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**GATE SOLVED PAPER
Mechanical Engineering
2007**

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GATE SOLVED PAPER - ME

2007

- Q. 1 The minimum value of function $y = x^2$ in the interval $[1, 5]$ is
GATE ME 2007 ONE MARK (A) 0 (B) 1
(C) 25 (D) undefined

Sol. 1 Option (B) is correct.
Given : $y = x^2 \dots(i)$
And interval $[1, 5]$
At $x = 1 \Rightarrow y = 1$
And at $x = 5 \Rightarrow y = (5)^2 = 25$
Here the interval is bounded between 1 & 5
So, the minimum value at this interval is 1.

- Q. 2 If a square matrix A is real and symmetric, then the eigen values
GATE ME 2007 ONE MARK (A) are always real (B) are always real and positive
(C) are always real and non-negative (D) occur in complex conjugate pairs

Sol. 2 Option (A) is correct
Let square matrix

$$A = \begin{bmatrix} x & y \\ y & x \end{bmatrix}$$

We know that the characteristic equation for the eigen values is given by

$$\begin{aligned} |A - \lambda I| &= 0 \\ \begin{vmatrix} x - \lambda & y \\ y & x - \lambda \end{vmatrix} &= 0 \\ (x - \lambda)^2 - y^2 &= 0 \\ (x - \lambda)^2 &= y^2 \\ x - \lambda &= \pm y \\ \lambda &= x \pm y \end{aligned}$$

So, eigen values are real if matrix is real and symmetric.

- Q. 3 If $\varphi(x, y)$ and $\psi(x, y)$ are functions with continuous second derivatives, then
GATE ME 2007 ONE MARK $\varphi(x, y) + i\psi(x, y)$ can be expressed as an analytic function of $x + iy$ ($i = \sqrt{-1}$), when

- (A) $\frac{\partial \varphi}{\partial x} = -\frac{\partial \psi}{\partial x}, \frac{\partial \varphi}{\partial y} = \frac{\partial \psi}{\partial y}$ (B) $\frac{\partial \varphi}{\partial y} = -\frac{\partial \psi}{\partial x}, \frac{\partial \varphi}{\partial x} = \frac{\partial \psi}{\partial y}$
(C) $\frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2} = \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = 1$ (D) $\frac{\partial \varphi}{\partial x} + \frac{\partial \varphi}{\partial y} = \frac{\partial \psi}{\partial x} + \frac{\partial \psi}{\partial y} = 0$

Sol. 3 Option (B) is correct.

We know from the Cauchy-Reimann equation, the necessary condition for a function $f(z)$ to be analytic is

$$\begin{aligned} \frac{\partial \varphi}{\partial x} &= \frac{\partial \psi}{\partial y} \\ \frac{\partial \varphi}{\partial y} &= -\frac{\partial \psi}{\partial x} \end{aligned}$$

When $\frac{\partial \varphi}{\partial x}$, $\frac{\partial \varphi}{\partial y}$, $\frac{\partial \psi}{\partial y}$, $\frac{\partial \psi}{\partial x}$ exist.

Q. 4

GATE ME 2007
ONE MARK

The partial differential equation

$$\frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2} + \frac{\partial \varphi}{\partial x} + \frac{\partial \varphi}{\partial y} = 0 \text{ has}$$

- (A) degree 1 order 2 (B) degree 1 order 1
(C) degree 2 order 1 (D) degree 2 order 2

Sol. 4

Option (A) is correct.

$$\text{Given : } \frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2} + \frac{\partial \varphi}{\partial x} + \frac{\partial \varphi}{\partial y} = 0$$

Order → It is determined by the order of the highest derivative present in it.

Degree → It is determined by the degree of the highest order derivative present in it after the differential equation is cleared of radicals & fractions.

So, degree = 1 & order = 2

Q. 5

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ONE MARK

Which of the following relationships is valid only for reversible processes undergone by a closed system of simple compressible substance (neglect changes in kinetic and potential energy ?)

- (A) $\delta Q = dU + \delta W$ (B) $Tds = dU + pdv$
(C) $Tds = dU + \delta W$ (D) $\delta Q = dU + pdv$

Sol. 5

Option (D) is correct.

In this question we discuss on all the four options.

(A) $\delta Q = dU + \delta W$

This equation holds good for any process undergone by a closed stationary system.

(B) $Tds = dU + pdv$

This equation holds good for any process reversible or irreversible, undergone by a closed system.

(C) $Tds = dU + \delta W$

This equation holds good for any process, reversible or irreversible, and for any system.

(D) $\delta Q = dU + pdv$

This equation holds good for a closed system when only pdv work is present. This is true only for a reversible (quasi-static) process.

Q. 6

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Water has a critical specific volume of $0.003155 \text{ m}^3/\text{kg}$. A closed and rigid steel tank of volume 0.025 m^3 contains a mixture of water and steam at 0.1 MPa . The mass of the mixture is 10 kg . The tank is now slowly heated. The liquid level inside the tank

- (A) will rise
(B) will fall
(C) will remain constant
(D) may rise or fall depending on the amount of heat transferred

Sol. 6

Option (A) is correct.

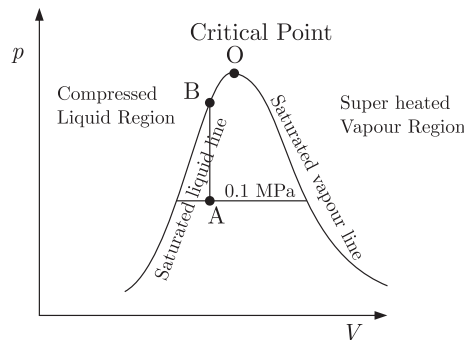
$$\text{Given : } \nu_{cri} = 0.003155 \text{ m}^3/\text{kg}, \nu = 0.025 \text{ m}^3, p = 0.1 \text{ MPa and } m = 10 \text{ kg}$$

We know, Rigid means volume is constant.

Specific volume,
$$\nu_s = \frac{\nu}{m}$$

$$= \frac{0.025}{10} = 0.0025 \text{ m}^3/\text{kg}$$

We see that the critical specific volume is more than the specific volume and during the heating process, both the temperature and the pressure remain constant, but the specific volume increases to the critical volume (i.e. critical point). The critical point is defined as the point at which the saturated liquid and saturated vapour states are identical.



So, point (B) will touch the saturated liquid line and the liquid line will rise at the point O.

Q. 7

GATE ME 2007
ONE MARK

Consider an incompressible laminar boundary layer flow over a flat plate of length L , aligned with the direction of an incoming uniform free stream. If F is the ratio of the drag force on the front half of the plate to the drag force on the rear half, then

- (A) $F < 1/2$
- (B) $F = 1/2$
- (C) $F = 1$
- (D) $F > 1$

Sol. 7

Option (D) is correct.

$$F_D = C_D \times \frac{\rho AV^2}{2} = \frac{1.33}{\sqrt{\text{Re}_L}} \times \frac{\rho AV^2}{2} C_D = \frac{1.33}{\sqrt{\text{Re}_L}}$$

$$= \frac{1.33}{\sqrt{\frac{\rho VL}{\mu}}} \times \frac{1}{2} \rho \times bLV^2 \quad \text{Re}_L = \frac{\rho VL}{\mu}$$

$$= \frac{1.33}{\sqrt{\frac{\rho V}{\mu}}} \times \frac{1}{2} \rho bV^2 \sqrt{L} \quad \dots (i)$$

So from equation (i)

$$F_D \propto \sqrt{L} \quad \dots (ii)$$

We can say that Drag force on front half of plate

$$F_{D/2} = \sqrt{\frac{L}{2}} = \frac{F_D}{\sqrt{2}} \quad \text{From Equation (ii)}$$

Drag on rear half,

$$F'_{D/2} = F_D - F_{D/2} = \left(1 - \frac{1}{\sqrt{2}}\right) F_D$$

Now ratio of $F_{D/2}$ & $F'_{D/2}$ is

$$F = \frac{F_{D/2}}{F'_{D/2}} = \frac{\frac{F_D}{\sqrt{2}}}{\left(1 - \frac{1}{\sqrt{2}}\right)F_D} = \frac{1}{\sqrt{2} - 1} > 1$$

Q. 8

GATE ME 2007
ONE MARK

In a steady flow through a nozzle, the flow velocity on the nozzle axis is given by $v = u_0(1 + 3x/L)$, where x is the distance along the axis of the nozzle from its inlet plane and L is the length of the nozzle. The time required for a fluid particle on the axis to travel from the inlet to the exit plane of the nozzle is

- (A) $\frac{L}{u_0}$
- (B) $\frac{L}{3u_0} \ln 4$
- (C) $\frac{L}{4u_0}$
- (D) $\frac{L}{2.5u_0}$

Sol. 8

Option (B) is correct.

Given :

$$v = u_0 \left(1 + \frac{3x}{L}\right)$$

$$\frac{dx}{dt} = u_0 \left(1 + \frac{3x}{L}\right) = \frac{u_0}{L} (L + 3x)$$

$$dt = \frac{L}{u_0} \times \frac{1}{(L + 3x)} dx$$

On integrating both the sides within limits $t \Rightarrow 0$ to t and $x \Rightarrow 0$ to L , we get

$$\int_0^t dt = \frac{L}{u_0} \int_0^L \frac{1}{(L + 3x)} dx$$

$$[t]_0^t = \frac{L}{3u_0} [\ln(L + 3x)]_0^L$$

$$t = \frac{L}{3u_0} [\ln 4L - \ln L] = \frac{L}{3u_0} \ln 4$$

Q. 9

GATE ME 2007
ONE MARK

Consider steady laminar incompressible anti-symmetric fully developed viscous flow through a straight circular pipe of constant cross-sectional area at a Reynolds number of 5. The ratio of inertia force to viscous force on a fluid particle is

- (A) 5
- (B) 1/5
- (C) 0
- (D) ∞

Sol. 9

Option (A) is correct.

Reynolds Number,

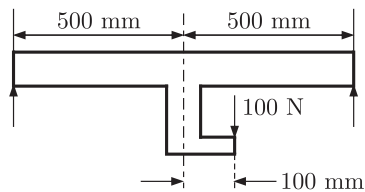
$$Re = \frac{\text{Inertia force}}{\text{Viscous force}} = \frac{\rho A V^2}{\mu \times \frac{V}{L} \times A}$$

$$= \frac{\rho V L}{\mu} = 5 = \frac{I.F.}{V.F.}$$

Q. 10

GATE ME 2007
ONE MARK

In a simply-supported beam loaded as shown below, the maximum bending moment in Nm is



- (A) 25
- (B) 30
- (C) 35
- (D) 60

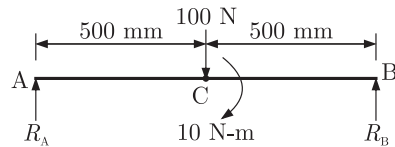
Sol. 10

Option (B) is correct.

