal/alloy (which is about one-half the absolute melting temperature.)

- Q. 26 Quality screw threads are produced by
GATE ME 2003 (A) thread milling
ONE MARK (B) thread chasing
(C) thread cutting with single point tool
(D) thread casting

Sol. 26 Option (D) is correct.
Quality screw threads are produced by only thread casting.
Quality screw threads are made by die-casting and permanent mould casting are very accurate and of high finish, if properly made.

- Q. 27 As tool and work are not in contact in EDM process
GATE ME 2003 (A) no relative motion occurs between them
ONE MARK (B) no wear of tool occurs
(C) no power is consumed during metal cutting
(D) no force between tool and work occurs

Sol. 27

Option (D) is correct.

In EDM, the thermal energy is employed to melt and vaporize tiny particles of work-material by concentrating the heat energy on a small area of the work-piece. A powerful spark, such as at the terminals of an automobile battery, will cause pitting or erosion of the metal at both anode & cathode. No force occurs between tool & work.

Q. 28

GATE ME 2003
ONE MARKThe dimensional limits on a shaft of $25h7$ are

- (A) 25.000, 25.021 mm (B) 25.000, 24.979 mm
(C) 25.000, 25.007 mm (D) 25.000, 24.993 mm

Sol. 28

Option (B) is correct.

Since 25 mm lies in the diameter step 18 & 30 mm, therefore the geometric mean diameter,

$$D = \sqrt{18 \times 30} = 23.24 \text{ mm}$$

We know that standard tolerance unit,

$$i(\text{microns}) = 0.45^3 \sqrt{D} + 0.001D$$

$$i = 0.45^3 \sqrt{23.24} + 0.001 \times 23.24 = 1.31 \text{ microns}$$

Standard tolerance for hole 'h' of grade 7 (IT7),

$$IT7 = 16i = 16 \times 1.31 = 20.96 \text{ microns}$$

$$\begin{aligned} \text{Hence, lower limit for shaft} &= \text{Upper limit of shaft} - \text{Tolerance} \\ &= 25 - 20.96 \times 10^{-3} \text{ mm} = 24.979 \text{ mm} \end{aligned}$$

Q. 29

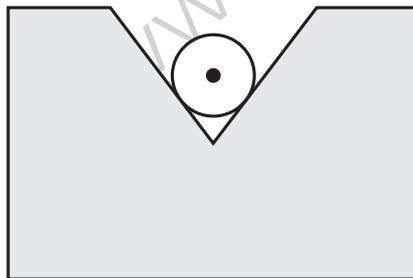
GATE ME 2003
ONE MARK

When a cylinder is located in a Vee-block, the number of degrees of freedom which are arrested is

- (A) 2 (B) 4
(C) 7 (D) 8

Sol. 29

Option (B) is correct.



We clearly see from the figure that cylinder can either revolve about x -axis or slide along x -axis & all the motions are restricted.

Hence, Number of degrees of freedom = 2 & movability includes the six degrees of freedom of the device as a whole, as the ground link were not fixed. So, 4 degrees of freedom are constrained or arrested.

Q. 30

GATE ME 2003
ONE MARK

The symbol used for Transport in work study is

- (A) \Rightarrow (B) T
(C) \square (D) ∇

Sol. 30

Option (A) is correct.

The symbol used for transport in work study is given by, \Rightarrow

Q. 31

Consider the system of simultaneous equations

GATE ME 2003
TWO MARK

$$\begin{aligned}x + 2y + z &= 6 \\2x + y + 2z &= 6 \\x + y + z &= 5\end{aligned}$$

This system has

- (A) unique solution
(C) no solution

- (B) infinite number of solutions
(D) exactly two solutions

Sol. 31

Option (C) is correct.

Given :

$$\begin{aligned}x + 2y + z &= 6 \\2x + y + 2z &= 6 \\x + y + z &= 5\end{aligned}$$

Comparing to $Ax = B$, we get

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 2 \\ 1 & 1 & 1 \end{bmatrix}, B = \begin{bmatrix} 6 \\ 6 \\ 5 \end{bmatrix}$$

Write the system of simultaneous equations in the form of Augmented matrix,

$$[A:B] = \begin{bmatrix} 1 & 2 & 1 & : & 6 \\ 2 & 1 & 2 & : & 6 \\ 1 & 1 & 1 & : & 5 \end{bmatrix}$$

Applying $R_2 \rightarrow R_2 - 2R_1$ and $R_3 \rightarrow 2R_3 - R_2$

$$= \begin{bmatrix} 1 & 2 & 1 & : & 6 \\ 0 & -3 & 0 & : & -6 \\ 0 & 1 & 0 & : & 4 \end{bmatrix}$$

Applying $R_3 \rightarrow 3R_3 + R_2$

$$= \begin{bmatrix} 1 & 2 & 1 & : & 6 \\ 0 & -3 & 0 & : & -6 \\ 0 & 0 & 0 & : & 6 \end{bmatrix}$$

It is a echelon form of matrix.

Since $\rho[A] = 2$ and $\rho[A:B] = 3$

$$\rho[A] \neq \rho[A:B]$$

So, the system has no solution and system is inconsistent.

Q. 32

The area enclosed between the parabola $y = x^2$ and the straight line $y = x$ isGATE ME 2003
TWO MARK

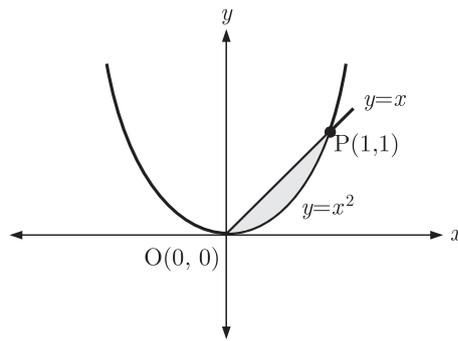
- (A) 1/8
(B) 1/6
(C) 1/3
(D) 1/2

Sol. 32

Option (B) is correct.

Given : $y = x^2$ & $y = x$.

The shaded area is show the area, which is bounded by the both curves (common area)



On solving given equation, we get the intersection points as,

$$\begin{aligned} y = x^2 \text{ put } y = x \\ x = x^2 \\ x^2 - x = 0 \\ x(x - 1) = 0 \\ x = 0, 1 \end{aligned}$$

Then from $y = x$

For $x = 0 \Rightarrow y = 0$

& $x = 1 \Rightarrow y = 1$

We can see that curve $y = x^2$ and $y = x$ intersects at point $(0, 0)$ and $(1, 1)$

So, the area bounded by both the curves is

$$\begin{aligned} A &= \int_{x=0}^{x=1} \int_{y=x}^{y=x^2} dy dx \\ &= \int_{x=0}^{x=1} dx \int_{y=x}^{y=x^2} dy = \int_{x=0}^{x=1} dx [y]_x^{x^2} \end{aligned}$$

After substituting the limit, we have

$$= \int_{x=0}^{x=1} (x^2 - x)$$

Integrating the equation, we get

$$\begin{aligned} &= \left[\frac{x^3}{3} - \frac{x^2}{2} \right]_0^1 = \frac{1}{3} - \frac{1}{2} = -\frac{1}{6} \\ &= \frac{1}{6} \text{ unit}^2 \end{aligned}$$

Area is never negative

Q. 33

GATE ME 2003
TWO MARK

The solution of the differential equation $\frac{dy}{dx} + y^2 = 0$ is

(A) $y = \frac{1}{x+c}$

(B) $y = -\frac{x^3}{3} + c$

(C) ce^x
linear

(D) unsolvable as equation is non-linear

Sol. 33

Option (A) is correct.

$$\frac{dy}{dx} + y^2 = 0$$

$$\frac{dy}{dx} = -y^2$$

$$-\frac{dy}{y^2} = dx$$

Integrating both the sides, we have

$$-\int \frac{dy}{y^2} = \int dx$$

$$y^{-1} = x + c$$

$$\frac{1}{y} = x + c \quad \Rightarrow y = \frac{1}{x+c}$$

Q. 34

GATE ME 2003
TWO MARKThe vector field is $\mathbf{F} = x\mathbf{i} - y\mathbf{j}$ (where \mathbf{i} and \mathbf{j} are unit vector) is

- (A) divergence free, but not irrotational
 (B) irrotational, but not divergence free
 (C) divergence free and irrotational
 (D) neither divergence free nor irrotational

Sol. 34

Option (C) is correct.

Given : $\mathbf{F} = x\mathbf{i} - y\mathbf{j}$

First Check divergence, for divergence,

$$\text{Grade } \mathbf{F} = \nabla \cdot \mathbf{F}$$

$$= \left[\frac{\partial}{\partial x} \mathbf{i} + \frac{\partial}{\partial y} \mathbf{j} + \frac{\partial}{\partial z} \mathbf{k} \right] \cdot [x\mathbf{i} - y\mathbf{j}] = 1 - 1 = 0$$

So we can say that \mathbf{F} is divergence free.Now we checking the irrationality. For irritation the curl $\mathbf{F} = 0$

$$\text{Curl } \mathbf{F} = \nabla \times \mathbf{F}$$

$$= \left[\frac{\partial}{\partial x} \mathbf{i} + \frac{\partial}{\partial y} \mathbf{j} + \frac{\partial}{\partial z} \mathbf{k} \right] \times [x\mathbf{i} - y\mathbf{j}]$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x & -y & 0 \end{vmatrix} = \mathbf{i}[0 - 0] - \mathbf{j}[0 - 0] + \mathbf{k}[0 - 0] = 0$$

So, vector field is irrotational. We can say that the vector field is divergence free and irrotational.

Q. 35

GATE ME 2003
TWO MARKLaplace transform of the function $\sin \omega t$ is

- (A) $\frac{s}{s^2 + \omega^2}$ (B) $\frac{\omega}{s^2 + \omega^2}$
 (C) $\frac{s}{s^2 - \omega^2}$ (D) $\frac{\omega}{s^2 - \omega^2}$

Sol. 35

Option (B) is correct.

Let $f(t) = \sin \omega t$

From the definition of Laplace transformation

$$\mathcal{L}[F(t)] = \int_0^{\infty} e^{-st} f(t) dt = \int_0^{\infty} e^{-st} \sin \omega t dt$$

$$= \int_0^{\infty} e^{-st} \left(\frac{e^{i\omega t} - e^{-i\omega t}}{2i} \right) dt$$

$$\sin \omega t = \frac{e^{i\omega t} - e^{-i\omega t}}{2i}$$

$$= \frac{1}{2i} \int_0^{\infty} (e^{-st} e^{i\omega t} - e^{-st} e^{-i\omega t}) dt$$

$$= \frac{1}{2i} \int_0^{\infty} [e^{(-s+i\omega)t} - e^{-(s+i\omega)t}] dt$$

On integrating above equation, we get

$$= \frac{1}{2i} \left[\frac{e^{(-s+i\omega)t}}{-s+i\omega} + \frac{e^{-(s+i\omega)t}}{(s+i\omega)} \right]_0^\infty = \frac{1}{2i} \left[\frac{e^{(-s+i\omega)t}}{-s+i\omega} - \frac{e^{-(s+i\omega)t}}{-(s+i\omega)} \right]_0^\infty$$

Substitute the limits, we get

$$\begin{aligned} &= \frac{1}{2i} \left[0 + 0 - \left(\frac{e^0}{(-s+i\omega)} + \frac{e^{-0}}{s+i\omega} \right) \right] \\ &= -\frac{1}{2i} \left[\frac{s+i\omega+i\omega-s}{(-s+i\omega)(s+i\omega)} \right] \\ &= -\frac{1}{2i} \times \frac{2i\omega}{(i\omega)^2 - s^2} = \frac{-\omega}{-\omega^2 - s^2} = \frac{\omega}{\omega^2 + s^2} \end{aligned}$$

Alternate :

From the definition of Laplace transformation

$$\mathcal{L}[F(t)] = \int_0^\infty e^{-st} \sin \omega t dt$$

We know $\int e^{at} \sin bt dt = \frac{e^{at}}{a^2 + b^2} [a \sin bt - b \cos bt]$ $\left(\begin{array}{l} a = -s \text{ and} \\ b = \omega \end{array} \right)$

Then, $\mathcal{L}[\sin \omega t] = \left[\frac{e^{-st}}{s^2 + \omega^2} (-s \sin \omega t - \omega \cos \omega t) \right]_0^\infty$

$$\begin{aligned} &= \left[\frac{e^{-\infty}}{s^2 + \omega^2} (-s \sin \infty - \omega \cos \infty) \right] - \left[\frac{e^{-0}}{s^2 + \omega^2} (-s \sin 0 - \omega \cos 0) \right] \\ &= 0 - \frac{1}{s^2 + \omega^2} [0 - \omega] = -\frac{1}{s^2 + \omega^2} (-\omega) \end{aligned}$$

$$\mathcal{L}[\sin \omega t] = \frac{\omega}{s^2 + \omega^2}$$

Q. 36

GATE ME 2003
TWO MARK

A box contains 5 black and 5 red balls. Two balls are randomly picked one after another from the box, without replacement. The probability for balls being red is

- (A) 1/90 (B) 1/2
(C) 19/90 (D) 2/9

Sol. 36

Option (D) is correct.