Due to 100 N force, bending moment occurs at point C & magnitude of this bending moment is,

$$M_C = 100 \times (0.1) = 10 \text{ N-m (in clock wise direction)}$$

We have to make a free body diagram of the given beam,



Where R_A & R_B are the reactions acting at point A & B

For equilibrium of forces,

$$R_A + R_B = 100 \text{ N} \quad \dots (i)$$

Taking the moment about point A ,

$$100 \times 0.5 + 10 = R_B \times 1 \Rightarrow R_B = 60 \text{ N}$$

From equation (i),

$$R_A = 100 - R_B = 100 - 60 = 40 \text{ N}$$

Maximum bending moment occurs at point C ,

$$\begin{aligned} M_C &= R_A \times 0.5 + 10 \\ &= 40 \times 0.5 + 10 = 20 + 10 = 30 \text{ N-m} \end{aligned}$$

Q. 11

GATE ME 2007
ONE MARK

A ball bearing operating at a load F has 8000 hours of life. The life of the bearing, in hours, when the load is doubled to $2F$ is

- (A) 8000 (B) 6000
(C) 4000 (D) 1000

Sol. 11

Option (D) is correct.

Given : $W_1 = F$, $W_2 = 2F$, $L_1 = 8000 \text{ hr}$

We know that, life of bearing is given by

$$L = \left(\frac{C}{W}\right)^k \times 10^6 \text{ revolution}$$

For ball bearing, $k = 3$, $L = \left(\frac{C}{W}\right)^3 \times 10^6 \text{ revolution}$

For initial condition life is,

$$L_1 = \left(\frac{C}{F}\right)^3 \times 10^6$$

$$8000 \text{ hr} = \left(\frac{C}{F}\right)^3 \times 10^6 \quad \dots (i)$$

For final load,

$$L_2 = \left(\frac{C}{2F}\right)^3 \times 10^6 = \frac{1}{8} \times \left(\frac{C}{F}\right)^3 \times 10^6$$

$$= \frac{1}{8} (8000 \text{ hr}) = 1000 \text{ hr} \quad \text{From equation (i)}$$

Q. 12

GATE ME 2007
ONE MARK

During inelastic collision of two particles, which one of the following is conserved ?

- (A) Total linear momentum only
(B) Total kinetic energy only
(C) Both linear momentum and kinetic energy
(D) Neither linear momentum nor kinetic energy

Sol. 12

Option (A) is correct.

In both elastic & in inelastic collision total linear momentum remains conserved. In the inelastic collision loss in kinetic energy occurs because the coefficient of restitution is less than one and loss in kinetic energy is given by the relation,

$$\Delta K.E. = \frac{m_1 m_2}{2(m_1 + m_2)} (u_1 - u_2)^2 (1 - e^2)$$

Q. 13

GATE ME 2007
ONE MARK

A steel rod of length L and diameter D , fixed at both ends, is uniformly heated to a temperature rise of ΔT . The Young's modulus is E and the co-efficient of linear expansion is α . The thermal stress in the rod is

- (A) 0 (B) $\alpha \Delta T$
(C) $E\alpha \Delta T$ (D) $E\alpha \Delta TL$

Sol. 13

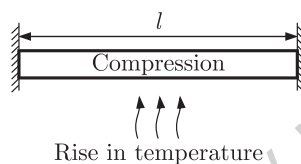
Option (C) is correct.

Let,

 l = original length of the bar α = Co-efficient of linear expansion of the bar material ΔT = Rise or drop in temperature of the bar δl = Change in length which would have occurred due to difference of temperature if the ends of the bar were free

to

expand or contract,



$$\alpha = \frac{\delta l}{l \times \Delta T}$$

or,

$$\delta l = l \times \alpha \times \Delta T$$

And temperature strain,

$$\varepsilon = \frac{\delta l}{l} = \frac{l \times \alpha \times \Delta T}{l} = \alpha \times \Delta T$$

Basically temperature stress and strain are longitudinal (i.e. tensile or compressive) stress and strain

$$E = \frac{\text{Stress}}{\text{Strain}} = \frac{\sigma}{\varepsilon}$$

$$\sigma = E\varepsilon = E\alpha \Delta T$$

Q. 14

GATE ME 2007
ONE MARK

For an under damped harmonic oscillator, resonance

- (A) occurs when excitation frequency is greater than undamped natural frequency
(B) occurs when excitation frequency is less than undamped natural frequency
(C) occurs when excitation frequency is equal to undamped natural frequency
(D) never occurs

Sol. 14

Option (C) is correct.

For an under damped harmonic oscillator resonance occurs when excitation frequency is equal to the undamped natural frequency

$$\omega_d = \omega_n$$

- Q. 15 If a particular Fe-C alloy contains less than 0.83% carbon, it is called
 (A) high speed steel (B) hypoeutectoid steel
 (C) hypereutectoid steel (D) cast iron

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ONE MARK

Sol. 15 Option (B) is correct.

The carbon alloy having less than 2% carbon are called “steels” and those containing over 2% carbon are called cast irons.

Now, steel may further be classified into two groups.

- (i) Steels having less than 0.83% carbon are called “hypo-eutectoid steels”
 (ii) Those having more than 0.83% carbon called “hyper-eutectoid steels”

- Q. 16 Which of the following engineering materials is the most suitable candidate for hot chamber die casting ?

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- (A) low carbon steel (B) titanium
 (C) copper (D) tin

Sol. 16 Option (D) is correct.

The hot chamber die casting process is used for low melting temperature alloys. Tin is a low melting temperature alloy.

- Q. 17 Which one of the following is a solid state joining process ?

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- (A) gas tungsten arc welding (B) resistance spot welding
 (C) friction welding (D) submerged arc welding

Sol. 17 Option (C) is correct.

Friction welding is defined as “ A solid state welding process wherein coalescence is produced by heat obtained from mechanically induced sliding motion between rubbing surfaces.

- Q. 18 In orthogonal turning of a low carbon steel bar of diameter 150 mm with uncoated carbide tool, the cutting velocity is 90 m/min. The feed is 0.24 mm/rev and the depth of cut is 2 mm. The chip thickness obtained is 0.48 mm. If the orthogonal rake angle is zero and the principle cutting edge angle is 90° , the shear angle in degree is

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ONE MARK

- (A) 20.56 (B) 26.56
 (C) 30.56 (D) 36.56

Sol. 18 Option (B) is correct.

Given : $D = 150$ mm, $V = 90$ m/min, $f = 0.24$ mm/rev.

$d = 2$ mm, $t_c = 0.48$ mm, $\alpha = 0^\circ$, $\lambda = 90^\circ$

Uncut chip thickness, $t = f \sin \lambda = 0.24 \times \sin 90^\circ = 0.24$ mm

Chip thickness ratio, $r = \frac{t}{t_c} = \frac{0.24}{0.48} = \frac{1}{2}$

From merchant’s theory,

Shear angle, $\tan \phi = \frac{r \cos \alpha}{1 - r \sin \alpha} = \frac{0.5 \cos 0^\circ}{1 - 0.5 \times \sin 0^\circ} = 0.5$

$\phi = \tan^{-1}(0.5) = 26.56^\circ$

Q. 19 Which type of motor is NOT used in axis or spindle drives of CNC machine tools ?

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- (A) induction motor (B) dc servo motor
(C) stepper motor (D) linear servo motor

Sol. 19 Option (C) is correct.

A spindle motor is a small, high precision, high reliability electric motor that is used to rotate the shaft or spindle used in machine tools for performing a wide rang of tasks like drilling, grinding, milling etc.

A stepper motor have not all these characteristic due to change of direction of rotation with time interval.

Q. 20 Volume of a cube of side 'l' and volume of a sphere of radius 'r' are equal. Both the cube and the sphere are solid and of same material. They are being cast. The ratio of the solidification time of the cube to the same of the sphere is

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ONE MARK

- (A) $\left(\frac{4\pi}{6}\right)^3 \left(\frac{l}{l}\right)^6$ (B) $\left(\frac{4\pi}{6}\right) \left(\frac{l}{l}\right)^2$
(C) $\left(\frac{4\pi}{6}\right)^2 \left(\frac{l}{l}\right)^3$ (D) $\left(\frac{4\pi}{6}\right)^2 \left(\frac{l}{l}\right)^4$

Sol. 20 Option (D) is correct.

According to Caine's relation

Solidification time, $(T) = q \left(\frac{V}{A}\right)^2$

Where : $V =$ Volume, $A =$ Surface area, $Q =$ Flow rate

$q =$ constant of proportionality depends upon composition of cast metal

Using the subscript c for the cube and subscript s for the sphere.

Given : $V_c = V_s$ So, $T \propto \frac{1}{A^2}$

$$\text{So, } \frac{T_c}{T_s} = \left(\frac{A_s}{A_c}\right)^2$$

$$= \left(\frac{4\pi r^2}{6l^2}\right)^2 = \left(\frac{4\pi}{6}\right)^2 \left(\frac{l}{l}\right)^4$$

Q. 21 If $y = x + \sqrt{x + \sqrt{x + \sqrt{x + \dots \infty}}}$, then $y(2) =$

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TWO MARK

- (A) 4 or 1 (B) 4 only
(C) 1 only (D) undefined

Sol. 21 Option (B) is correct.

Given $y = x + \sqrt{x + \sqrt{x + \sqrt{x + \dots \infty}}}$... (i)

$$y - x = \sqrt{x + \sqrt{x + \sqrt{x + \dots \infty}}}$$

Squaring both the sides,

$$(y - x)^2 = x + \sqrt{x + \sqrt{x + \dots \infty}}$$

$$(y - x)^2 = y \quad \text{From equation (i)}$$

$$y^2 + x^2 - 2xy = y \quad \dots \text{(ii)}$$

We have to find $y(2)$, put $x = 2$ in equation (ii),

$$y^2 + 4 - 4y = y$$

$$y^2 - 5y + 4 = 0$$

$$(y - 4)(y - 1) = 0$$

$$y = 1, 4$$

From Equation (i) we see that

$$\text{For } y(2) \quad y = 2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \dots \infty}}} > 2$$

Therefore, $y = 4$

Q. 22

GATE ME 2007
TWO MARK

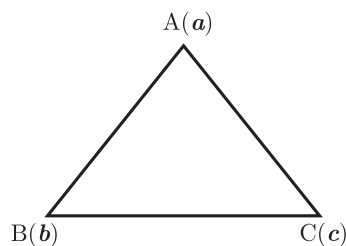
The area of a triangle formed by the tips of vectors \vec{a} , \vec{b} and \vec{c} is

(A) $\frac{1}{2}(\vec{a} - \vec{b}) \cdot (\vec{a} - \vec{c})$ (B) $\frac{1}{2}|(\vec{a} - \vec{b}) \times (\vec{a} - \vec{c})|$

(C) $\frac{1}{2}|\vec{a} \times \vec{b} \times \vec{c}|$ (D) $\frac{1}{2}(\vec{a} \times \vec{b}) \cdot \vec{c}$

Sol. 22

Option (B) is correct.



Vector area of ΔABC ,

$$A = \frac{1}{2} \mathbf{BC} \times \mathbf{BA}$$

$$= \frac{1}{2}(\mathbf{c} - \mathbf{b}) \times (\mathbf{a} - \mathbf{b})$$

$$A = \frac{1}{2}[\mathbf{c} \times \mathbf{a} - \mathbf{c} \times \mathbf{b} - \mathbf{b} \times \mathbf{a} + \mathbf{b} \times \mathbf{b}]$$

$$A = \frac{1}{2}[\mathbf{c} \times \mathbf{a} + \mathbf{b} \times \mathbf{c} + \mathbf{a} \times \mathbf{b}]$$

$$\mathbf{b} \times \mathbf{b} = 0 \text{ \& } \mathbf{c} \times \mathbf{b} = -(\mathbf{b} \times \mathbf{c})$$

$$A = \frac{1}{2}[(\mathbf{a} - \mathbf{b}) \times (\mathbf{a} - \mathbf{c})]$$

Q. 23

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TWO MARK

The solution of $\frac{dy}{dx} = y^2$ with initial value $y(0) = 1$ bounded in the interval

(A) $-\infty \leq x \leq \infty$ (B) $-\infty \leq x \leq 1$

(C) $x < 1, x > 1$ (D) $-2 \leq x \leq 2$

Sol. 23

Option (C) is correct.

Given : $\frac{dy}{dx} = y^2$

$$\frac{dy}{y^2} = dx$$

Integrating both the sides

$$\int \frac{dy}{y^2} = \int dx$$

$$-\frac{1}{y} = x + C \quad \dots(i)$$

Given $y(0) = 1$ at $x = 0 \Rightarrow y = 1$

Put in equation (i) for the value of C

$$-\frac{1}{1} = 0 + C \Rightarrow C = -1$$

From equation (i),

$$-\frac{1}{y} = x - 1$$

$$y = -\frac{1}{x - 1}$$

For this value of y , $x - 1 \neq 0$

$$x \neq 1$$

And $x < 1$ or $x > 1$

Q. 24

GATE ME 2007
TWO MARK

If $F(s)$ is the Laplace transform of function $f(t)$, then Laplace transform of

$\int_0^t f(\tau) d\tau$ is

(A) $\frac{1}{s}F(s)$ (B) $\frac{1}{s}F(s) - f(0)$

(C) $sF(s) - f(0)$ (D) $\int F(s) ds$

Sol. 24

Option (A) is correct.

Let $\phi(t) = \int_0^t f(t) dt$ and $\phi(0) = 0$ then $\phi'(t) = f(t)$

We know the formula of Laplace transforms of $\phi'(t)$ is

$$L[\phi'(t)] = sL[\phi(t)] - \phi(0)$$

$$L[\phi'(t)] = sL[\phi(t)] \quad \phi(0) = 0$$

$$L[\phi(t)] = \frac{1}{s}L[\phi'(t)]$$

Substitute the values of $\phi(t)$ & $\phi'(t)$, we get

$$L\left[\int_0^t f(t) dt\right] = \frac{1}{s}L[f(t)]$$

or $L\left[\int_0^t f(t) dt\right] = \frac{1}{s}F(s)$

Q. 25

GATE ME 2007
TWO MARK

A calculator has accuracy up to 8 digits after decimal place. The value of $\int_0^{2\pi} \sin x dx$ when evaluated using the calculator by trapezoidal method with 8 equal intervals, to 5 significant digits is

- (A) 0.00000 (B) 1.0000
(C) 0.00500 (D) 0.00025

Sol. 25

Option (A) is correct.

From the Trapezoidal Method

$$\int_a^b f(x) dx = \frac{h}{2}[f(x_0) + 2f(x_1) + 2f(x_2) + \dots + 2f(x_{n-1}) + f(x_n)]$$

... (i)

$$\text{Interval } h = \frac{2\pi - 0}{8} = \frac{\pi}{4}$$

Find $\int_0^{2\pi} \sin x dx$ Here $f(x) = \sin x$

Then we have to make the table for the interval of $\pi/4$

Angle θ	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$	$\frac{7\pi}{4}$	2π
$f(x) = \sin x$	0	0.707	1	0.707	0	-0.707	-1	-0.707	0

Now from equation(i),

$$\int_0^{2\pi} \sin x dx = \frac{\pi}{8} [0 + 2(0.707 + 1 + 0.707 + 0 - 0.707 - 1 - 0.707 + 0)]$$

$$= \frac{\pi}{8} \times 0 = 0$$

Q. 26

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TWO MARK

Let X and Y be two independent random variables. Which one of the relations between expectation (E), variance (Var) and covariance (Cov) given below is FALSE ?

- (A) $E(XY) = E(X)E(Y)$ (B) $\text{Cov}(X, Y) = 0$
 (C) $\text{Var}(X + Y) = \text{Var}(X) + \text{Var}(Y)$ (D) $E(X^2 Y^2) = (E(X))^2 (E(Y))^2$

Sol. 26

Option (D) is correct.

The X and Y be two independent random variables.

So, $E(XY) = E(X)E(Y)$ (i)

& covariance is defined as

$$\begin{aligned} \text{Cov}(X, Y) &= E(XY) - E(X)E(Y) \\ &= E(X)E(Y) - E(X)E(Y) \quad \text{From eqn. (i)} \\ &= 0 \end{aligned}$$

For two independent random variables

$$\text{Var}(X + Y) = \text{Var}(X) + \text{Var}(Y)$$

& $E(X^2 Y^2) = E(X^2)E(Y^2)$

So, option (D) is incorrect.

Q. 27

GATE ME 2007
TWO MARK

$$\lim_{x \rightarrow 0} \frac{e^x - \left(1 + x + \frac{x^2}{2}\right)}{x^3} =$$

- (A) 0 (B) 1/6
(C) 1/3 (D) 1

Sol. 27

Option (B) is correct.

Let, $f(x) = \lim_{x \rightarrow 0} \frac{e^x - \left(1 + x + \frac{x^2}{2}\right)}{x^3} \quad \frac{0}{0} \text{ form}$

Applying the L-Hospital rule,

$$= \lim_{x \rightarrow 0} \frac{e^x - (1 + x)}{3x^2} \quad \frac{0}{0} \text{ form}$$

Again applying L-Hospital Rule,

$$= \lim_{x \rightarrow 0} \frac{e^x - 1}{6x} \quad \frac{0}{0} \text{ form}$$

Again applying L-Hospital Rule,

$$\begin{aligned} &= \lim_{x \rightarrow 0} \frac{e^x}{6} \\ &= \frac{e^0}{6} = \frac{1}{6} \end{aligned}$$

- Q. 28 The number of linearly independent eigen vectors of $\begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}$ is
 (A) 0 (B) 1
 (C) 2 (D) infinite

GATE ME 2007
 TWO MARK

Sol. 28 Option (B) is correct.

Let, $A = \begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}$

Let λ is the eigen value of the given matrix then characteristic matrix is

$$|A - \lambda I| = 0 \quad \text{Here } I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \text{Identity matrix}$$

$$\begin{vmatrix} 2 - \lambda & 1 \\ 0 & 2 - \lambda \end{vmatrix} = 0$$

$$(2 - \lambda)^2 = 0$$

$$\lambda = 2, 2$$

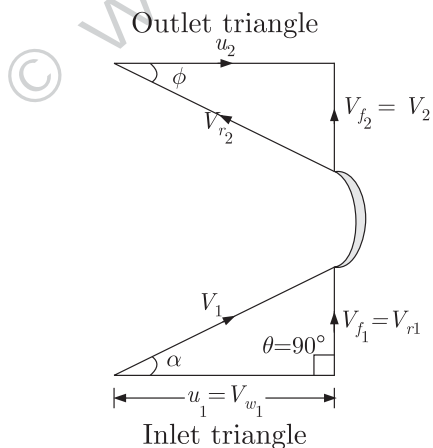
So, only one eigen vector.

- Q. 29 The inlet angle of runner blades of a Francis turbine is 90° . The blades are so shaped that the tangential component of velocity at blade outlet is zero. The flow velocity remains constant throughout the blade passage and is equal to half of the blade velocity at runner inlet. The blade efficiency of the runner is
 (A) 25% (B) 50%
 (C) 80% (D) 89%

GATE ME 2007
 TWO MARK

Sol. 29 Option (C) is correct.

Given figure shows the velocity triangle for the pelton wheel.



Velocity triangle for Francis turbine

Given :

Flow velocity at Inlet V_{f1} = flow velocity at outlet V_{f2}

$$V_{f1} = V_{f2} = \frac{u_1}{2} \text{ (blade velocity)}$$

$$V_2 = V_{f2}$$

$$u_1 = V_{w1}$$

$$\theta = 90^\circ$$

From Inlet triangle,

$$V_1^2 = (V_h)^2 + (V_w)^2 = \left(\frac{u_1}{2}\right)^2 + (u_1)^2 = \frac{5}{4} u_1^2$$

$$\begin{aligned} \text{Blade efficiency} &= \frac{V_1^2 - V_2^2}{V_1^2} \times 100 = \frac{\frac{5}{4} u_1^2 - \frac{u_1^2}{4}}{\frac{5}{4} u_1^2} \times 100 \\ &= \frac{u_1^2}{\frac{5}{4} u_1^2} \times 100 = 80\% \end{aligned}$$

Q. 30

GATE ME 2007
TWO MARK

The temperature distribution within the thermal boundary layer over a heated isothermal flat plate is given by $\frac{T - T_w}{T_\infty - T_w} = \frac{3}{2} \left(\frac{y}{\delta_t}\right) - \frac{1}{2} \left(\frac{y}{\delta_t}\right)^3$, where T_w and T_∞ are the temperature of plate and free stream respectively, and y is the normal distance measured from the plate. The local Nusselt number based on the thermal boundary layer thickness δ_t is given by

- (A) 1.33 (B) 1.50
(C) 2.0 (D) 4.64

Sol. 30

Option (B) is correct.