Given : black balls = 5, Red balls = 5, Total balls = 10

Here, two balls are picked from the box randomly one after the other without replacement. So the probability of both the balls are red is

$$\begin{aligned} P &= \frac{{}^5C_0 \times {}^5C_2}{{}^{10}C_2} & {}^n C_r &= \frac{n!}{r!(n-r)!} \\ &= \frac{5! \times 5!}{10!} \times \frac{5!}{3!2!} = \frac{1 \times 10}{45} = \frac{10}{45} = \frac{2}{9} \end{aligned}$$

Common Data For Q. Alternate method

Given : Black balls = 5,
Red balls = 5
Total balls = 10

The probability of drawing a red ball,

$$P_1 = \frac{5}{10} = \frac{1}{2}$$

Ball is not replaced, then box contains 9 balls.

So, probability of drawing the next red ball from the box.

$$P_2 = \frac{4}{9}$$

Hence, probability for both the balls being red is,

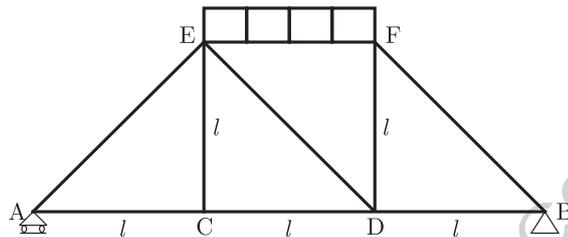
$$P = P_1 \times P_2$$

$$P = \frac{1}{2} \times \frac{4}{9} = \frac{2}{9}$$

Q. 37

GATE ME 2003
ONE MARK

A truss consists of horizontal members (AC, CD, DB and EF) and vertical members (CE and DF) having length l each. The members AE, DE and BF are inclined at 45° to the horizontal. For the uniformly distributed load “ p ” per unit length on the member EF of the truss shown in figure given below, the force in the member CD is



- (A) $\frac{pl}{2}$ (B) pl
(C) 0 (D) $\frac{2pl}{3}$

Sol. 37

Option (A) is correct.

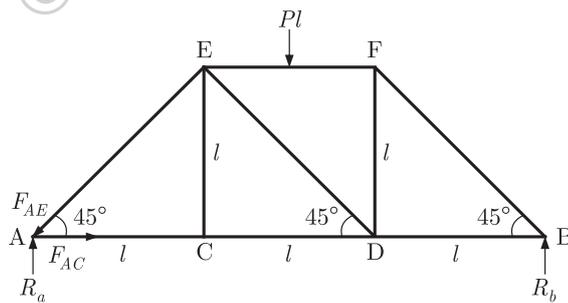
Given : $AC = CD = DB = EF = CE = DF = l$

At the member EF uniform distributed load is acting, the U.D.L. is given as “ p ” per unit length.

So, the total load acting on the element EF of length l

$$= \text{Load per unit length} \times \text{Total length of element}$$

$$= p \times l = pl$$



This force acting at the mid point of EF .

We made the FBD of the object. At A & B reactions are acting because of the roller supports, in the upward direction.

In equilibrium condition,

Upward force = Downward forces

$$R_a + R_b = pl \quad \dots (i)$$

And take the moment about point A ,

$$pl \times \left(l + \frac{l}{2} \right) = R_b (l + l + l)$$

$$pl \times \frac{3}{2}l = R_b \times 3l \Rightarrow R_b = \frac{pl}{2}$$

Substitute the value of R_b in equation (i), we get

$$R_a + \frac{pl}{2} = pl$$

$$R_a = pl - \frac{pl}{2} = \frac{pl}{2}$$

So, $R_a = R_b = \frac{pl}{2}$

At point A we use the principal of resolution of forces in the y -direction, $\sum F_y = 0$

$$F_{AE} \sin 45^\circ = R_a = \frac{pl}{2}$$

$$F_{AE} = \frac{pl}{2} \times \frac{1}{\sin 45^\circ} = \frac{pl}{2} \times \sqrt{2} = \frac{pl}{\sqrt{2}}$$

And $F_{AC} = F_{AE} \cos 45^\circ = \frac{pl}{\sqrt{2}} \times \frac{1}{\sqrt{2}} = \frac{pl}{2}$

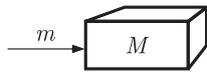
At C , No external force is acting. So,

$$F_{AC} = \frac{pl}{2} = F_{CD}$$

Q. 38

GATE ME 2003
ONE MARK

A bullet of mass " m " travels at a very high velocity v (as shown in the figure) and gets embedded inside the block of mass " M " initially at rest on a rough horizontal floor. The block with the bullet is seen to move a distance " s " along the floor. Assuming μ to be the coefficient of kinetic friction between the block and the floor and " g " the acceleration due to gravity what is the velocity v of the bullet ?



- (A) $\frac{M+m}{m} \sqrt{2\mu gs}$ (B) $\frac{M-m}{m} \sqrt{2\mu gs}$
 (C) $\frac{\mu(M+m)}{m} \sqrt{2\mu gs}$ (D) $\frac{M}{m} \sqrt{2\mu gs}$

Sol. 38

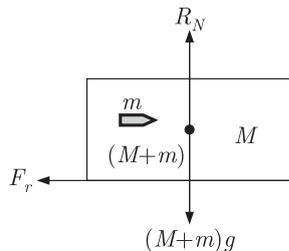
Option (A) is correct.

Given : Mass of bullet = m
 Mass of block = M
 Velocity of bullet = v

Coefficient of Kinematic friction = μ

Let, Velocity of system (Block + bullet) after striking the bullet = u

We have to make the FBD of the box after the bullet strikes,



Friction Force (Retardation) = F_r

By Applying principal of conservation of linear momentum, $\frac{dP}{dt} = 0$ or $P = mV = \text{constant}$.

So, $mv = (M + m)u$

$$u = \frac{mv}{M + m} \quad \dots (i)$$

And, from the FBD the vertical force (reaction force),

$$R_N = (M + m)g$$

$$F_r = \mu R_N = \mu(M + m)g$$

Frictional retardation $a = \frac{-F_r}{(m + M)} = \frac{-\mu(M + m)g}{M + m} = -\mu g \quad \dots (ii)$

Negative sign show the retardation of the system (acceleration in opposite direction).

From the Newton's third law of motion,

$$V_f^2 = u^2 + 2as$$

V_f = Final velocity of system (block + bullet)

= 0

$$u^2 + 2as = 0$$

$$u^2 = -2as$$

$$u^2 = -2 \times (-\mu g) \times s = 2\mu gs$$

From equation (ii)

Substitute the value of u from equation (i), we get

$$\left(\frac{mv}{M + m}\right)^2 = 2\mu gs$$

$$\frac{m^2 v^2}{(M + m)^2} = 2\mu gs$$

$$v^2 = \frac{2\mu gs(M + m)^2}{m^2}$$

$$v = \sqrt{2\mu gs} \times \left(\frac{M + m}{m}\right) = \frac{M + m}{m} \sqrt{2\mu gs}$$

Q. 39

GATE ME 2003
TWO MARK

A simply supported laterally loaded beam was found to deflect more than a specified value. Which of the following measures will reduce the deflection ?

- (A) Increase the area moment of inertia
- (B) Increase the span of the beam
- (C) Select a different material having lesser modulus of elasticity
- (D) Magnitude of the load to be increased

Sol. 39

Option (A) is correct.

We know, differential equation of flexure for the beam is,

$$EI \frac{d^2 y}{dx^2} = M \Rightarrow \frac{d^2 y}{dx^2} = \frac{M}{EI}$$

Integrating both sides,

$$\frac{dy}{dx} = \frac{1}{EI} \int M dx = \frac{1}{EI} Mx + c_1$$

Again integrating,

$$y = \frac{1}{EI} \left(\frac{Mx^2}{2}\right) + c_1 x + c_2 \quad \dots (i)$$

Where, y gives the deflection at the given point.

It is easily shown from the equation (i), If we increase the value of E & I , then deflection reduces.

Q. 40

GATE ME 2003
TWO MARK

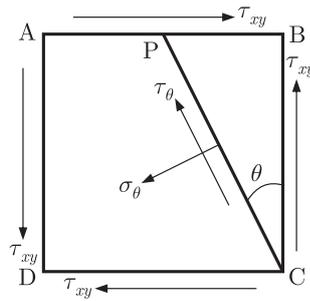
A shaft subjected to torsion experiences a pure shear stress τ on the surface. The maximum principal stress on the surface which is at 45° to the axis will have a value

- (A) $\tau \cos 45^\circ$ (B) $2\tau \cos 45^\circ$
(C) $\tau \cos^2 45^\circ$ (D) $2\tau \sin 45^\circ \cos 45^\circ$

Sol. 40

Option (D) is correct.

Given figure shows stresses on an element subjected to pure shear.



Let consider a element to which shear stress have been applied to the sides AB and DC .

Complementary stress of equal value but of opposite effect are then setup on sides AD and BC in order to prevent rotation of the element. So, applied and complementary shears are represented by symbol τ_{xy} .

Consider the equilibrium of portion PBC . Resolving normal to PC assuming unit depth.

$$\begin{aligned}\sigma_\theta \times PC &= \tau_{xy} \times BC \sin \theta + \tau_{xy} \times PB \cos \theta \\ &= \tau_{xy} \times PC \cos \theta + \tau_{xy} \times PC \sin \theta \cos \theta \\ &= \tau_{xy} (2 \sin \theta \cos \theta) \times PC \\ \sigma_\theta &= 2\tau_{xy} \sin \theta \cos \theta\end{aligned}$$

The maximum value of σ_θ is τ_{xy} when $\theta = 45^\circ$.

$$\sigma_\theta = 2\tau \sin 45^\circ \cos 45^\circ \quad \text{Given } (\tau_{xy} = \tau)$$

Q. 41

GATE ME 2003
TWO MARK

For a certain engine having an average speed of 1200 rpm, a flywheel approximated as a solid disc, is required for keeping the fluctuation of speed within 2% about the average speed. The fluctuation of kinetic energy per cycle is found to be 2 kJ. What is the least possible mass of the flywheel if its diameter is not to exceed 1 m ?

- (A) 40 kg (B) 51 kg
(C) 62 kg (D) 73 kg

Sol. 41

Option (B) is correct.

Given $N = 1200$ rpm, $\Delta E = 2$ kJ = 2000 J, $D = 1$ m, $C_s = 0.02$

Mean angular speed of engine,

$$\begin{aligned}\omega &= \frac{2\pi N}{60} \\ &= \frac{2 \times 3.14 \times 1200}{60} \\ &= 125.66 \text{ rad/sec}\end{aligned}$$

Fluctuation of energy of the flywheel is given by,

$$\Delta E = I\omega^2 C_s = \frac{1}{2} mR^2 \omega^2 C_s \text{ For solid disc } I = \frac{mR^2}{2}$$

$$m = \frac{2\Delta E}{R^2 \omega^2 C_s} \quad \dots (i)$$

Substitute the values in equation (i),

$$\begin{aligned} &= \frac{2 \times 2000}{\left(\frac{1}{2}\right)^2 \times (125.66)^2 \times 0.02} \\ &= \frac{4 \times 2 \times 2000}{(125.66)^2 \times 0.02} = 50.66 \text{ kg} \approx 51 \text{ kg} \end{aligned}$$

Q. 42

GATE ME 2003
TWO MARK

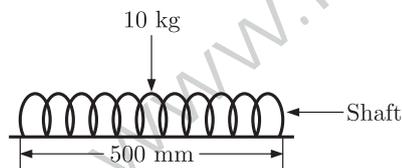
A flexible rotor-shaft system comprises of a 10 kg rotor disc placed in the middle of a mass-less shaft of diameter 30 mm and length 500 mm between bearings (shaft is being taken mass-less as the equivalent mass of the shaft is included in the rotor mass) mounted at the ends. The bearings are assumed to simulate simply supported boundary conditions. The shaft is made of steel for which the value of E 2.1×10^{11} Pa. What is the critical speed of rotation of the shaft ?

- (A) 60 Hz (B) 90 Hz
(C) 135 Hz (D) 180 Hz

Sol. 42

Option (B) is correct.