The region beyond the thermal entrance region in which the dimensionless temperature profile expressed as $\left(\frac{T - T_w}{T_\infty - T_w}\right)$ remains unchanged is called thermally fully developed region.

Nusselt Number is given by,

$$N_u = \frac{hL}{k} = \left(\frac{\partial T}{\partial y'}\right)_{\text{at } y'=0} \quad \dots (i)$$

Here, $T = \frac{T - T_w}{T_\infty - T_w}$ & $y' = \frac{y}{\delta_t}$

$$\begin{aligned} \text{So, } N_u &= \frac{\partial}{\partial y'} \left[\frac{3}{2} \left(\frac{y}{\delta_t}\right) - \frac{1}{2} \left(\frac{y}{\delta_t}\right)^3 \right]_{y'=0} = \frac{\partial}{\partial y'} \left[\frac{3}{2} y' - \frac{1}{2} (y')^3 \right]_{y'=0} \\ &= \left[\frac{3}{2} - \frac{3}{2} \left(\frac{y}{\delta_t}\right)^2 \right]_{y'=0} = \frac{3}{2} = 1.5 \end{aligned}$$

Q. 31

GATE ME 2007
TWO MARK

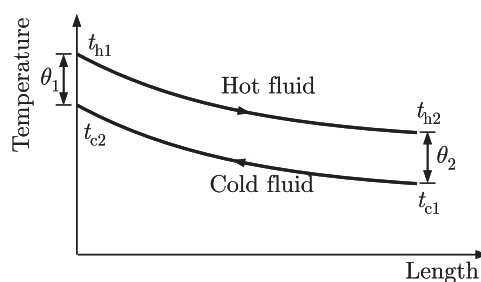
In a counter flow heat exchanger, hot fluid enters at 60°C and cold fluid leaves at 30°C . Mass flow rate of the fluid is 1 kg/s and that of the cold fluid is 2 kg/s . Specific heat of the hot fluid is 10 kJ/kgK and that of the cold fluid is 5 kJ/kgK . The Log Mean Temperature Difference (LMTD) for the heat exchanger in $^\circ\text{C}$ is

- (A) 15 (B) 30
(C) 35 (D) 45

Sol. 31

Option (B) is correct.

The counter flow arrangement of the fluid shown below :



Given: for hot fluid :

$$t_{h1} = 60^\circ \text{C}, \dot{m}_h = 1 \text{ kg/sec}, c_h = 10 \text{ kJ/kg K}$$

And for cold fluid :

$$t_{c2} = 30^\circ \text{C}, \dot{m}_c = 2 \text{ kg/sec}, c_c = 5 \text{ kJ/kg K}$$

Heat capacity of Hot fluid,

$$C_h = \dot{m}_h c_h = 1 \times 10 = 10 \text{ kJ/k. sec}$$

And heat capacity of cold fluid,

$$C_c = \dot{m}_c c_c = 2 \times 5 = 10 \text{ kJ/k sec}$$

By energy balance for the counter flow

$$\dot{m}_h c_h (t_{h1} - t_{h2}) = \dot{m}_c c_c (t_{c2} - t_{c1})$$

$$C_h (t_{h1} - t_{h2}) = C_c (t_{c2} - t_{c1})$$

$$C_h = C_c$$

$$t_{h1} - t_{c2} = t_{h2} - t_{c1}$$

$$\theta_1 = \theta_2$$

$$\text{LMTD, } \theta_m = \frac{\theta_1 - \theta_2}{\ln\left(\frac{\theta_1}{\theta_2}\right)} \quad \dots (i)$$

Let, $\frac{\theta_1}{\theta_2} = x$ θ_1 is equal to θ_2 & θ_m is undetermined

$$\theta_1 = x\theta_2$$

Substituting θ_1 in equation (i), we get,

$$\theta_m = \lim_{x \rightarrow 1} \frac{x\theta_2 - \theta_2}{\ln(x)} = \lim_{x \rightarrow 1} \frac{\theta_2(x-1)}{\ln(x)}$$

$\left(\frac{0}{0}\right)$ form, So we apply L-hospital rule,

$$\theta_m = \lim_{x \rightarrow 1} \frac{\theta_2 \times 1}{\frac{1}{x}} = \lim_{x \rightarrow 1} x\theta_2$$

$$\theta_m = \theta_2 = \theta_1 \Rightarrow \theta_1 = t_{h1} - t_{c2} \\ = 60 - 30 = 30^\circ \text{C}$$

Q. 32

GATE ME 2007
TWO MARK

The average heat transfer co-efficient on a thin hot vertical plate suspended in still air can be determined from observations of the change in plate temperature with time as it cools. Assume the plate temperature to be uniform at any instant of time and radiation heat exchange with the surroundings negligible. The ambient temperature is 25°C , the plate has a total surface area of 0.1 m^2 and a mass of 4 kg . The specific heat of the plate material is 2.5 kJ/kgK . The convective heat transfer co-efficient in $\text{W/m}^2 \text{ K}$, at the instant when the plate temperature is 225°C and the change in plate temperature with time $dT/dt = -0.02 \text{ K/s}$, is

(A) 200

(B) 20

(C) 15

(D) 10

Sol. 32

Option (D) is correct.

Given : $T_1 = 25^\circ \text{C} = (273 + 25) = 298 \text{ K}$, $A = 0.1 \text{ m}^2$, $m = 4 \text{ kg}$, $c = 2.5 \text{ kJ/kg K}$
 $h = ?$, $T_2 = 225^\circ \text{C} = 273 + 225 = 498 \text{ K}$

$$\text{Temperature Gradient, } \frac{dT}{dt} = -0.02 \text{ K/s}$$

Here negative sign shows that plate temperature decreases with the time.

From the given condition,

Heat transfer by convection to the plate = Rate of change of internal energy

$$hA(T_2 - T_1) = -mc \frac{dT}{dt}$$

$$h = -\frac{mc}{A(T_2 - T_1)} \times \frac{dT}{dt}$$

$$= -\frac{4 \times 2.5 \times 10^3}{0.1(498 - 298)} \times (-0.02) = 10 \text{ W/m}^2 \text{ K}$$

Q. 33

GATE ME 2007
TWO MARK

A model of a hydraulic turbine is tested at a head of $1/4^{\text{th}}$ of that under which the full scale turbine works. The diameter of the model is half of that of the full scale turbine. If N is the RPM of the full scale turbine, the RPM of the model will be

- (A) $N/4$ (B) $N/2$
(C) N (D) $2N$

Sol. 33

Option (C) is correct.

$$u = \frac{\pi DN}{60} = \sqrt{2gH}$$

From this equation,

$$\sqrt{H} \propto DN$$

$$\frac{\sqrt{H}}{DN} = \text{Constant}$$

So using this relation for the given model or prototype,

$$\left(\frac{\sqrt{H}}{DN}\right)_p = \left(\frac{\sqrt{H}}{DN}\right)_m$$

$$\frac{N_p}{N_m} = \sqrt{\frac{H_p}{H_m}} \times \frac{D_m}{D_p} \quad \dots (i)$$

$$\text{Given : } H_m = \frac{1}{4} H_p, D_m = \frac{1}{2} D_p, N_p = N$$

$$\frac{N}{N_m} = \sqrt{\frac{H_p}{\frac{1}{4} H_p}} \times \frac{\frac{1}{2} D_p}{D_p} = \sqrt{4} \times \frac{1}{2} = 1$$

$$\text{So, } N_m = N$$

Q. 34

GATE ME 2007
TWO MARK

The stroke and bore of a four stroke spark ignition engine are 250 mm and 200 mm respectively. The clearance volume is 0.001 m^3 . If the specific heat ratio $\gamma = 1.4$, the air-standard cycle efficiency of the engine is

- (A) 46.40% (B) 56.10%
(C) 58.20% (D) 62.80%

Sol. 34

Option (C) is correct.

$$\text{Given : } L = 250 \text{ mm} = 0.25 \text{ m}, D = 200 \text{ mm} = 0.2 \text{ m},$$

$$\nu_c = 0.001 \text{ m}^3, \gamma = \frac{C_p}{C_v} = 1.4$$

Swept volume

$$\nu_s = A \times L = \frac{\pi}{4} (D)^2 \times L$$

$$= \frac{\pi}{4} (0.2)^2 \times 0.25 = 0.00785 \text{ m}^3$$

Compression ratio

$$r = \frac{\nu_T}{\nu_c} = \frac{\nu_c + \nu_s}{\nu_c} = \frac{0.001 + 0.00785}{0.001} = 8.85$$

Air standard efficiency

$$\eta = 1 - \frac{1}{(r)^{\gamma-1}} = 1 - \frac{1}{(8.85)^{1.4-1}}$$

$$= 1 - \frac{1}{2.39} = 1 - 0.418 = 0.582 \text{ or } 58.2\%$$

Q. 35

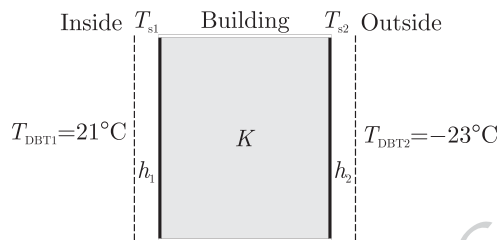
GATE ME 2007
TWO MARK

A building has to be maintained at 21°C (dry bulb) and 14.5°C (wet bulb). The dew point temperature under these conditions is 10.17°C . The outside temperature is -23°C (dry bulb) and the internal and external surface heat transfer coefficients are $8\text{ W/m}^2\text{K}$ and $23\text{ W/m}^2\text{K}$ respectively. If the building wall has a thermal conductivity of 1.2 W/m K , the minimum thickness (in m) of the wall required to prevent condensation is

- (A) 0.471 (B) 0.407
(C) 0.321 (D) 0.125

Sol. 35

Option (B) is correct.



Let h_1 & h_2 be the internal and external surface heat transfer coefficients respectively and building wall has thermal conductivity k .

Given : $h_1 = 8\text{ W/m}^2\text{K}$, $h_2 = 23\text{ W/m}^2\text{K}$, $k = 1.2\text{ W/m K}$, $T_{DPT} = 10.17^\circ\text{C}$

Now to prevent condensation, temperature of inner wall should be more than or equal to the dew point temperature. It is the limiting condition to prevent condensation

So, $T_{s1} = 10.17^\circ\text{C}$

Here T_{s1} & T_{s2} are internal & external wall surface temperature of building.