Given $m = 10$ kg, $d = 30$ mm = 0.03 m, $l = 500$ mm = 0.5 m, E_{shaft}
= 2.1×10^{11} Pa



We know that, static deflection due to 10 kg of Mass at the centre is given by,

$$\delta = \frac{Wl^3}{48EI} = \frac{mgl^3}{48EI} \quad \dots (i)$$

The moment of inertia of the shaft,

$$I = \frac{\pi}{64} d^4 = \frac{\pi}{64} (0.03)^4 = 3.974 \times 10^{-8} \text{ m}^4 \dots (ii)$$

Substitute values in equation (i), we get

$$\begin{aligned} \delta &= \frac{10 \times 9.81 \times (0.5)^3}{48 \times 2.1 \times 10^{11} \times 3.974 \times 10^{-8}} \\ &= \frac{12.2625}{400.58 \times 10^3} = 3.06 \times 10^{-5} \text{ m} \end{aligned}$$

If ω_c is the critical or whirling speed in r.p.s. then,

$$\begin{aligned} \omega_c &= \sqrt{\frac{g}{\delta}} \Rightarrow 2\pi f_c = \sqrt{\frac{g}{\delta}} \\ f_c &= \frac{1}{2\pi} \sqrt{\frac{g}{\delta}} = \frac{1}{2 \times 3.14} \sqrt{\frac{9.81}{3.06 \times 10^{-5}}} \\ &= \frac{1}{6.28} \sqrt{\frac{9.81}{30.6 \times 10^{-6}}} = 90.16 \text{ Hz} \approx 90 \text{ Hz} \end{aligned}$$

Q. 43

GATE ME 2003
TWO MARK

Square key of side " $d/4$ " each and length ' l ' is used to transmit torque " T " from the shaft of diameter " d " to the hub of a pulley. Assuming the length of the key to be equal to the thickness of pulley, the average shear stress developed in the key is given by

- (A) $\frac{4T}{ld}$ (B) $\frac{16T}{ld^2}$
(C) $\frac{8T}{ld^2}$ (D) $\frac{16T}{\pi d^3}$

Sol. 43

Option (C) is correct.

Given : Diameter of shaft = d
Torque transmitted = T
Length of the key = l

We know that, width and thickness of a square key are equal.

i.e. $w = t = \frac{d}{4}$

Force acting on circumference of shaft

$$F = \frac{T}{r} = \frac{2T}{d} \quad (r = d/2)$$

Shearing Area, $A = \text{width} \times \text{length} = \frac{d}{4} \times l = \frac{dl}{4}$

Average shear stress, $\tau = \frac{\text{Force}}{\text{shearing Area}} = \frac{2T/d}{dl/4} = \frac{8T}{ld^2}$

Q. 44

GATE ME 2003
TWO MARK

In a band brake the ratio of tight side band tension to the tension on the slack side is 3. If the angle of overlap of band on the drum is 180° , the coefficient of friction required between drum and the band is

- (A) 0.20 (B) 0.25
(C) 0.30 (D) 0.35

Sol. 44

Option (D) is correct.

Let, $T_1 \rightarrow$ Tension in the tight side of the band,
 $T_2 \rightarrow$ Tension in the slack side of the band
 $\theta \rightarrow$ Angle of lap of the band on the drum

Given : $\frac{T_1}{T_2} = 3$, $\theta = 180^\circ = \frac{\pi}{180} \times 180 = \pi$ radian

For band brake, the limiting ratio of the tension is given by the relation,

$$\frac{T_1}{T_2} = e^{\mu\theta} \text{ or } 2.3 \log\left(\frac{T_1}{T_2}\right) = \mu\theta$$

$$2.3 \times \log(3) = \mu \times \pi$$

$$2.3 \times 0.4771 = \mu \times 3.14$$

$$\mu = \frac{1.09733}{3.14} = 0.349 \approx 0.35$$

Q. 45

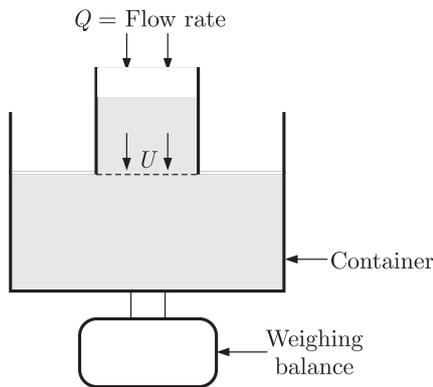
GATE ME 2003
TWO MARK

A water container is kept on a weighing balance. Water from a tap is falling vertically into the container with a volume flow rate of Q ; the velocity of the water when it hits the water surface is U . At a particular instant of time the total mass of the container and water is m . The force registered by the weighing balance at this instant of time is

- (A) $mg + \rho QU$ (B) $mg + 2\rho QU$
(C) $mg + \rho QU^2/2$ (D) $\rho QU^2/2$

Sol. 45

Option (A) is correct.



Given : Flow rate = Q

Velocity of water when it strikes the water surface = U

Total Mass (container + water) = m

Force on weighing balance due to water strike = Change in momentum

$$\Delta P = \text{Initial Momentum} - \text{Final momentum}$$

$$= \rho Q U - \rho Q (0) = \rho Q U \text{ Final velocity is zero}$$

Weighing balance also experience the weight of the container & water.

So, Weight of container & water = mg

Therefore, total force on weighing Balance = $\rho Q U + mg$

Q. 46

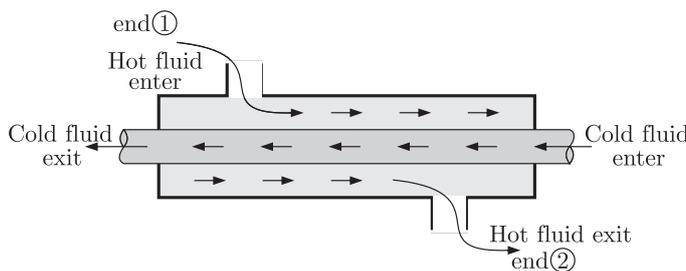
GATE ME 2003
TWO MARK

In a counter flow heat exchanger, for the hot fluid the heat capacity = 2 kJ/kgK, mass flow rate = 5 kg/s, inlet temperature = 150°C, outlet temperature = 100°C. For the cold fluid, heat capacity = 4 kJ/kgK, mass flow rate = 10 kg/s, inlet temperature = 20°C. Neglecting heat transfer to the surroundings, the outlet temperature of the cold fluid in °C is

- (A) 7.5
- (B) 32.5
- (C) 45.5
- (D) 70.0

Sol. 46

Option (B) is correct.



In counter flow, hot fluid enters at the point 1 & exits at the point 2 or cold fluid enter at the point 2 & exit at the point 1.

Given : for hot fluid,

$$c_h = 2 \text{ kJ/kg K}, \dot{m}_h = 5 \text{ kg/sec}, t_{h1} = 150^\circ \text{C}, t_{h2} = 100^\circ \text{C}$$

and for cold fluid,

$$c_c = 4 \text{ kJ/kg K}, \dot{m}_c = 10 \text{ kg/sec}, t_{c2} = 20^\circ \text{C}, t_{c1} = ?$$

From the energy balance,

Heat transferred by the hot fluid = Heat gain by the cold fluid

$$\dot{m}_h c_h (t_{h1} - t_{h2}) = \dot{m}_c c_c (t_{c1} - t_{c2})$$

$\frac{\pi}{4} D_t^2 V_1 = \frac{\pi}{4} D^2 U$ $V_2 = U$. Let at point (1) velocity = V_1

$$V_1 = \left(\frac{D}{D_t}\right)^2 \times U \quad \dots (ii)$$

Substitute the value of V_1 from equation (ii) into the equation (i),

$$p_1 - p_2 = \frac{\rho}{2} \left[U^2 - \left(\frac{D}{D_t}\right)^4 U^2 \right] = \frac{\rho}{2} U^2 \left[1 - \left(\frac{D}{D_t}\right)^4 \right] \quad \dots (iii)$$

From the figure, we have

Spring force = Pressure force due to air

$$\begin{aligned} -kx &= A_s(p_1 - p_2) = \frac{\pi}{4} D_s^2 \times (p_1 - p_2) \\ &= \frac{\pi}{4} D_s^2 \times \frac{\rho}{2} U^2 \left[1 - \left(\frac{D}{D_t}\right)^4 \right] \text{ From equation (iii)} \\ kx &= \frac{\pi}{8} D_s^2 \rho U^2 \left[\left(\frac{D}{D_t}\right)^4 - 1 \right] \\ x &= \frac{\rho U^2}{8k} \left[\left(\frac{D}{D_t}\right)^4 - 1 \right] \pi D_s^2 \end{aligned}$$

Q. 48

GATE ME 2003
TWO MARK

Consider a laminar boundary layer over a heated flat plate. The free stream velocity is U_∞ . At some distance x from the leading edge the velocity boundary layer thickness is δ_v and the thermal boundary layer thickness is δ_T . If the Prandtl number is greater than 1, then

- (A) $\delta_v > \delta_T$
- (B) $\delta_T > \delta_v$
- (C) $\delta_v \approx \delta_T \sim (U_\infty x)^{-1/2}$
- (D) $\delta_v \approx \delta_T \sim x^{-1/2}$

Sol. 48

Option (A) is correct.

The non-dimensional Prandtl Number for thermal boundary layer is given by,

$$\frac{\delta_v}{\delta_T} = (\text{Pr})^{1/3}$$

- (i) When $\text{Pr} = 1$ $\delta_v = \delta_T$
- (ii) When $\text{Pr} > 1$ $\delta_v > \delta_T$
- (iii) When $\text{Pr} < 1$ $\delta_v < \delta_T$

So for $\text{Pr} > 1$, $\delta_v > \delta_T$

Q. 49

GATE ME 2003
TWO MARK

Considering the relationship $Tds = dU + pdv$ between the entropy (s), internal energy (U), pressure (p), temperature (T) and volume (v), which of the following statements is correct ?

- (A) It is applicable only for a reversible process
- (B) For an irreversible process, $Tds > dU + pdv$
- (C) It is valid only for an ideal gas
- (D) It is equivalent to Ist law, for a reversible process

Sol. 49

Option (D) is correct.

The Tds equation considering a pure, compressible system undergoing an internally reversible process.

From the first law of thermodynamics

$$(\delta Q)_{rev} = dU + (\delta W)_{rev} \quad \dots (i)$$

By definition of simple compressible system, the work is

$$(\delta W)_{rev} = p d\nu$$

And entropy changes in the form of

$$ds = \left(\frac{\delta Q}{T}\right)_{rev}$$

$$(\delta Q)_{rev} = T ds$$

From equation (i), we get

$$T ds = dU + p d\nu$$

This equation is equivalent to the F^{st} law, for a reversible process.

Q. 50

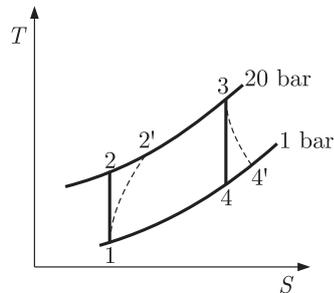
GATE ME 2003
TWO MARK

In a gas turbine, hot combustion products with the specific heats $c_p = 0.98$ kJ/kgK, and $c_v = 0.7538$ kJ/kgK enters the turbine at 20 bar, 1500 K exit at 1 bar. The isentropic efficiency of the turbine is 0.94. The work developed by the turbine per kg of gas flow is

- (A) 689.64 kJ/kg (B) 794.66 kJ/kg
(C) 1009.72 kJ/kg (D) 1312.00 kJ/kg

Sol. 50

Option (A) is correct.



Given : $c_p = 0.98$ kJ/kgK, $\eta_{isen} = 0.94$, $c_v = 0.7538$ kJ/kgK, $T_3 = 1500$ K
 $p_3 = 20$ bar = 20×10^5 N/m², $p_4 = 1$ bar = 1×10^5 N/m²

$$\gamma = \frac{c_p}{c_v} = \frac{0.98}{0.7538} = 1.3$$

Apply general Equation for the reversible adiabatic process between point 3 and 4 in T - s diagram,

$$\left(\frac{T_3}{T_4}\right) = \left(\frac{p_3}{p_4}\right)^{\frac{\gamma-1}{\gamma}}$$

$$\frac{1500}{T_4} = \left(\frac{20 \times 10^5}{1 \times 10^5}\right)^{\frac{1.3-1}{1.3}} = (20)^{\frac{0.3}{1.3}}$$

$$T_4 = \frac{1500}{(20)^{\frac{0.3}{1.3}}} = 751.37 \text{ K}$$

And $\eta_{isentropic} = \frac{\text{Actual output}}{\text{Ideal output}} = \frac{T_3 - T_4'}{T_3 - T_4}$

$$0.94 = \frac{1500 - T_4'}{1500 - 751.37}$$

$$0.94 \times 748.63 = 1500 - T_4'$$

$$T_4' = 1500 - 703.71 = 796.3 \text{ K}$$

Turbine work, $W_t = c_p(T_3 - T_4')$
 $= 0.98(1500 - 796.3) = 698.64$ kJ/kg

Q. 51

GATE ME 2003
TWO MARK

An automobile engine operates at a fuel air ratio of 0.05, volumetric efficiency of 90% and indicated thermal efficiency of 30%. Given that the calorific value of the fuel is 45 MJ/kg and the density of air at intake is 1 kg/m³, the indicated mean

effective pressure for the engine is

- (A) 6.075 bar (B) 6.75 bar
(C) 67.5 bar (D) 243 bar

Sol. 51

Option (A) is correct.

$$\text{Given : } \phi = \frac{F}{A} = \frac{m_f}{m_a} = 0.05, \eta_v = 90\% = 0.90, \eta_{ith} = 30\% = 0.3$$

$$CV_{fuel} = 45 \text{ MJ/kg}, \rho_{air} = 1 \text{ kg/m}^3$$

We know that, volumetric efficiency is given by,

$$\eta_v = \frac{\text{Actual Volume}}{\text{Swept Volume}} = \frac{v_{ac}}{v_s}$$

$$v_{ac} = \eta_v v_s = 0.90 v_s \quad \dots (i)$$

$$\text{Mass of air, } m_a = \rho_{air} \times v_{ac} = 1 \times 0.9 v_s = 0.9 v_s$$

$$m_f = 0.05 \times m_a = 0.045 v_s$$

$$\eta_{ith} = \frac{I.P.}{m_f \times CV} = \frac{p_{im} LAN}{m_f \times CV} \quad I.P. = p_{im} LAN$$

$$p_{im} = \frac{\eta_{ith} \times m_f \times CV}{LAN} \quad LAN = v_s$$

$$\frac{0.30 \times 0.045 \times v_s \times 45 \times 10^6}{v_s} = 0.6075 \times 10^6$$

$$= 6.075 \times 10^5 \text{ Pa} = 6.075 \text{ bar} \quad 1 \text{ bar} = 10^5 \text{ Pa}$$

Q. 52

GATE ME 2003
TWO MARK

For an engine operating on air standard Otto cycle, the clearance volume is 10% of the swept volume. The specific heat ratio of air is 1.4. The air standard cycle efficiency is

- (A) 38.3% (B) 39.8%
(C) 60.2% (D) 61.7%

Sol. 52

Option (D) is correct.

$$\text{Given: } v_c = 10\% \text{ of } v_s = 0.1 v_s$$

$$\frac{v_s}{v_c} = \frac{1}{0.1} = 10$$

And specific heat ratio $c_p/c_v = \gamma = 1.4$

We know compression ratio,

$$r = \frac{v_T}{v_c} = \frac{v_c + v_s}{v_c} = 1 + \frac{v_s}{v_c} \\ = 1 + 10 = 11$$

Efficiency of Otto cycle,

$$\eta_{Otto} = 1 - \frac{1}{(r)^{\gamma-1}} = 1 - \frac{1}{(11)^{1.4-1}} \\ = 1 - \frac{1}{(11)^{0.4}} = 1 - 0.3832 = 0.6168 \approx 61.7\%$$

Q. 53

GATE ME 2003
TWO MARK

A centrifugal pump running at 500 rpm and at its maximum efficiency is delivering a head of 30 m at a flow rate of 60 litres per minute. If the rpm is changed to 1000, then the head H in metres and flow rate Q in litres per minute at maximum efficiency are estimated to be

- (A) $H = 60, Q = 120$ (B) $H = 120, Q = 120$
(C) $H = 60, Q = 480$ (D) $H = 120, Q = 30$

Sol. 53

Option (B) is correct.

$$\text{Given : } N_1 = 500 \text{ rpm, } H_1 = 30 \text{ meter, } N_2 = 1000 \text{ rpm, } Q_1 = 60 \text{ litres per minute}$$

From the general relation,

$$U = \frac{\pi DN}{60} = \sqrt{2gH}$$

$$DN \propto \sqrt{H} \Rightarrow N \propto \frac{\sqrt{H}}{D}$$

Centrifugal pump is used for both the cases. So $D_1 = D_2$

$$N \propto \sqrt{H}$$

$$\frac{H_1}{H_2} = \frac{N_1^2}{N_2^2}$$

$$H_2 = \frac{N_2^2}{N_1^2} \times H_1 = \frac{(1000)^2}{(500)^2} \times 30 = 120 \text{ m}$$

The specific speed will be constant for centrifugal pump & relation is given by,

$$N_s = \frac{N\sqrt{Q}}{H^{3/4}} = \text{Constant}$$

So, $\frac{N_1\sqrt{Q_1}}{H_1^{3/4}} = \frac{N_2\sqrt{Q_2}}{H_2^{3/4}}$ For both cases

$$\begin{aligned} \sqrt{Q_2} &= \frac{N_1}{N_2} \times \left(\frac{H_2}{H_1}\right)^{3/4} \times \sqrt{Q_1} = \frac{500}{1000} \times \left(\frac{120}{30}\right)^{3/4} \times \sqrt{60} \\ &= \frac{1}{2} \times (2)^{3/2} \times \sqrt{60} \end{aligned}$$

Squaring both sides

$$Q_2 = \frac{1}{4} \times 8 \times 60 = 120 \text{ litre/min}$$

Alternate :

From unit quantities

Unit speed

$$N_u = \frac{N_1}{\sqrt{H_1}} = \frac{N_2}{\sqrt{H_2}}$$

$$\frac{N_1}{\sqrt{H_1}} = \frac{N_2}{\sqrt{H_2}}$$

$$\sqrt{H_2} = \frac{N_2\sqrt{H_1}}{N_1}$$

or $H_2 = \frac{N_2^2 \times H_1}{N_1^2} = \frac{(1000)^2 \times 30}{(500)^2}$

$$H_2 = 120 \text{ m}$$

Unit discharge

$$Q_u = \frac{Q_1}{\sqrt{H_1}} = \frac{Q_2}{\sqrt{H_2}}$$

$$\frac{Q_1}{\sqrt{H_1}} = \frac{Q_2}{\sqrt{H_2}}$$

or $Q_2 = \frac{Q_1\sqrt{H_2}}{\sqrt{H_1}} = \frac{60 \times \sqrt{120}}{\sqrt{30}}$

$$Q_2 = 120 \text{ litre/min}$$

Q. 54

GATE ME 2003
TWO MARK

Hardness of steel greatly improves with

(A) annealing

(B) cyaniding

(C) normalizing

(D) tempering

Sol. 54

Option (B) is correct.

Hardness is greatly depend on the carbon content present in the steel.

Cyaniding is case-hardening with powered potassium cyanide or potassium ferrocyanide mixed with potassium bichromate, substituted for carbon. Cyaniding produces a thin but very hard case in a very short time.

Q. 55

GATE ME 2003
TWO MARK

With a solidification factor of $0.97 \times 10^6 \text{ s/m}^2$, the solidification time (in seconds) for a spherical casting of 200 mm diameter is

- (A) 539
- (B) 1078
- (C) 4311
- (D) 3233

Sol. 55

Option (B) is correct.

Given : $q = 0.97 \times 10^6 \text{ s/m}^2$, $D = 200 \text{ mm} = 0.2 \text{ m}$

From the chaine's relation solidification time, $T = q \left(\frac{V}{A} \right)^2$

Volume $V = \frac{4}{3} \pi R^3$

Surface Area $A = 4\pi R^2$

So,
$$T = 0.97 \times 10^6 \left(\frac{\frac{4}{3} \pi R^3}{4\pi R^2} \right)^2 = 0.97 \times 10^6 \left(\frac{R}{3} \right)^2$$

$$= \frac{0.97}{9} \times 10^6 \left(\frac{0.2}{2} \right)^2 = 1078 \text{ sec}$$

Q. 56

GATE ME 2003
TWO MARK

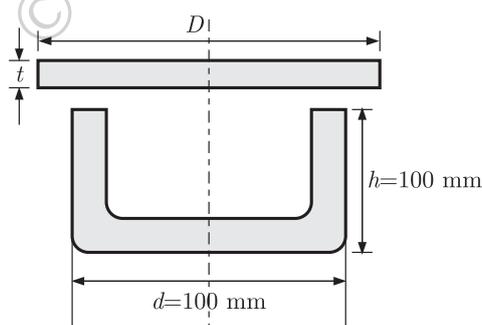
A shell of 100 mm diameter and 100 mm height with the corner radius of 0.4 mm is to be produced by cup drawing. The required blank diameter is

- (A) 118 mm
- (B) 161 mm
- (C) 224 mm
- (D) 312 mm

Sol. 56

Option (C) is correct.

Given : $d = 100 \text{ mm}$, $h = 100 \text{ mm}$, $R = 0.4 \text{ mm}$



Here we see that $d > 20r$

If $d \geq 20r$, blank diameter in cup drawing is given by,

$$D = \sqrt{d^2 + 4dh}$$

Where,

D = diameter of flat blank

d = diameter of finished shell

h = height of finished shell

Substitute the values, we get

$$D = \sqrt{(100)^2 + 4 \times 100 \times 100} = \sqrt{50000}$$

$$= 223.61 \text{ mm} \simeq 224 \text{ mm}$$

- Q. 57 A brass billet is to be extruded from its initial diameter of 100 mm to a final diameter of 50 mm. The working temperature of 700°C and the extrusion constant is 250 MPa. The force required for extrusion is
 (A) 5.44 MN (B) 2.72 MN
 (C) 1.36 MN (D) 0.36 MN

Sol. 57 Option (B) is correct.
 Given : $d_i = 100\text{ mm}$, $d_f = 50\text{ mm}$, $T = 700^\circ\text{C}$, $k = 250\text{ MPa}$
 Extrusion force is given by,

$$F_e = kA_i \ln\left(\frac{A_i}{A_f}\right)$$

$$= k\frac{\pi}{4}d_i^2 \ln\left(\frac{\frac{\pi}{4}d_i^2}{\frac{\pi}{4}d_f^2}\right) = k\frac{\pi}{4}d_i^2 \ln\left(\frac{d_i}{d_f}\right)^2$$

Substitute the values, we get

$$F_e = 250 \times \frac{\pi}{4} (0.1)^2 \ln\left(\frac{0.1}{0.05}\right)^2$$

$$= 1.96 \ln 4 = 2.717\text{ MN} \approx 2.72\text{ MN}$$

- Q. 58 A metal disc of 20 mm diameter is to be punched from a sheet of 2 mm thickness. The punch and the die clearance is 3%. The required punch diameter is
 (A) 19.88 mm (B) 19.84 mm
 (C) 20.06 mm (D) 20.12 mm

Sol. 58 Option (A) is correct.
 Given : $D = 20\text{ mm}$, $t = 2\text{ mm}$, Punch or diameter clearance = 3%
 Required punch diameter will be,

$$d = D - 2 \times (\text{3\% of thickness})$$

$$= 20 - 2 \times \frac{3}{100} \times 2 = 19.88\text{ mm}$$

- Q. 59 A batch of 10 cutting tools could produce 500 components while working at 50 rpm with a tool feed of 0.25 mm/rev and depth of cut of 1mm. A similar batch of 10 tools of the same specification could produce 122 components while working at 80 rpm with a feed of 0.25 mm/rev and 1 mm depth of cut. How many components can be produced with one cutting tool at 60 rpm ?
 (A) 29 (B) 31
 (C) 37 (D) 42

Sol. 59 Option (A) is correct.
 Given : For case (I) :
 $N = 50\text{ rpm}$, $f = 0.25\text{ mm/rev.}$, $d = 1\text{ mm}$
 Number of cutting tools = 10
 Number of components produce = 500
 So, Velocity $V_1 = N \times f = 50 \times 0.25$
 $= 12.5\text{ mm/min.}$
 For case (II) :
 $N = 80\text{ rpm}$, $f = 0.25\text{ mm/rev.}$, $d = 1\text{ mm}$
 Number of cutting tools, = 10

Number of components produce = 122

So, Velocity $V_2 = N \times f = 80 \times 0.25 = 20 \text{ mm/min}$

From the tool life equation between cutting speed & tool life, $VT^n = C$,

$$V_1 T_1^n = V_2 T_2^n \quad \text{where } C = \text{constant} \quad \dots (i)$$

Tool life = Number of components produce \times Tool constant

For case (I), $T_1 = 500k$ $k = \text{tool constant}$

For case (II), $T_2 = 122k$

From equation (i),

$$12.5 \times (500k)^n = 20 \times (122k)^n$$

$$\left(\frac{500k}{122k}\right)^n = \frac{20}{12.5} = 1.6$$

Taking log both the sides,

$$n \ln\left(\frac{500}{122}\right) = \ln(1.6)$$

$$n(1.41) = 0.47$$

$$n = 0.333$$

Let the number of components produced be n_1 by one cutting tool at 60 r.p.m.