Hence, heat flux per unit area inside the building,

$$q_i = \frac{Q}{A} = h_1(T_{DBT1} - T_{s1})$$

$$q_i = 8(21 - 10.17) = 8 \times 10.83 = 86.64\text{ W/m}^2 \dots (i)$$

& Heat flux per unit area outside the building is

$$q_o = h_2(T_{s2} - T_{DBT2}) = 23(T_{s2} + 23) \dots (ii)$$

Heat flow will be same at inside & outside the building. So from equation (i) & (ii)

$$\begin{aligned} q_i &= q_o \\ 86.64 &= 23(T_{s2} + 23) \\ T_{s2} + 23 &= 3.767 \\ T_{s2} &= 3.767 - 23 = -19.23^\circ\text{C} \end{aligned}$$

For minimum thickness of the wall, use the fourier's law of conduction for the building. Heat flux through wall,

$$q = \frac{k(T_{s1} - T_{s2})}{x} = \frac{1.2 \times (10.17 + 19.23)}{x}$$

Substitute the value of q_i from equation (i), we get

$$\begin{aligned} 86.64 &= \frac{1.2 \times 29.4}{x} \\ x &= \frac{35.28}{86.64} = 0.407\text{ m} \end{aligned}$$

Note :- Same result is obtained with the value of q_o

Q. 36

GATE ME 2007
TWO MARK

Atmospheric air at a flow rate of 3 kg/s (on dry basis) enters a cooling and dehumidifying coil with an enthalpy of 85 kJ/kg of dry air and a humidity ratio of 19 grams/kg of dry air. The air leaves the coil with an enthalpy of 43 kJ/kg of dry air and a humidity ratio of 8 grams/kg of dry air. If the condensate water leaves the coil with an enthalpy of 67 kJ/kg, the required cooling capacity of the coil in kW is

- (A) 75.0
(B) 123.8
(C) 128.2
(D) 159.0

Sol. 36

Option (C) is correct.

Given : $\dot{m}_a = 3$ kg/sec,

Using subscript 1 and 2 for the inlet and outlet of the coil respectively.

$h_1 = 85$ kJ/kg of dry air, $W_1 = 19$ grams/kg of dry air = 19×10^{-3} kg/kg of dry air

$h_2 = 43$ kJ/kg of dry air, $W_2 = 8$ grams/kg of dry air = 8×10^{-3} kg/kg of dry air

$h_3 = 67$ kJ/kg

Mass flow rate of water vapour at the inlet of the coil is,

$$\dot{m}_{v1} = W_1 \times \dot{m}_a \quad W = \frac{\dot{m}_v}{\dot{m}_a}$$

$$\dot{m}_{v1} = 19 \times 10^{-3} \times 3 = 57 \times 10^{-3} \text{ kg/sec}$$

And mass flow rate of water vapour at the outlet of coil is,

$$\begin{aligned} \dot{m}_{v2} &= W_2 \times \dot{m}_a \\ &= 8 \times 10^{-3} \times 3 = 24 \times 10^{-3} \text{ kg/sec} \end{aligned}$$

So, mass of water vapour condensed in the coil is,

$$\begin{aligned} \dot{m}_v &= \dot{m}_{v1} - \dot{m}_{v2} \\ &= (57 - 24) \times 10^{-3} = 33 \times 10^{-3} \text{ kg/sec} \end{aligned}$$

Therefore, required cooling capacity of the coil = change in enthalpy of dry air + change in enthalpy of condensed water

$$\begin{aligned} &= (85 - 43) \times 3 + 67 \times 33 \times 10^{-3} \\ &= 128.211 \text{ kW} \end{aligned}$$

Q. 37

GATE ME 2007
TWO MARK

A heat transformer is device that transfers a part of the heat, supplied to it at an intermediate temperature, to a high temperature reservoir while rejecting the remaining part to a low temperature heat sink. In such a heat transformer, 100 kJ of heat is supplied at 350 K. The maximum amount of heat in kJ that can be transferred to 400 K, when the rest is rejected to a heat sink at 300 K is

- (A) 12.50
(B) 14.29
(C) 33.33
(D) 57.14

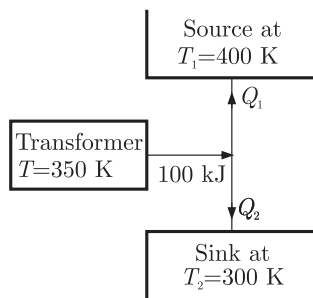
Sol. 37

Option (D) is correct.

Given : $T_1 = 400$ K, $T_2 = 300$ K, $T = 350$ K, $Q = 100$ kJ

Q_1 → Heat transferred to the source by the transformer

Q_2 → Heat transferred to the sink by the transformer



Applying energy balance on the system,

$$Q = Q_1 + Q_2$$

$$Q_2 = Q - Q_1 = 100 - Q_1 \quad \dots (i)$$

Apply Clausius inequality on the system,

$$\frac{Q}{T} = \frac{Q_1}{T_1} + \frac{Q_2}{T_2}$$

$$\frac{100}{350} = \frac{Q_1}{400} + \frac{Q_2}{300}$$

Substitute the value of Q_2 from equation (i),

$$\frac{100}{350} = \frac{Q_1}{400} + \left(\frac{100 - Q_1}{300}\right) = \frac{Q_1}{400} + \frac{100}{300} - \frac{Q_1}{300}$$

$$\frac{100}{350} - \frac{100}{300} = Q_1 \left[\frac{1}{400} - \frac{1}{300} \right]$$

$$-\frac{1}{21} = -\frac{Q_1}{1200}$$

So, $Q_1 = \frac{1200}{21} = 57.14 \text{ kJ}$

Therefore the maximum amount of heat that can be transferred at 400 K is 57.14 kJ.

Q. 38

GATE ME 2007
TWO MARK

Which combination of the following statements is correct ?

The incorporation of reheater in a steam power plant :

P : always increases the thermal efficiency of the plant.

Q : always increases the dryness fraction of steam at condenser inlet

R : always increases the mean temperature of heat addition.

S : always increases the specific work output.

(A) P and S

(B) Q and S

(C) P, R and S

(D) P, Q, R and S

Sol. 38

Option (B) is correct.

We know, dryness fraction or quality of the liquid vapour mixture,

$$x = \frac{m_v}{m_v + m_l} = \frac{1}{1 + m_l/m_v} \quad \dots (i)$$

Where,

$m_v \rightarrow$ Mass of vapour and $m_l \rightarrow$ Mass of liquid

The value of x varies between 0 to 1. Now from equation (i) if incorporation of reheater in a steam power plant adopted then Mass of vapour m_v increase & Mass of liquid m_l decreases So, dryness fraction x increases.

In practice the use of reheater only gives a small increase in cycle efficiency, but it increases the net work output by making possible the use of higher pressure.

Q. 39

GATE ME 2007
TWO MARK

Which combination of the following statements is correct ?

P : A gas cools upon expansion only when its Joule-Thomson coefficient is positive in the temperature range of expansion.

Q : For a system undergoing a process, its entropy remains constant only when the process is reversible.

R : The work done by closed system in an adiabatic is a point function.

S : A liquid expands upon freezing when the slope of its fusion curve on pressure-Temperature diagram is negative.

(A) R and S

(B) P and Q

(C) Q, R and S

(D) P, Q and R

Sol. 39

Option (A) is correct.

Following combination is correct

(R) The work done by a closed system in an adiabatic is a point function.

(S) A liquid expands upon freezing when the slope of its fusion curve on pressure-temperature diagram is negative.

Q. 40

GATE ME 2007
ONE MARK

Which combination of the following statements about steady incompressible forced vortex flow is correct ?

P: Shear stress is zero at all points in the flow.

Q: Vorticity is zero at all points in the flow.

R: Velocity is directly proportional to the radius from the center of the vortex.

S: Total mechanical energy per unit mass is constant in the entire flow field.

(A) P and Q

(B) R and S

(C) P and R

(D) P and S

Sol. 40

Option (B) is correct.

For forced Vortex flow the relation is given by,

$$V = r\omega \quad \dots(i)$$

From equation (i) it is shown easily that velocity is directly proportional to the radius from the centre of the vortex (Radius of fluid particle from the axis of rotation)

And also for forced vortex flow,

$$\frac{1}{2}\rho\omega^2(r_2^2 - r_1^2) - \rho g(z_2 - z_1) = 0$$

$$\Delta K.E. - \Delta P.E. = 0 \Rightarrow \Delta K.E. = \Delta P.E.$$

Now total mechanical energy per unit mass is constant in the entire flow field.

Q. 41

GATE ME 2007
TWO MARK

Match List-I with List-II and select the correct answer using the codes given below the lists :

	List-I		List-II
P.	Centrifugal compressor	1.	Axial flow
Q.	Centrifugal pump	2.	Surging
R.	Pelton wheel	3.	Priming
S.	Kaplan turbine	4.	Pure impulse

Codes :

	P	Q	R	S
(A)	2	3	4	1
(B)	2	3	1	4
(C)	3	4	1	2
(D)	1	2	3	4

Sol. 41

Option (A) is correct.

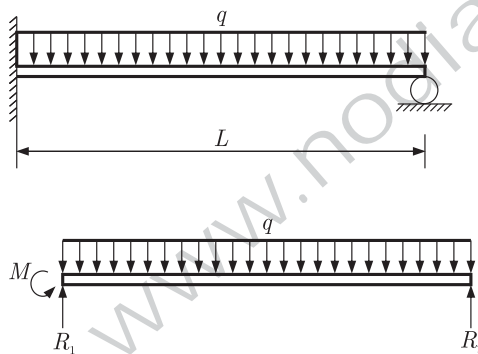
	List-I		List-II
P.	Centrifugal compressor	2.	Surging
Q.	Centrifugal pump	3.	Priming
R.	Pelton wheel	4.	Pure Impulse
S.	Kaplan Turbine	1.	Axial Flow

So, correct pairs are P-2, Q-3, R-4, S-1

Q. 42

GATE ME 2007
TWO MARK

A uniformly loaded propped cantilever beam and its free body diagram are shown below. The reactions are

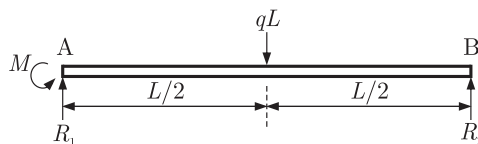


- (A) $R_1 = \frac{5qL}{8}, R_2 = \frac{3qL}{8}, M = \frac{qL^2}{8}$ (B) $R_1 = \frac{3qL}{8}, R_2 = \frac{5qL}{8}, M = \frac{qL^2}{8}$
 (C) $R_1 = \frac{5qL}{8}, R_2 = \frac{3qL}{8}, M = 0$ (D) $R_1 = \frac{3qL}{8}, R_2 = \frac{5qL}{8}, M = 0$

Sol. 42

Option (A) is correct.