So,

Tool life, $T_3 = n_1 k$

Velocity, $V_3 = 60 \times 0.25 = 15 \text{ mm/min}$ feed remains same

Now, tool life T_1 if only 1 component is used,

$$T_1' = \frac{500k}{10}$$

So, $V_1 (T_1')^n = V_3 (T_3)^n$

Substitute the values, we get

$$V_1 \left(\frac{500k}{10}\right)^n = 15 (n_1 k)^n$$

$$\left(\frac{50k}{n_1 k}\right)^n = \frac{15}{12.5}$$

$$\frac{50}{n_1} = (1.2)^{1/0.333} = 1.73$$

$$n_1 = \frac{50}{1.73} = 28.90 \approx 29$$

Q. 60

GATE ME 2003
TWO MARK

A thread nut of M16 ISO metric type, having 2 mm pitch with a pitch diameter of 14.701 mm is to be checked for its pitch diameter using two or three number of balls or rollers of the following sizes

(A) Rollers of 2 mm φ

(B) Rollers of 1.155 mm φ

(C) Balls of 2 mm φ

(D) Balls of 1.155 mm φ

Sol. 60

Option (B) is correct.

Given : $p = 2 \text{ mm}$, $d = 14.701 \text{ mm}$

We know that, in case of ISO metric type threads,

$$2\theta = 60^\circ \Rightarrow \theta = 30^\circ$$

And in case of threads, always rollers are used.

For best size of rollers, $d = \frac{p}{2} \sec \theta$

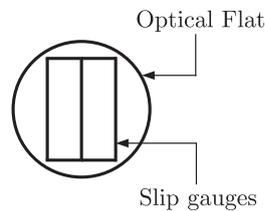
$$d = \frac{2}{2} \sec 30^\circ = 1.155 \text{ mm}$$

Hence, rollers of 1.155 mm diameter (1.155 ϕ) is used.

Q. 61

GATE ME 2003
TWO MARK

Two slip gauges of 10 mm width measuring 1.000 mm and 1.002 mm are kept side by side in contact with each other lengthwise. An optical flat is kept resting on the slip gauges as shown in the figure. Monochromatic light of wavelength 0.0058928 mm is used in the inspection. The total number of straight fringes that can be observed on both slip gauges is



- (A) 2 (B) 6
(C) 8 (D) 13

Sol. 61

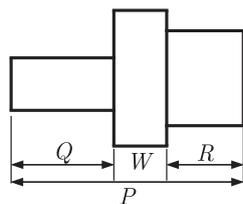
Option (D) is correct.

The total number of straight fringes that can be observed on both slip gauges is 13.

Q. 62

GATE ME 2003
TWO MARK

A part shown in the figure is machined to the sizes given below



$$P = 35.00 \pm 0.08 \text{ mm}, Q = 12.00 \pm 0.02 \text{ mm}, R = 13.00^{+0.04}_{-0.02} \text{ mm}$$

With 100% confidence, the resultant dimension W will have the specification

- (A) $9.99 \pm 0.03 \text{ mm}$
(B) $9.99 \pm 0.13 \text{ mm}$
(C) $10.00 \pm 0.03 \text{ mm}$
(D) $10.00 \pm 0.13 \text{ mm}$

Sol. 62

Option (A) is correct.

$$\text{Given : } P = 35.00 \pm 0.08 \text{ mm}, Q = 12.00 \pm 0.02 \text{ mm}$$

$$R = 13.00^{+0.04}_{-0.02} \text{ mm} = 13.01 \pm 0.03 \text{ mm}$$

From the given figure, we can say

$$P = Q + W + R$$

$$W = P - (Q + R)$$

$$W = (35.00 \pm 0.08) - [(12.00 \pm 0.02) + (13.01 \pm 0.03)]$$

$$W = (35 - 12 - 13.01)^{+0.08-0.02-0.03}_{-0.08+0.02+0.03}$$

$$= 9.99^{+0.03}_{-0.03} = 9.99 \pm 0.03 \text{ mm}$$

Q. 63

GATE ME 2003
TWO MARK

Two machines of the same production rate are available for use. On machine 1, the fixed cost is Rs. 100 and the variable cost is Rs. 2 per piece produced. The corresponding numbers for the machine 2 are Rs. 200 and Re. 1 respectively. For certain strategic reasons both the machines are to be used concurrently. The sales price of the first 800 units is Rs. 3.50 per unit and subsequently it is only Rs. 3.00 . The breakeven production rate for each machine is

- (A) 75
(B) 100
(C) 150
(D) 600

Sol. 63

Option (A) is correct.

Given :

For machine $M1$:

$$\text{Fixed cost} = 100 \text{ Rs.}$$

$$\text{Variable cost} = 2 \text{ Rs. per piece}$$

For machine $M2$:

$$\text{Fixed cost} = 200 \text{ Rs.}$$

$$\text{Variable cost} = 1 \text{ Rs. per piece}$$

Let, n number of units are produced per machine, when both the machines are to be used concurrently.

We know that,

Total cost

$$= \text{Fixed cost} + \text{Variable cost} \times \text{Number of units}$$

For $M1$

$$\text{Total cost of production} = 100 + 2 \times n$$

For $M2$

$$\text{Total cost of production} = 200 + n$$

Hence,

Total cost of production on machine $M1$ & $M2$ is

$$= 100 + 2n + 200 + n$$

$$= 300 + 3n$$

We know, Breakeven point is the point, where total cost of production is equal to the total sales price.

Assuming that Number of units produced are less than 800 units and selling price is Rs. 3.50 per unit.

So at breakeven point,

$$300 + 3n = 3.50(n + n)$$

$$300 + 3n = 3.50 \times 2n$$

$$300 = 4n$$

$$n = \frac{300}{4}$$

$$n = 75 \text{ units}$$

Q. 64

GATE ME 2003
TWO MARK

A residential school stipulates the study hours as 8.00 pm to 10.30 pm. Warden makes random checks on a certain student 11 occasions a day during the study hours over a period of 10 days and observes that he is studying on 71 occasions. Using 95% confidence interval, the estimated minimum hours of his study during that 10 day period is

- (A) 8.5 hours (B) 13.9 hours
(C) 16.1 hours (D) 18.4 hours

Sol. 64

Option (C) is correct.

Warden checks the student 11 occasions a day during the study hours over a period of 10 days.

So, Total number of observations in 10 days.

$$= 11 \times 10 = 110 \text{ observations}$$

Study hours as 8.00 pm to 10.30 pm.

So, total study hours in 10 days

$$= 2.5 \times 10$$

$$= 25 \text{ hours.}$$

Number of occasions when student studying

$$= 71$$

So, Probability of studying

$$P = \frac{\text{No. of observations when student studying}}{\text{Total observations}}$$

$$= \frac{71}{110} = 0.645$$

Hence,

Minimum hours of his study during 10 day period is

$$T = P \times \text{Total study hours in 10 days}$$

$$= 0.645 \times 25$$

$$= 16.1 \text{ hours}$$

Q. 65

GATE ME 2003
TWO MARK

The sale of cycles in a shop in four consecutive months are given as 70, 68, 82, 95. Exponentially smoothing average method with a smoothing factor of 0.4 is used in forecasting. The expected number of sales in the next month is

- (A) 59 (B) 72
(C) 86 (D) 136

Sol. 65

Option (B) is correct.

We know, from the exponential and smoothing average method, the exponential smoothed average $u_{(t+1)}$ which is the forecast for the next period $(t+1)$ is given by

$$u_{(t+1)} = \alpha u_t + \alpha(1-\alpha)u_{t-1} + \dots + \alpha(1-\alpha)^n u_{t-n} + \dots + \infty$$

Now, for sales of the fifth month put $t = 4$ in the above equation,

So,

$$u_5 = \alpha u_4 + \alpha(1-\alpha)u_3 + \alpha(1-\alpha)^2 u_2 + \alpha(1-\alpha)^3 u_1$$

where u_1, u_2, u_3 and u_4 are 70, 68, 82, and 95 respectively and $\alpha = 0.4$

Hence

$$u_5$$

$$= 0.4 \times 95 + 0.4(1 - 0.4)82 + 0.4(1 - 0.4)^2 \times 68 + 0.4(1 - 0.4)^3 \times 70$$

$$u_5 = 38 + 19.68 + 9.792 + 6.048$$

$$u_5 = 73.52$$

Q. 66

GATE ME 2003
TWO MARK

Market demand for springs is 8,00,000 per annum. A company purchases these springs in lots and sells them. The cost of making a purchase order is Rs. 1200. The cost of storage of springs is Rs. 120 per stored piece per annum. The economic order quantity is

- (A) 400 (B) 2,828
(C) 4,000 (D) 8,000

Sol. 66

Option (C) is correct.

Given :

$$D = 800000 \text{ per annum}$$

$$C_o = 1200 \text{ Rs.}$$

$$C_h = 120 \text{ per piece per annum}$$

We know that,

$$\text{Economic order quantity (EOQ)} = N = \sqrt{\frac{2C_o D}{C_h}}$$

$$N = \sqrt{\frac{2 \times 1200 \times 800000}{120}}$$

$$= \sqrt{16 \times 10^6}$$

$$= 4 \times 10^3 = 4000$$

Q. 67

GATE ME 2003
TWO MARK

A manufacturer produces two types of products, 1 and 2, at production levels of x_1 and x_2 respectively. The profit is given is $2x_1 + 5x_2$. The production constraints are

$$x_1 + 3x_2 \leq 40$$

$$3x_1 + x_2 \leq 24$$

$$x_1 + x_2 \leq 10$$

$$x_1 > 0, x_2 > 0$$

The maximum profit which can meet the constraints is

- (A) 29 (B) 38
(C) 44 (D) 75

Sol. 67

Option (A) is correct.

Given : Objective function,

$$Z = 2x_1 + 5x_2$$

And

$$x_1 + 3x_2 \leq 40$$

$$3x_1 + x_2 \leq 24$$

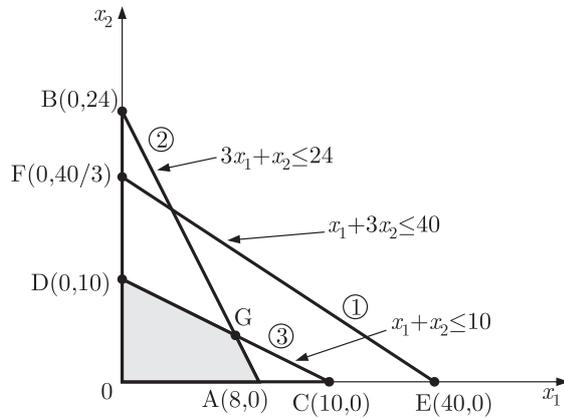
$$x_1 + x_2 \leq 10$$

$$x_1 > 0$$

$$x_2 > 0$$

First we have to make a graph from the given constraints. For draw the graph, substitute alternatively x_1 & x_2 equal to zero in each constraints to find the point on the x_1 & x_2 axis.

Now shaded area shows the common area. Note that the constraint $x_1 + 3x_2 \leq 40$ does not affect the solution space and it is the redundant constraint. Finding the coordinates of point G by the equations.



$$3x_1 + x_2 = 24$$

$$x_1 + x_2 = 10$$

Subtract these equations,

$$(3x_1 - x_1) + 0 = 24 - 10$$

$$2x_1 = 14 \Rightarrow x_1 = 7$$

$$x_2 = 10 - x_1 = 10 - 7 = 3$$

So, point $G(7, 3)$

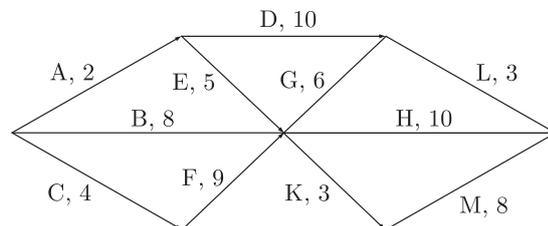
So, maximum profit which can meet the constraints at $G(7, 3)$ is

$$\begin{aligned} Z_{\max} &= 2 \times 7 + 5 \times 3 \\ &= 14 + 15 \\ &= 29 \end{aligned}$$

Q. 68

GATE ME 2003
TWO MARK

A project consists of activities A to M shown in the net in the following figure with the duration of the activities marked in days



The project can be completed

- (A) between 18, 19 days
- (B) between 20, 22 days
- (C) between 24, 26 days
- (D) between 60, 70 days

Sol. 68

Option (C) is correct.