First of all, we have to make a FBD of the beam. We know that a UDL acting at the mid-point of the beam and its magnitude is equal to $(q \times L)$. So,



In equilibrium of forces,

$$R_1 + R_2 = qL \quad \dots (i)$$

This cantilever beam is subjected to two types of load.

First load is due to UDL and second load is due to point load at B. Due to this deflection occurs at B, which is equal in amount.

So, deflection occurs at B due to the UDL alone,

$$\delta_{UDL} = \frac{qL^4}{8EI}$$

Also, deflection at B due to point load,

$$\delta_{PL} = \frac{R_2 L^3}{3EI}$$

We know, deflections are equal at B,

$$\delta_{UDL} = \delta_{PL}$$

$$\frac{qL^4}{8EI} = \frac{R_2 L^3}{3EI} \Rightarrow R_2 = \frac{3qL}{8}$$

And from equation (i), we have

$$R_1 = qL - R_2 = qL - \frac{3qL}{8} = \frac{5qL}{8}$$

For M , taking the moment about B,

$$-qL \times \frac{L}{2} + R_1 \times L - M = 0$$

$$-\frac{qL^2}{2} + \frac{5qL^2}{8} - M = 0$$

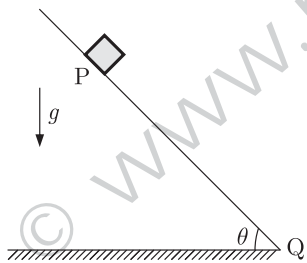
$$M = \frac{qL^2}{8}$$

Therefore, $R_1 = \frac{5qL}{8}$, $R_2 = \frac{3qL}{8}$ and $M = \frac{qL^2}{8}$

Q. 43

GATE ME 2007
TWO MARK

A block of mass M is released from point P on a rough inclined plane with inclination angle θ , shown in the figure below. The co-efficient of friction is μ . If $\mu < \tan \theta$, then the time taken by the block to reach another point Q on the inclined plane, where $PQ = s$, is



(A) $\sqrt{\frac{2s}{g \cos \theta (\tan \theta - \mu)}}$

(B) $\sqrt{\frac{2s}{g \cos \theta (\tan \theta + \mu)}}$

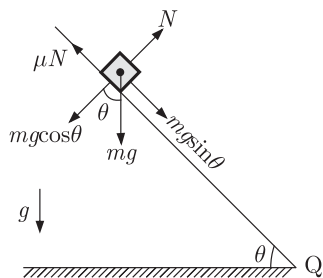
(C) $\sqrt{\frac{2s}{g \sin \theta (\tan \theta - \mu)}}$

(D) $\sqrt{\frac{2s}{g \sin \theta (\tan \theta + \mu)}}$

Sol. 43

Option (A) is correct.

First of all we resolve all the force which are acting on the block.



Given : $PQ = s$ where $N =$ Normal fraction force

$$\mu < \tan \theta$$

Now from Newton's second law,

$$F = ma$$

$$mg \sin \theta - \mu N = ma$$

$a =$ Acceleration of block

$$mg \sin \theta - \mu mg \cos \theta = ma$$

$$N = mg \cos \theta$$

$$g \sin \theta - \mu g \cos \theta = a$$

$$a = g \cos \theta \left[\frac{\sin \theta}{\cos \theta} - \mu \right] = g \cos \theta (\tan \theta - \mu) \quad \dots(i)$$

From the Newton's second law of Motion,

$$s = ut + \frac{1}{2}at^2$$

$$s = 0 + \frac{1}{2}g \cos \theta (\tan \theta - \mu) t^2 \quad u = 0$$

$$t = \sqrt{\frac{2s}{g \cos \theta (\tan \theta - \mu)}}$$

Q. 44

GATE ME 2007
TWO MARK

A $200 \times 100 \times 50$ mm steel block is subjected to a hydrostatic pressure of 15 MPa.

The Young's modulus and Poisson's ratio of the material are 200 GPa and 0.3 respectively. The change in the volume of the block in mm^3 is

- (A) 85
- (B) 90
- (C) 100
- (D) 110

Sol. 44

Option (B) is correct.

Given :

$$V = 200 \times 100 \times 50 \text{ mm}^3 = 10^6 \text{ mm}^3$$

$$p = 15 \text{ MPa} = 15 \times 10^6 \text{ N/m}^2 = 15 \text{ N/mm}^2$$

$$E = 200 \text{ GPa} = 200 \times 10^3 \text{ N/mm}^2$$

$$\left(\nu \text{ or } \frac{1}{m} \right) = 0.3$$

We know the relation between volumetric strain, young's modulus & Poisson's ration is given by,

$$\frac{\Delta V}{V} = \frac{3p}{E} (1 - 2\nu)$$

Substitute the values, we get

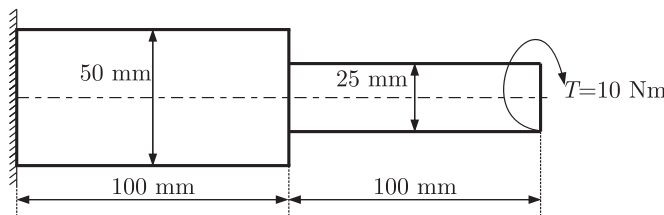
$$\frac{\Delta V}{10^6} = \frac{3 \times 15}{200 \times 10^3} (1 - 2 \times 0.3)$$

$$\Delta V = \frac{45 \times 10}{2} (1 - 0.6) = 225 \times 0.4 = 90 \text{ mm}^3$$

Q. 45

GATE ME 2007
TWO MARK

A stepped steel shaft shown below is subjected to 10 Nm torque. If the modulus of rigidity is 80 GPa, the strain energy in the shaft in N-mm is



- (A) 4.12
- (B) 3.46
- (C) 1.73
- (D) 0.86

Sol. 45

Option (C) is correct.

Given : $T = 10 \text{ N-m} = 10^4 \text{ N-mm}$, $G = 80 \text{ GPa} = 80 \times 10^3 \text{ N/mm}^2$ $L_1 = L_2 = 100 \text{ mm}$, $d_1 = 50 \text{ mm}$, $d_2 = 25 \text{ mm}$ We know that for a shaft of length l and polar moment of inertia J , subjected to a torque T with an angle of twist θ . The expression of strain energy,

$$U = \frac{1}{2} \frac{T^2 l}{GJ} \quad U = \frac{1}{2} T\theta, \text{ \& } \theta = \frac{Tl}{GJ}$$

So Total strain energy,

$$U = \frac{T^2 L}{2GJ_1} + \frac{T^2 L}{2GJ_2}$$

$$U = \frac{T^2 L}{2G} \left[\frac{1}{J_1} + \frac{1}{J_2} \right] \quad J = \frac{\pi}{32} d^4$$

Substitute the values, we get

$$\begin{aligned} U &= \frac{(10^4)^2 \times 100}{2 \times 80 \times 10^3} \left[\frac{1}{\frac{\pi}{32} (50)^4} + \frac{1}{\frac{\pi}{32} (25)^4} \right] \\ &= \frac{10^6}{16} \times \frac{32}{\pi} \left[\frac{1}{625 \times 10^4} + \frac{1}{390625} \right] \\ &= \frac{10^6}{16 \times 10^4} \times \frac{32}{\pi} \left[\frac{1}{625} + \frac{1}{39.0625} \right] \\ &= 63.69 \times [0.0016 + 0.0256] \\ &= 63.69 \times 0.0272 = 1.73 \text{ N-mm} \end{aligned}$$

Q. 46

GATE ME 2007
TWO MARK

A thin spherical pressure vessel of 200 mm diameter and 1 mm thickness is subjected to an internal pressure varying from 4 to 8 MPa. Assume that the yield, ultimate and endurance strength of material are 600, 800 and 400 MPa respectively. The factor of safety as per Goodman's relation is

- (A) 2.0 (B) 1.6
(C) 1.4 (D) 1.2

Sol. 46

Option (B) is correct.

Given : $d = 200 \text{ mm}$, $t = 1 \text{ mm}$, $\sigma_u = 800 \text{ MPa}$, $\sigma_e = 400 \text{ MPa}$

Circumferential stress induced in spherical pressure vessel is,

$$\sigma = \frac{p \times r}{2t} = \frac{p \times 100}{2 \times 1} = 50p \text{ MPa}$$

Given that, pressure vessel is subject to an internal pressure varying from 4 to 8 MPa.

So,

$$\sigma_{\min} = 50 \times 4 = 200 \text{ MPa}$$

$$\sigma_{\max} = 50 \times 8 = 400 \text{ MPa}$$

Mean stress,

$$\sigma_m = \frac{\sigma_{\min} + \sigma_{\max}}{2} = \frac{200 + 400}{2} = 300 \text{ MPa}$$

Variable stress,

$$\sigma_v = \frac{\sigma_{\max} - \sigma_{\min}}{2} = \frac{400 - 200}{2} = 100 \text{ MPa}$$

From the Goodman method,

$$\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_u} + \frac{\sigma_v}{\sigma_e} = \frac{300}{800} + \frac{100}{400}$$

$$\frac{1}{F.S.} = \frac{3}{8} + \frac{1}{4} = \frac{5}{8} \Rightarrow F.S. = \frac{8}{5} = 1.6$$

Q. 47

GATE ME 2007
TWO MARK

A natural feed journal bearing of diameter 50 mm and length 50 mm operating at 20 revolution/ second carries a load of 2 kN. The lubricant used has a viscosity of 20 mPas. The radial clearance is 50 μm. The Sommerfeld number for the bearing is

- (A) 0.062
- (B) 0.125
- (C) 0.250
- (D) 0.785

Sol. 47

Option (B) is correct.

Given : $d = 50 \text{ mm}$, $l = 50 \text{ mm}$, $N = 20 \text{ rps}$, $Z = 20 \text{ mPa-sec} = 20 \times 10^{-3} \text{ Pa-sec}$

Radial clearance = $50 \mu\text{m} = 50 \times 10^{-3} \text{ mm}$, Load = 2 kN

We know that,

$p =$ Bearing Pressure on the projected bearing

area

$$= \frac{\text{Load on the journal}}{l \times d}$$

$$= \frac{2 \times 10^3}{50 \times 50} = 0.8 \text{ N/mm}^2 = 0.8 \times 10^6 \text{ N/m}^2$$

Sommerfeld Number = $\frac{ZN}{p} \left(\frac{d}{c}\right)^2$ $c =$ diametral clearance
 = $2 \times$ radial clearance

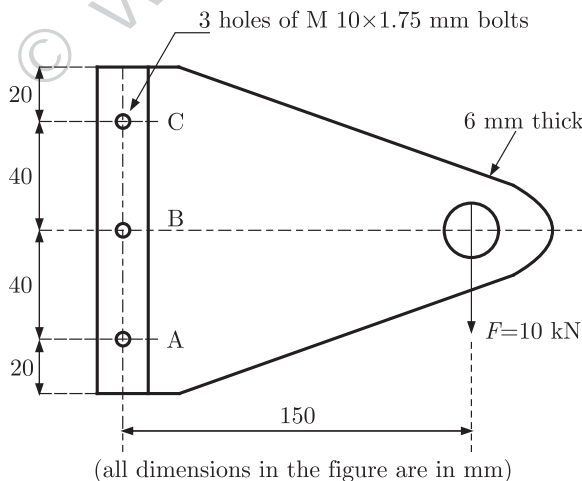
$$S.N. = \frac{20 \times 10^{-3} \times 20}{0.8 \times 10^6} \times \left(\frac{50}{100 \times 10^{-3}}\right)^2$$

$$= \frac{20 \times 10^{-3} \times 20}{0.8 \times 10^6} \times \left(\frac{1}{2}\right)^2 \times 10^6 = 0.125$$

Q. 48

GATE ME 2007
TWO MARK

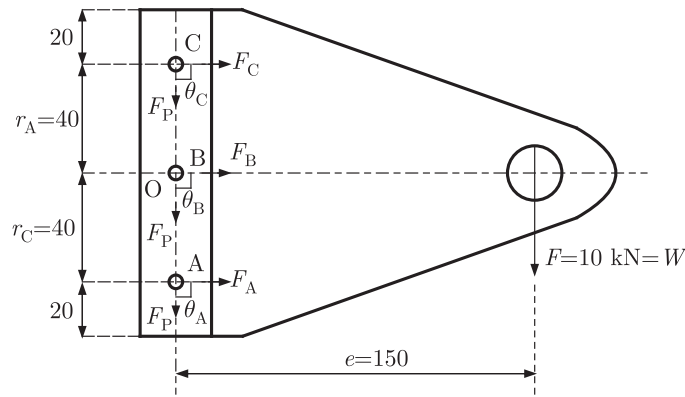
A bolted joint is shown below. The maximum shear stress, in MPa in the bolts at A and B, respectively are



- (A) 242.6, 42.5
- (B) 42.5, 242.6
- (C) 42.5, 42.5
- (D) 18.75, 343.64

Sol. 48

Option (A) is correct.



Given : Diameter of bolt $d = 10$ mm, $F = 10$ kN, No. of bolts $n = 3$

Direct or Primary shear load of each rivet

$$F_P = \frac{F}{n} = \frac{10 \times 10^3}{3} \text{ N}$$

$$F_P = 3333.33 \text{ N}$$

The centre of gravity of the bolt group lies at O. (due to symmetry of figure).

$$e = 150 \text{ mm} \quad (\text{eccentricity given})$$

Turning moment produced by the load F due to eccentricity

$$= F \times e = 10 \times 10^3 \times 150$$

$$= 1500 \times 10^3 \text{ N-mm}$$

Secondary shear load on bolts from fig. $r_A = r_C = 40$ mm and $r_B = 0$

We know that $F \times e = \frac{F_A}{r_A} [(r_A)^2 + (r_B)^2 + (r_C)^2]$

$$= \frac{F_A}{r_A} \times [2(r_A)^2] \quad (r_A = r_C \text{ and } r_B = 0)$$

$$1500 \times 10^3 = \frac{F_A}{40} \times [2(40)^2] = 80F_A$$

$$F_A = \frac{1500 \times 10^3}{80} = 18750 \text{ N}$$

$$F_B = 0$$

$$(r_B = 0)$$

$$F_C = F_A \times \frac{r_C}{r_A} = 18750 \times \frac{40}{40}$$

$$= 18750 \text{ N}$$

From fig we find that angle between

$$F_A \text{ and } F_P = \theta_A = 90^\circ$$

$$F_B \text{ and } F_P = \theta_B = 90^\circ$$

$$F_C \text{ and } F_P = \theta_C = 90^\circ$$

Resultant load on bolt A,

$$\begin{aligned} R_A &= \sqrt{(F_P)^2 + (F_A)^2 + 2F_P \times F_A \cos \theta_A} \\ &= \sqrt{(3333.33)^2 + (18750)^2 + 2 \times 3333.33 \times 18750 \times \cos 90^\circ} \\ R_A &= 19044 \text{ N} \end{aligned}$$

Maximum shear stress at A

$$\tau_A = \frac{R_A}{\frac{\pi}{4}(d)^2} = \frac{19044}{\frac{\pi}{4}(10)^2}$$

$$\tau_A = 242.6 \text{ MPa}$$

Resultant load on Bolt B,

$$R_B = F_P = 3333.33 \text{ N}$$

$$(F_B = 0)$$

Maximum shear stress at B ,

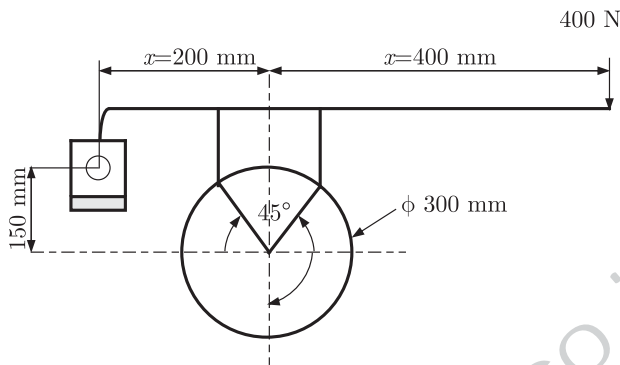
$$\tau_B = \frac{R_B}{\frac{\pi}{4}(d)^2} = \frac{3333.33}{\frac{\pi}{4} \times (10)^2}$$

$$\tau_B = 42.5 \text{ MPa}$$

Q. 49

GATE ME 2007
TWO MARK

A block-brake shown below has a face width of 300 mm and a mean co-efficient of friction of 0.25. For an activating force of 400 N, the braking torque in Nm is

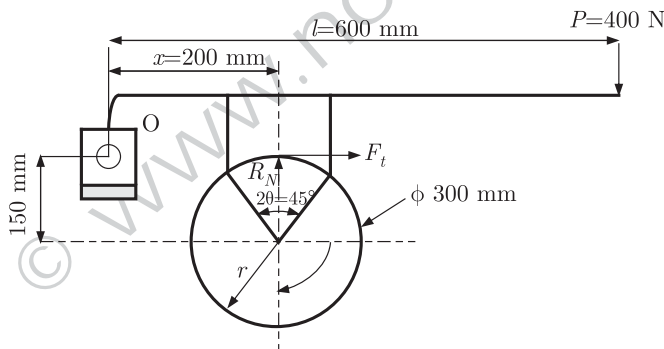


- (A) 30
- (C) 45

- (B) 40
- (D) 60

Sol. 49

Option (C) is correct.



Given : $P = 400 \text{ N}$, $r = \frac{300}{2} \text{ mm} = 150 \text{ mm}$, $l = 600 \text{ mm}$

$x = 200 \text{ mm}$, $\mu = 0.25$ and $2\theta = 45^\circ$

Let, $R_N \rightarrow$ Normal force pressing the brake block on the wheel

$F_t \rightarrow$ Tangential braking force or the frictional force acting at the

contact surface of the block & the wheel.

Here the line of action of tangential braking force F_t passes through the fulcrum O of the lever and brake wheel rotates clockwise. Then for equilibrium, Taking the moment about the fulcrum O ,

$$R_N \times x = P \times l$$

$$R_N = \frac{P \times l}{x} = \frac{400 \times 0.6}{0.2} = 1200 \text{ N}$$

Tangential braking force on the wheel,

$$F_t = \mu R_N$$

Braking Torque,

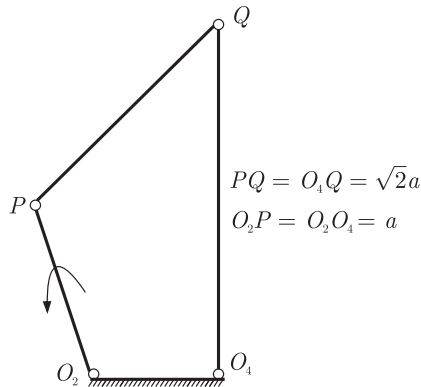
$$T_B = F_t \times r = \mu R_N \times r$$

$$= 0.25 \times 1200 \times 0.15 = 45 \text{ N-m}$$

Q. 50

GATE ME 2007
TWO MARK

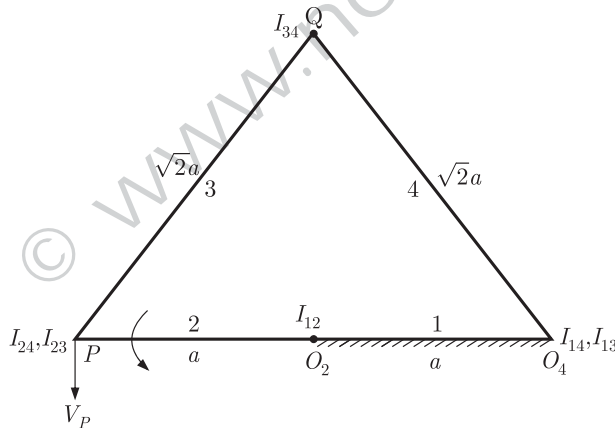
The input link O_2P of a four bar linkage is rotated at 2 rad/s in counter clockwise direction as shown below. The angular velocity of the coupler PQ in rad/s, at an instant when $\angle O_4O_2P = 180^\circ$, is



- (A) 4
- (B) $2\sqrt{2}$
- (C) 1
- (D) $\frac{1}{\sqrt{2}}$

Sol. 50

Option (C) is correct.
 Given, $\angle O_4O_2P = 180^\circ$, $\omega_{O_2P} = 2 \text{ rad/sec}$
 The instantaneous centre diagram is given below,



Let, velocity of point P on link O_2P is V_P ,

$$V_P = \omega_{O_2P} \times O_2P = \omega_{O_2P} \times (I_{12}I_{23}) = 2a \quad \dots(i)$$

And P is also a point on link QP ,

So,

$$V_P = \omega_{PQ} \times O_4P = \omega_{PQ} \times (I_{13}I_{23})$$

$$= \omega_{PQ} \times 2a \quad \dots(ii)$$

Both the links O_2P and QP are runs at the same speed
 From equation (i) and (ii), we get

$$2a = \omega_{PQ} \times 2a$$

or,

$$\omega_{PQ} = 1 \text{ rad/sec}$$

Q. 51

GATE ME 2007
TWO MARK

The speed of an engine varies from 210 rad/s to 190 rad/s. During the cycle the change in kinetic energy is found to be 400 Nm. The inertia of the flywheel in kg/m² is

- (A) 0.10
- (B) 0.20
- (C) 0.30
- (D) 0.40

Sol. 51

Option (A) is correct.