The various path and their duration are :-

Path	Duration (days)
<i>A-D-L</i>	$2 + 10 + 3 = 15$
<i>A-E-G-L</i>	$2 + 5 + 6 + 3 = 16$
<i>A-E-H</i>	$2 + 5 + 10 = 17$
<i>B-H</i>	$8 + 10 = 18$
<i>C-F-K-M</i>	$4 + 9 + 3 + 8 = 24$
<i>C-F-H</i>	$4 + 9 + 10 = 23$
<i>A-E-K-M</i>	$2 + 5 + 3 + 8 = 18$
<i>B-G-L</i>	$8 + 6 + 3 = 17$
<i>B-K-M</i>	$8 + 3 + 8 = 19$
<i>C-F-G-L</i>	$4 + 9 + 6 + 3 = 22$

Here maximum time along the path *C-F-K-M*. So, it is a critical path and project can be completed in 24 days.

Q. 69

GATE ME 2003
TWO MARK

Match List-I with the List-II and select the correct answer using the codes given below the lists :

	List-I		List-II
P	Curtis	1.	Reaction steam turbine
Q	Rateau	2.	Gas turbine
R	Kaplan	3.	Velocity compounding
S	Francis	4.	Pressure compounding
		5.	Impulse water turbine
		6.	Axial turbine
		7.	Mixed flow turbine
		8.	Centrifugal pump

Codes :

	P	Q	R	S
(A)	2	1	1	6
(B)	3	1	5	7
(C)	1	3	1	5
(D)	3	4	7	6

Sol. 69

None of these is correct.

	List-I		List-II
P.	Curtis	3.	Velocity compounding
Q.	Rateau	4.	Pressure compounding
R.	Kaplan	6.	Axial flow turbine
S.	Francis	7.	Mixed flow turbine

So, correct pairs are P-3, Q-4, R-6, S-7.

Q. 70

Match the following

GATE ME 2003
TWO MARK

	Working material		Type of Joining
P.	Aluminium	1.	Submerged Arc Welding
Q.	Die steel	2.	Soldering
R.	Copper wire	3.	Thermit Welding
S.	Titanium sheet	4.	Atomic Hydrogen Welding
		5.	Gas Tungsten Arc Welding
		6.	Laser Beam Welding

- (A) P-2 Q-5 R-1 S-3
 (B) P-6 Q-3 R-4 S-1
 (C) P-4 Q-1 R-6 S-2
 (D) P-5 Q-4 R-2 S-6

Sol. 70

Option (D) is correct.

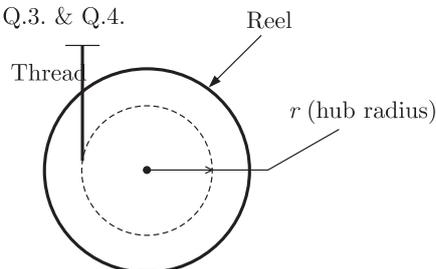
	Working material		Type of Joining
P.	Aluminium	5.	Gas Tungsten Arc Welding
Q.	Die steel	4.	Atomic Hydrogen Welding
R.	Copper Wire	2.	Soldering
S.	Titanium sheet	6.	Laser Beam Welding

So, correct pairs are, P - 5, Q - 4, R - 2, S - 6

Common Data For Q. Data for Q. 71 & 72 are given below. Solve the problems and choose correct answers.

A reel of mass " m " and radius of gyration " k " is rolling down smoothly from rest with one end of the thread wound on it held in the ceiling as depicted in the figure. Consider the thickness of thread and its mass negligible in comparison with the radius " r " of the hub and the reel mass " m ". Symbol " g " represents the acceleration due to gravity.

Q.3. & Q.4.



Q. 71

The linear acceleration of the reel is

GATE ME 2003
TWO MARK

- (A) $\frac{gr^2}{(r^2 + k^2)}$ (B) $\frac{gk^2}{(r^2 + k^2)}$
 (C) $\frac{grk}{(r^2 + k^2)}$ (D) $\frac{mgr^2}{(r^2 + k^2)}$

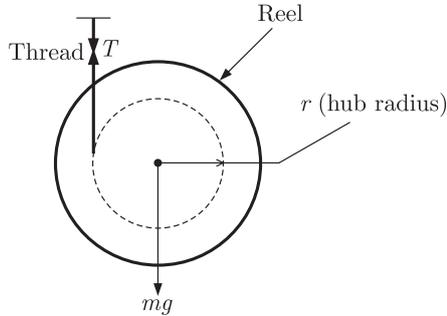
Sol. 71

Option (A) is correct.

Given : Mass of reel = m

Radius of gyration = k

We have to make FBD of the system,



Where, T = Tension in the thread

mg = Weight of the system

Here the reel is rolling down. So Angular acceleration (α) comes in the action

From FBD, For vertical translation motion,

$$mg - T = ma \quad \dots(i)$$

& for rotational motion,

$$\Sigma M_G = I_G \alpha$$

$$T \times r = mk^2 \times \frac{a}{r} \quad I_G = mk^2, \alpha = a/r$$

$$T = \frac{mk^2}{r^2} \times a \quad \dots(ii)$$

From equation (i) & (ii) Substitute the value of T in equation (i), we get



$$mg - \frac{mk^2}{r^2} \times a = ma$$

$$mg = a \left[\frac{mk^2}{r^2} + m \right]$$

$$a = \frac{gr^2}{k^2 + r^2} \quad \dots(iii)$$

Q. 72

GATE ME 2003
TWO MARK

The tension in the thread is

(A) $\frac{mgr^2}{(r^2 + k^2)}$

(B) $\frac{mgrk}{(r^2 + k^2)}$

(C) $\frac{mgk^2}{(r^2 + k^2)}$

(D) $\frac{mg}{(r^2 + k^2)}$

Sol. 72

Option (C) is correct.

From previous question,

$$T = mg - ma$$

Substitute the value of a from equation (iii), we get

$$\begin{aligned} T &= mg - m \times \frac{gr^2}{(k^2 + r^2)} \\ &= \frac{mg(k^2 + r^2) - mgr^2}{(k^2 + r^2)} = \frac{mgk^2}{k^2 + r^2} \end{aligned}$$

Common Data For Q. Data for Q. 73 and 74 are given below. Solve the problems and choose correct answers.

The state of stress at a point “ P ” in a two dimensional loading is such that the Mohr’s circle is a point located at 175 MPa on the positive normal stress axis.

Q. 73

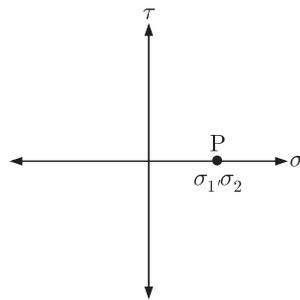
GATE ME 2003
TWO MARK

The maximum and minimum principal stresses respectively from the Mohr’s circle are

- (A) +175 MPa, –175 MPa (B) +175 MPa, +175 MPa
(C) 0, –175 MPa (D) 0, 0

Sol. 73

Option (B) is correct.



Given, Mohr’s circle is a point located at 175 MPa on the positive Normal stress (at point P)

So, $\sigma_1 = \sigma_2 = 175$ MPa, and $\tau_{\max} = 0$

So, both maximum and minimum principal stresses are equal.

Alternate Method

$$\sigma_x = 175 \text{ MPa} \quad \sigma_y = 175 \text{ MPa} \quad \& \quad \tau_{xy} = 0$$

Maximum principal stress

$$\sigma_1 = \frac{1}{2}[(\sigma_x + \sigma_y) + \sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}]$$

$$\sigma_1 = \frac{1}{2}[(175 + 175) + 0]$$

$$\sigma_1 = 175 \text{ MPa}$$

Minimum principal stress

$$\sigma_2 = \frac{1}{2}[(\sigma_x + \sigma_y) - \sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}]$$

$$\sigma_2 = \frac{1}{2}[(175 + 175) - 0]$$

$$\sigma_2 = 175 \text{ MPa}$$

Q. 74

GATE ME 2003
TWO MARK

The directions of maximum and minimum principal stresses at the point “ P ” from the Mohr’s circle are

- (A) $0^\circ, 90^\circ$ (B) $90^\circ, 0^\circ$
(C) $45^\circ, 135^\circ$ (D) all directions

Sol. 74

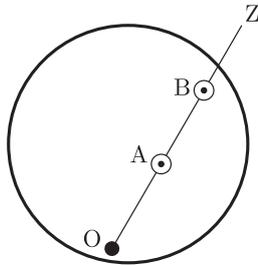
Option (D) is correct.

Mohr’s circle is a point, and a point will move in every direction. So, the directions of maximum and minimum principal stresses at point P is in all directions.

Every value of θ will give the same result of 175 MPa in all directions.

Common Data For Q. Data for Q. 75 and 76 are given below. Solve the problems and choose correct answers.

The circular disc shown in its plan view in the figure rotates in a plane parallel to the horizontal plane about the point O at a uniform angular velocity ω . Two other points A and B are located on the line OZ at distances r_A and r_B from O respectively.



Q. 75

GATE ME 2003
TWO MARK

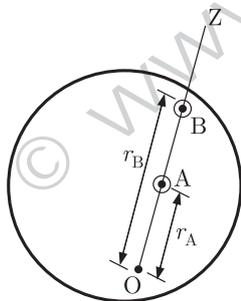
The velocity of Point B with respect to point A is a vector of magnitude

- (A) 0
 (B) $\omega(r_B - r_A)$ and direction opposite to the direction of motion of point B
 (C) $\omega(r_B - r_A)$ and direction same as the direction of motion of point B
 (D) $\omega(r_B - r_A)$ and direction being from O to Z

Sol. 75

Option (C) is correct.

Given, the circular disc rotates about the point O at a uniform angular velocity ω .



Let v_A is the linear velocity of point A & v_B is the linear velocity of point B.

$$v_A = \omega r_A \text{ and } v_B = \omega r_B$$

Velocity of point B with respect to point A is given by,

$$v_{BA} = v_B - v_A = \omega r_B - \omega r_A = \omega(r_B - r_A)$$

From the given figure,

$$r_B > r_A$$

So,

$$\omega r_B > \omega r_A$$

$$v_B > v_A$$

Therefore, relative velocity $\omega(r_B - r_A)$ in the direction of point B.

Q. 76

GATE ME 2003
TWO MARK

The acceleration of point B with respect to point A is a vector of magnitude

- (A) 0
 (B) $\omega(r_B - r_A)$ and direction same as the direction of motion of point B
 (C) $\omega^2(r_B - r_A)$ and direction opposite to be direction of motion of point B
 (D) $\omega^2(r_B - r_A)$ and direction being from Z to O

Sol. 76

Option (D) is correct.

Acceleration of point B with respect to point A is given by,

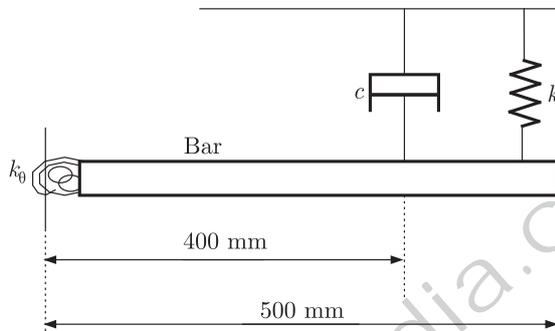
$$a_{BA} = \omega v_{BA} = \omega \times \omega (r_B - r_A) = \omega^2 (r_B - r_A) \dots (i)$$

This equation (i) gives the value of centripetal acceleration which acts always towards the centre of rotation.

So, a_{BA} acts towards to O i.e. its direction from Z to O

Common Data For Q. Data for Q. 77 and 78 are given below. Solve the problems and choose correct answer.

A uniform rigid cylinder bar of mass 10 kg, hinged at the left end is suspended with the help of spring and damper arrangement as shown in the figure where $k = 2 \text{ kN/m}$, $c = 500 \text{ Ns/m}$ and the stiffness of the torsional spring k_θ is 1 kN/m/rad . Ignore the hinge dimensions.



Q. 77

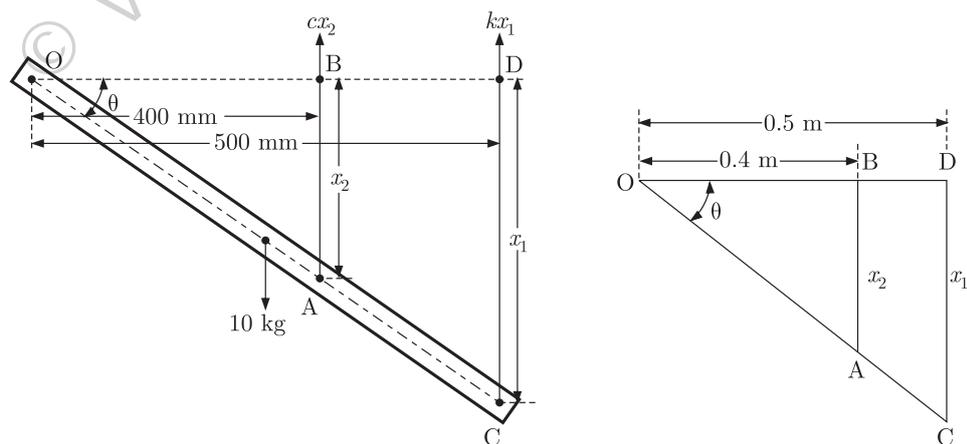
GATE ME 2003
TWO MARK

The undamped natural frequency of oscillations of the bar about the hinge point is

- (A) 42.43 rad/s
- (B) 30 rad/s
- (C) 17.32 rad/s
- (D) 14.14 rad/s

Sol. 77

Option (A) is correct.



Given $m = 10 \text{ kg}$, $k = 2 \text{ kN/m}$, $c = 500 \text{ Ns/m}$, $k_\theta = 1 \text{ kN/m/rad}$
 $l_1 = 0.5 \text{ m}$, $l_2 = 0.4 \text{ m}$

Let, the rigid slender bar twist downward at the angle θ . Now spring & damper exert a force kx_1 & cx_2 on the rigid bar in the upward direction.

From similar triangle OAB & OCD ,

$$\tan \theta = \frac{x_2}{0.4} = \frac{x_1}{0.5}$$

Let θ be very very small, then $\tan \theta \simeq \theta$,

$$\theta = \frac{x_2}{0.4} = \frac{x_1}{0.5}$$

$$x_2 = 0.4\theta \text{ or } x_1 = 0.5\theta \quad \dots (i)$$

On differentiating the above equation, we get

$$\dot{x}_2 = 0.4\dot{\theta} \text{ or } \dot{x}_1 = 0.5\dot{\theta} \quad \dots (ii)$$

We know, the moment of inertia of the bar hinged at the one end is,

$$I = \frac{ml_1^2}{3} = \frac{10 \times (0.5)^2}{3} = 0.833 \text{ kg} \cdot \text{m}^2$$

As no external force acting on the system. So, governing equation of motion from the Newton's law of motion is,

$$I\ddot{\theta} + c\dot{x}_2 l_2 + kx_1 l_1 + k_\theta \theta = 0$$

$$0.833\ddot{\theta} + 500 \times 0.4\dot{x}_2 + 2000 \times (0.5)x_1 + 1000\theta = 0$$

$$0.833\ddot{\theta} + 200\dot{x}_2 + 1000x_1 + 1000\theta = 0 \quad \dots (iii)$$

$$0.833\ddot{\theta} + 200 \times 0.4\dot{\theta} + 1000 \times 0.5\theta + 1000\theta = 0$$

$$0.833\ddot{\theta} + 80\dot{\theta} + 1500\theta = 0 \quad \dots (iv)$$

On comparing equation (iv) with its general equation,

$$I\ddot{\theta} + c\dot{\theta} + k\theta = 0$$

We get, $I = 0.833$, $c = 80$, $k = 1500$

So, undamped natural frequency of oscillations is given by

$$\omega_n = \sqrt{\frac{k}{I}} = \sqrt{\frac{1500}{0.833}} = \sqrt{1800.72} = 42.43 \text{ rad/sec}$$

Q. 78

GATE ME 2003
TWO MARK

The damping coefficient in the vibration equation is given by

- (A) 500 Nms/rad (B) 500 N/(m/s)
(C) 80 Nms/rad (D) 80 N/(m/s)

Sol. 78

Option (C) is correct.

From the previous part of the question

Damping coefficient, $c = 80 \text{ Nms/rad}$

Common Data For Q. Data for Q. 79 - 80 given below. Solve the problems and choose correct answers.

The overall gear ratio in a 2 stage speed reduction gear box (with all spur gears) is 12. The input and output shafts of the gear box are collinear. The counter shaft which is parallel to the input and output shafts has a gear (Z_2 teeth) and pinion ($Z_3 = 15$ teeth) to mesh with pinion ($Z_1 = 16$ teeth) on the input shaft and gear (Z_4 teeth) on the output shaft respectively. It was decided to use a gear ratio of 4 with 3 module in the first stage and 4 module in the second stage.

Q. 79

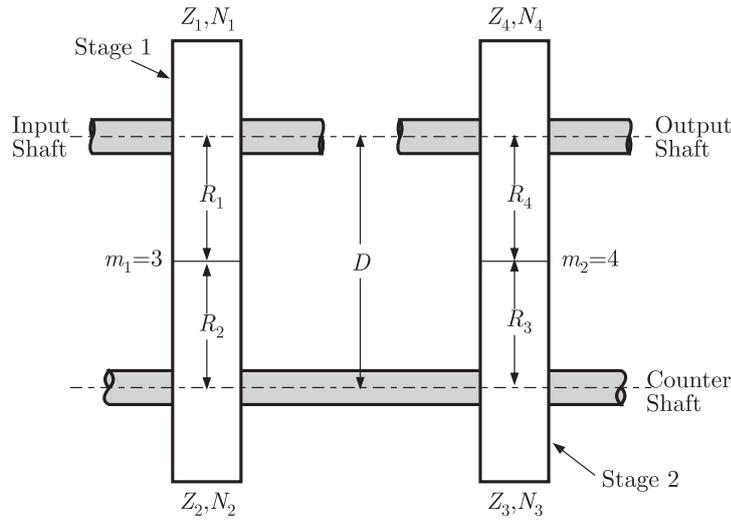
GATE ME 2003
TWO MARK

Z_2 and Z_4 are

- (A) 64 and 45
(B) 45 and 64
(C) 48 and 60
(D) 60 and 48

Sol. 79

Option (A) is correct.



Let N_1, N_2, N_3 and N_4 are the speeds of pinion 1, gear 2, pinion 3 and gear 4 respectively.

Given : $Z_1 = 16$ teeth , $Z_3 = 15$ teeth and $Z_4 = ?$, $Z_2 = ?$

Velocity ratio $\frac{N_1}{N_4} = \frac{Z_2/Z_1}{Z_3/Z_4} \quad N \propto 1/Z$

$$= \frac{Z_2}{Z_1} \times \frac{Z_4}{Z_3} = 12 \quad \dots(i)$$

But for stage 1, $\frac{N_1}{N_2} = \frac{Z_2}{Z_1} = 4 \quad \dots(ii)$

So, $4 \times \frac{Z_4}{Z_3} = 12 \quad \text{from eq. (i)}$

$$\frac{Z_4}{Z_3} = 3, \quad \Rightarrow Z_4 = 3 \times 15 = 45 \text{ teeth}$$

From equation (ii), $Z_2 = 4 \times Z_1 = 4 \times 16 = 64$ teeth

Q. 80

GATE ME 2003
TWO MARK

The centre distance in the second stage is

- (A) 90 mm
- (B) 120 mm
- (C) 160 mm
- (D) 240 mm

Sol. 80

Option (B) is correct.

Let centre distance in the second stage is D .

$$D = R_4 + R_3 = \frac{D_4 + D_3}{2}$$

But, $\frac{D_4}{Z_4} = \frac{D_3}{Z_3} = m \quad m = D/Z \text{ module}$

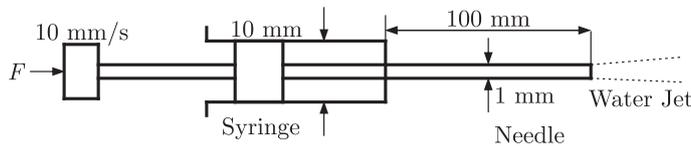
$$D_4 = 4 \times Z_4 = 4 \times 45 = 180$$

Or, $D_3 = 4 \times Z_3 = 4 \times 15 = 60$

So, $D = \frac{180 + 60}{2} = 120 \text{ mm}$

Common Data For Q. Data for Q. 81 & 82 are given below. Solve the problems and choose correct answers.

A syringe with a frictionless plunger contains water and has at its end a 100 mm long needle of 1 mm diameter. The internal diameter of the syringe is 10 mm. Water density is 1000 kg/m³. The plunger is pushed in at 10 mm/s and the water comes out as a jet



Q. 81

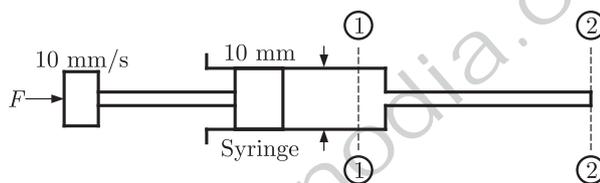
GATE ME 2003
TWO MARK

Assuming ideal flow, the force F in newtons required on the plunger to push out the water is

- (A) 0
- (B) 0.04
- (C) 0.13
- (D) 1.15

Sol. 81

Option (B) is correct.



Given : $L = 100 \text{ mm}$, $d = 1 \text{ mm}$, $D = 10 \text{ mm}$, $V_1 = 10 \text{ mm/sec}$

We have to take the two sections of the system (1) & (2).

Apply continuity equation on section (1) & (2),

$$A_1 V_1 = A_2 V_2 \quad Q = AV, Q = \text{flow rate}$$

$$V_2 = \left(\frac{A_1}{A_2}\right) \times V_1$$

$$= \frac{\pi/4 (0.01)^2}{\pi/4 (0.001)^2} \times 0.010 = 1 \text{ m/sec}$$

Again applying the Bernoulli's equation at section (1) & (2),

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

The syringe & the plunger is situated on the same plane so $z_1 = z_2$,

Take $p_2 = 0 = \text{Atmospheric pressure (Outside the needle)}$

$$\frac{p_1}{\rho g} = \frac{V_2^2 - V_1^2}{2g}$$

$$p_1 = \frac{\rho}{2} (V_2^2 - V_1^2) = \frac{1000}{2} [(1)^2 - (0.01)^2] = 499.95 \text{ N/m}^2$$

Force required on plunger,

$$F = p_1 \times A_1 = 499.95 \times \frac{\pi}{4} (0.01)^2 = 0.04 \text{ N}$$

Q. 82

GATE ME 2003
TWO MARK

Neglect losses in the cylinder and assume fully developed laminar viscous flow throughout the needle; the Darcy friction factor is $64/Re$. Where Re is the Reynolds number. Given that the viscosity of water is $1.0 \times 10^{-3} \text{ kg/s-m}$, the force F in newtons required on the plunger is

- (A) 0.13
- (B) 0.16
- (C) 0.3
- (D) 4.4

Sol. 82

Option (C) is correct.

Given : $f = \frac{64}{\text{Re}}$, $\mu = 1 \times 10^{-3}$ kg/s-m

$$\begin{aligned} \text{Re} &= \frac{\rho V d}{\mu} = \frac{\rho V_2 d_2}{\mu} && \text{For Needle} \\ &= \frac{1000 \times 1 \times 0.001}{1 \times 10^{-3}} = 1000 \end{aligned}$$

$$\text{And } f = \frac{64}{\text{Re}} = \frac{64}{1000} = 0.064$$

From the help of f we have to find Head loss in needle,

$$\begin{aligned} h_f &= \frac{f L V_2^2}{2 g d_2} \\ &= \frac{0.064 \times 0.1 \times (1)^2}{2 \times 9.81 \times 0.001} = 0.3265 \text{ m of water} \end{aligned}$$

Applying Bernoulli's equation at section (1) & (2) with the head loss in account.

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + h_f$$

$$\text{And } \begin{aligned} z_1 &= z_2 && \text{At the same plane} \\ p_2 &= 0 && \text{Atmospheric pressure} \end{aligned}$$

$$\frac{p_1}{\rho g} = \left(\frac{V_2^2 - V_1^2}{2g} \right) + h_f$$

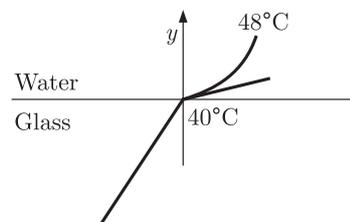
$$\begin{aligned} p_1 &= \frac{\rho}{2} (V_2^2 - V_1^2) + \rho g h_f \\ &= \frac{1000}{2} [(1)^2 - (0.01)^2] + 1000 \times 9.81 \times 0.3265 \\ &= 499.95 + 3202.965 = 3702.915 \text{ N/m}^2 \end{aligned}$$

Force required on plunger,

$$F = p_1 \times A_1 = 3702.915 \times \frac{\pi}{4} \times (0.01)^2 = 0.3 \text{ N}$$

Common Data For Q. 83 - 84 are given below. Solve the problems and choose correct answers.

Heat is being transferred by convection from water at 48°C to a glass plate whose surface that is exposed to the water is at 40°C . The thermal conductivity of water is 0.6 W/mK and the thermal conductivity of glass is 1.2 W/mK . The spatial gradient of temperature in the water at the water-glass interface is $dT/dy = 1 \times 10^4 \text{ K/m}$.



Q. 83

GATE ME 2003
TWO MARK

The value of the temperature gradient in the glass at the water-glass interface in K/m is

- (A) -2×10^4 (B) 0.0
(C) 0.5×10^4 (D) 2×10^4

Sol. 83

Option (C) is correct.

Given for water : $T_w = 48^\circ \text{C}$, $k_w = 0.6 \text{ W/mK}$ And for glass : $T_g = 40^\circ \text{C}$, $k_g = 1.2 \text{ W/mK}$ Spatial gradient $\left(\frac{dT}{dy}\right)_w = 1 \times 10^4 \text{ K/m}$

Heat transfer takes place between the water and glass interface by the conduction and convection. Heat flux would be same for water and glass interface. So, applying the conduction equation for water and glass interface.

$$k_w \left(\frac{dT}{dy}\right)_w = k_g \left(\frac{dT}{dy}\right)_g \quad q = \frac{Q}{A} = \frac{-kA \frac{dT}{dx}}{A} = -k \frac{dT}{dx}$$

$$\left(\frac{dT}{dy}\right)_g = \frac{k_w}{k_g} \left(\frac{dT}{dy}\right)_w$$

$$= \frac{0.6}{1.2} \times 10^4 = 0.5 \times 10^4 \text{ K/m}$$

Q. 84

GATE ME 2003
TWO MARKThe heat transfer coefficient h in $\text{W/m}^2 \text{K}$ is

(A) 0.0

(B) 4.8

(C) 6

(D) 750

Sol. 84

Option (D) is correct.

From the equation of convection,

Heat flux, $q = h[T_w - T_g]$... (i)Where, h = Heat transfer coefficientFirst find q , $q = k_w \left(\frac{dT}{dy}\right)_w = k_g \left(\frac{dT}{dy}\right)_g$

$$= 0.6 \times 10^4 = 6000 \text{ W/m}^2$$

Now from equation (i),

$$h = \frac{q}{T_w - T_g}$$

$$= \frac{6000}{48 - 40} = \frac{6000}{8} = 750 \text{ W/m}^2 \text{K}$$

Common Data For Q. 85 & 86 are given below. Solve the problems and choose correct answers.

Nitrogen gas (molecular weight 28) is enclosed in a cylinder by a piston, at the initial condition of 2 bar, 298 K and 1 m^3 . In a particular process, the gas slowly expands under isothermal condition, until the volume becomes 2 m^3 . Heat exchange occurs with the atmosphere at 298 K during this process.

Q. 85

GATE ME 2003
TWO MARK

The work interaction for the Nitrogen gas is

(A) 200 kJ

(B) 138.6 kJ

(C) 2 kJ

(D) -200 kJ

Sol. 85

Option (B) is correct.

Given : $p_1 = 2 \text{ bar} = 2 \times 10^5 \text{ N/m}^2$, $T_1 = 298 \text{ K} = T_2$, $\nu_1 = 1 \text{ m}^3$, $\nu_2 = 2 \text{ m}^3$

The process is isothermal,

$$\text{So, } W = p_1 \nu_1 \ln \frac{p_1}{p_2} = p_1 \nu_1 \ln \left(\frac{\nu_2}{\nu_1}\right)$$

$$= 2 \times 10^5 \times 1 \ln \left[\frac{2}{1}\right] = 2 \times 0.6931 \times 10^5$$

$$= 10^5 \times 1.3863 = 138.63 \text{ kJ} \approx 138.6 \text{ kJ}$$

Q. 86

GATE ME 2003
TWO MARK

The entropy changes for the Universe during the process in kJ/K is

- (A) 0.4652
- (B) 0.0067
- (C) 0
- (D) -0.6711

Sol. 86

Option (A) is correct.

Entropy,
$$\Delta S = \frac{\Delta Q}{T} \quad \dots (i)$$

From first law of thermodynamics,

$$\Delta Q = \Delta U + \Delta W$$

For isothermal process,

$$\Delta U = 0$$

$$\Delta Q = \Delta W$$

From equation (i),

$$\Delta S = \frac{\Delta W}{T} = \frac{138.63 \text{ kJ}}{298 \text{ K}} = 0.4652 \text{ kJ/K}$$

Common Data For Q.87 and Q. 88 are given below. Solve the problems and choose correct answers.

A refrigerator based on ideal vapour compression cycle operates between the temperature limits of -20°C and 40°C . The refrigerant enters the condenser as saturated vapour and leaves as saturated liquid. The enthalpy and entropy values for saturated liquid and vapour at these temperatures are given in the table below.

T (°C)	h_f (kJ/kg)	h_g (kJ/kg)	s_f (kJ/kg K)	s_g (kJ/kg K)
-20	20	180	0.07	0.7366
40	80	200	0.3	0.67

Q. 87

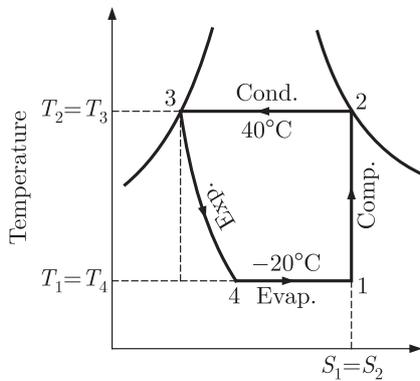
GATE ME 2003
TWO MARK

If refrigerant circulation rate is 0.025 kg/s, the refrigeration effect is equal to

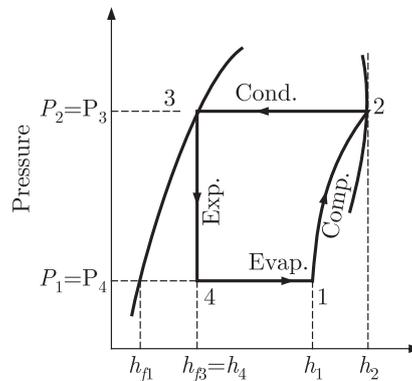
- (A) 2.1 kW
- (B) 2.5 kW
- (C) 3.0 kW
- (D) 4.0 kW

Sol. 87

Option (A) is correct.



(A) T-S Diagram



(B) p-h Diagram

Given : $T_1 = T_4 = -20^\circ\text{C} = (-20 + 273) \text{ K} = 253 \text{ K}$, $\dot{m} = 0.025 \text{ kg/sec}$

$$T_2 = T_3 = 40^\circ \text{C} = (40 + 273) \text{K} = 313 \text{K}$$

From the given table,

$$\text{At, } T_2 = 40^\circ \text{C, } h_2 = 200 \text{ kJ/kg}$$

$$\text{And } h_3 = h_4 = 80 \text{ kJ/kg}$$

From the given T - s curve

$$s_1 = s_2$$

$$s_2 = s_f + xs_{fg}$$

$x =$ Dryness fraction

{ s_2 is taken 0.67 because s_2 at the temperature 40°C & at 2 high temperature and pressure vapour refrigerant exist.}

$$0.67 = 0.07 + x(0.7366 - 0.07) \quad s_{fg} = s_g - s_f$$

$$0.67 - 0.07 = x \times 0.6666$$

$$0.6 = x \times 0.6666$$

$$x = \frac{0.6}{0.6666} = 0.90$$

And Enthalpy at point 1 is,

$$\begin{aligned} h_1 &= h_f + xh_{fg} = h_f + x(h_g - h_f) \\ &= 20 + 0.90(180 - 20) = 164 \text{ kJ/kg} \end{aligned}$$

Now refrigeration effect is produce in the evaporator.

Heat extracted from the evaporator or refrigerating effect,

$$R_E = \dot{m}(h_1 - h_4) = 0.025(164 - 80) = 2.1 \text{ kW}$$

Q. 88

GATE ME 2003
TWO MARK

The COP of the refrigerator is

(A) 2.0

(B) 2.33

(C) 5.0

(D) 6.0

Sol. 88

Option (B) is correct.

$$\begin{aligned} (COP)_{\text{refrigerator}} &= \frac{h_1 - h_4}{h_2 - h_1} = \frac{\text{Refrigerating effect}}{\text{Work done}} \\ &= \frac{164 - 80}{200 - 164} = \frac{84}{36} = 2.33 \end{aligned}$$

Data for Q. 89 - 90 are given below. Solve the problems and choose correct answers.

A cylinder is turned on a lathe with orthogonal machining principle. Spindle rotates at 200 rpm. The axial feed rate is 0.25 mm per revolution. Depth of cut is 0.4 mm. The rake angle is 10° . In the analysis it is found that the shear angle is 27.75° .

Q. 89

GATE ME 2003
TWO MARK

The thickness of the produced chip is

(A) 0.511 mm

(B) 0.528 mm

(C) 0.818 mm

(D) 0.846 mm

Sol. 89

Option (A) is correct

Given : $N = 200 \text{ rpm}$, $f = 0.25 \text{ mm/revolution}$, $d = 0.4 \text{ mm}$, $\alpha = 10^\circ$, $\phi = 27.75^\circ$

Uncut chip thickness,

$$t = f(\text{feed, mm/rev.}) = 0.25 \text{ mm/rev.}$$

Chip thickness ratio is given by,

$$r = \frac{t}{t_c} = \frac{\sin \phi}{\cos(\phi - \alpha)}$$

Where,

$t_c =$ thickness of the produced chip.

So,

$$t_c = \frac{t \times \cos(\phi - \alpha)}{\sin \phi}$$

$$= \frac{0.25 \times \cos(27.75 - 10)}{\sin(27.75)} = 0.511 \text{ mm}$$

Alternate :

We also find the value of t_c by the general relation,

$$\tan \phi = \frac{r \cos \alpha}{1 - r \sin \alpha} \quad \text{where}$$

$$r = \frac{t}{t_c}$$

Q. 90

GATE ME 2003
TWO MARK

In the above problem, the coefficient of friction at the chip tool interface obtained using Earnest and Merchant theory is

- (A) 0.18 (B) 0.36
(C) 0.71 (D) 0.98

Sol. 90

Option (D) is correct.

We know that angle of friction,

$$\beta = \tan^{-1} \mu$$

or,

$$\mu = \tan \beta \quad \dots(i)$$

For merchant and earnest circle, the relation between rake angle (α), shear angle (ϕ) and friction angle (β) is given by,

$$2\phi + \beta - \alpha = 90^\circ$$

$$\beta = 90^\circ + \alpha - 2\phi$$

$$= 90^\circ + 10 - 2 \times 27.75 = 44.5^\circ$$

Now, from equation (i),

$$\mu = \tan(44.5^\circ) = 0.98$$

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Answer Sheet									
1.	(A)	19.	(C)	37.	(A)	55.	(B)	73.	(B)
2.	(D)	20.	(C)	38.	(A)	56.	(C)	74.	(D)
3.	(C)	21.	(D)	39.	(A)	57.	(B)	75.	(C)
4.	(D)	22.	(D)	40.	(D)	58.	(A)	76.	(D)
5.	(D)	23.	(C)	41.	(B)	59.	(A)	77.	(A)
6.	(C)	24.	(B)	42.	(B)	60.	(B)	78.	(C)
7.	(B)	25.	(C)	43.	(C)	61.	(D)	79.	(A)
8.	(D)	26.	(D)	44.	(D)	62.	(A)	80.	(B)
9.	(A)	27.	(D)	45.	(A)	63.	(A)	81.	(B)
10.	(B)	28.	(B)	46.	(B)	64.	(C)	82.	(C)
11.	(C)	29.	(B)	47.	(D)	65.	(B)	83.	(C)
12.	(C)	30.	(A)	48.	(A)	66.	(C)	84.	(D)
13.	(D)	31.	(C)	49.	(D)	67.	(A)	85.	(B)
14.	(C)	32.	(B)	50.	(A)	68.	(C)	86.	(A)
15.	(C)	33.	(A)	51.	(A)	69.	(*)	87.	(A)
16.	(B)	34.	(C)	52.	(D)	70.	(D)	88.	(B)
17.	(B)	35.	(B)	53.	(B)	71.	(A)	89.	(A)
18.	(B)	36.	(D)	54.	(B)	72.	(C)	90.	(D)

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