Given $\omega_1 = 210$ rad/sec, $\omega_2 = 190$ rad/sec, $\Delta E = 400$ Nm

As the speed of flywheel changes from ω_1 to ω_2 , the maximum fluctuation of energy,

$$\Delta E = \frac{1}{2} I [(\omega_1)^2 - (\omega_2)^2]$$

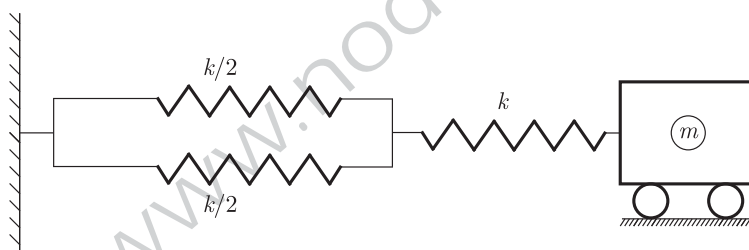
$$I = \frac{2\Delta E}{[(\omega_1)^2 - (\omega_2)^2]}$$

$$= \frac{2 \times 400}{[(210)^2 - (190)^2]} = \frac{800}{400 \times 20} = 0.10 \text{ kgm}^2$$

Q. 52

GATE ME 2007
TWO MARK

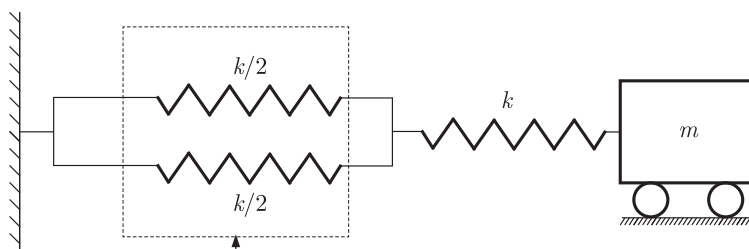
The natural frequency of the system shown below is



- (A) $\sqrt{\frac{k}{2m}}$
- (B) $\sqrt{\frac{k}{m}}$
- (C) $\sqrt{\frac{2k}{m}}$
- (D) $\sqrt{\frac{3k}{m}}$

Sol. 52

Option (A) is correct.



Springs are in Parallel Combination

The springs, with stiffness $\frac{k}{2}$ & $\frac{k}{2}$ are in parallel combination. So their resultant stiffness will be,

$$k_1 = \frac{k}{2} + \frac{k}{2} = k$$

As k_1 & k are in series, so the resultant stiffness will be,

$$k_{eq} = \frac{k \times k}{k + k} = \frac{k^2}{2k} = \frac{k}{2}$$

The general equation of motion for undamped free vibration is given as,

$$m\ddot{x} + k_{eq}x = 0$$

$$m\ddot{x} + \frac{k}{2}x = 0$$

$$\ddot{x} + \frac{k}{2m}x = 0$$

Compare above equation with general equation $\ddot{x} + \omega_n^2 x = 0$, we get
Natural frequency of the system is,

$$\omega_n^2 = \frac{k}{2m}$$

$$\omega_n = \sqrt{\frac{k}{2m}}$$

Alternatively :

$$k_{eq} = \frac{k}{2}$$

We know, for a spring mass system,

$$\omega_n = \sqrt{\frac{k_{eq}}{m}} = \sqrt{\frac{k/2}{m}} = \sqrt{\frac{k}{2m}}$$

Q. 53

GATE ME 2007
TWO MARK

The equation of motion of a harmonic oscillator is given by

$$\frac{d^2x}{dt^2} + 2\xi\omega_n \frac{dx}{dt} + \omega_n^2 x = 0$$

and the initial conditions at $t = 0$ are $x(0) = X$, $\frac{dx}{dt}(0) = 0$. The amplitude of $x(t)$ after n complete cycles is

- (A) $Xe^{-2n\pi(\frac{\xi}{\sqrt{1-\xi^2}})}$ (B) $Xe^{2n\pi(\frac{\xi}{\sqrt{1-\xi^2}})}$
(C) $Xe^{-2n\pi(\frac{\sqrt{1-\xi^2}}{\xi})}$ (D) X

Sol. 53

Option (A) is correct.

Given The equation of motion of a harmonic oscillator is

$$\frac{d^2x}{dt^2} + 2\xi\omega_n \frac{dx}{dt} + \omega_n^2 x = 0 \quad \dots(i)$$

$$\ddot{x} + 2\xi\omega_n \dot{x} + \omega_n^2 x = 0$$

Compare equation (i) with the general equation,

$$m\ddot{x} + c\dot{x} + kx = 0$$

$$\ddot{x} + \frac{c}{m}\dot{x} + \frac{k}{m}x = 0$$

We get, $\frac{c}{m} = 2\xi\omega_n \quad \dots(ii)$

$$\frac{k}{m} = \omega_n^2, \quad \Rightarrow \omega_n = \sqrt{\frac{k}{m}} \quad \dots(iii)$$

From equation (ii) & (iii),

$$\xi = \frac{c}{2m \times \sqrt{\frac{k}{m}}} = \frac{c}{2\sqrt{km}}$$

Logarithmic decrement, $\delta = \ln\left(\frac{x_1}{x_2}\right) = \frac{2\pi c}{\sqrt{c_c^2 - c^2}} \quad \dots(iv)$

$$\begin{aligned}\delta &= \ln\left(\frac{x_1}{x_2}\right) = \frac{2\pi \times 2\xi\sqrt{km}}{(2\sqrt{km})^2 - (2\xi\sqrt{km})^2} \\ &= \frac{4\pi\xi\sqrt{km}}{\sqrt{4km - 4\xi^2 km}} \\ &= \frac{2\pi\xi}{\sqrt{1 - \xi^2}} \\ \frac{x_1}{x_2} &= e^{\frac{2\pi\xi}{\sqrt{1 - \xi^2}}}\end{aligned}$$

If system executes n cycles, the logarithmic decrement δ can be written as

$$\begin{aligned}\delta &= \frac{1}{n} \log_e \frac{x_1}{x_{n+1}} \\ e^{n\delta} &= \frac{x_1}{x_{n+1}}\end{aligned}$$

Where x_1 = amplitude at the starting position.

x_{n+1} = Amplitude after n cycles

The amplitude of $x(t)$ after n complete cycles is,

$$\begin{aligned}e^{n\delta} &= \frac{X}{x(t)} \\ x(t) &= e^{-n\delta} \times X = Xe^{-\frac{n2\pi\xi}{\sqrt{1 - \xi^2}}}\end{aligned} \quad \text{From equation (iv)}$$

Q. 54

GATE ME 2007
TWO MARK

The piston rod of diameter 20 mm and length 700 mm in a hydraulic cylinder is subjected to a compressive force of 10 kN due to the internal pressure. The end conditions for the rod may be assumed as guided at the piston end and hinged at the other end. The Young's modulus is 200 GPa. The factor of safety for the piston rod is

- (A) 0.68 (B) 2.75
(C) 5.62 (D) 11.0

Sol. 54

Option (C) is correct.

Given : $d = 20$ mm, $l = 700$ mm,

$E = 200$ GPa = 200×10^9 N/m² = 200×10^3 N/mm²

Compressive or working Load = 10 kN

According to Euler's theory, the crippling or buckling load (W_{cr}) under various end conditions is given by the general equation,

$$W_{cr} = \frac{c\pi^2 EI}{l^2} \quad \dots(i)$$

Given that one end is guided at the piston end and hinged at the other end. So,

$$c = 2$$

From equation (i),

$$\begin{aligned}W_{cr} &= \frac{2\pi^2 EI}{l^2} = \frac{2\pi^2 E}{l^2} \times \frac{\pi}{64} d^4 \quad I = \frac{\pi}{64} d^4 \\ &= \frac{2 \times 9.81 \times 200 \times 10^3}{(700)^2} \times \frac{3.14}{64} \times (20)^4 \\ &= 62864.08 \text{ N} = 62.864 \text{ kN}\end{aligned}$$

We know that, factor of safety (FOS)

$$\text{FOS} = \frac{\text{Crippling Load}}{\text{Working Load}} = \frac{62.864}{10} = 6.28$$

The most appropriate option is (C).

Q. 55

GATE ME 2007
TWO MARK

In electrodischarge machining (EDM), if the thermal conductivity of tool is high and the specific heat of work piece is low, then the tool wear rate and material removal rate are expected to be respectively

- (A) high and high (B) low and low
(C) high and low (D) low and high

Sol. 55

Question (A) is correct.

Metal removal rate depends upon current density and it increases with current. The MRR increase with thermal conductivity also

$$\text{Wear ratio} = \frac{\text{Volume of metal removed work}}{\text{Volume of metal removed tool}}$$

The volume of metal removed from the tool is very less compare to the volume of metal removed from the work.

So, Wear ration \propto volume of metal removed work.

Hence, both the wear rate and MRR are expected to be high.

Q. 56

GATE ME 2007
TWO MARK

In orthogonal turning of medium carbon steel, the specific machining energy is 2.0 J/mm^3 . The cutting velocity, feed and depth of cut are 120 m/min , 0.2 mm/rev. and 2 mm respectively. The main cutting force in N is

- (A) 40 (B) 80
(C) 400 (D) 800

Sol. 56

Option (D) is correct.

Given : $E = 2 \text{ J/mm}^3$, $V = 120 \text{ m/min}$, $f = 0.2 \text{ mm/rev.} = t$, $d = 2 \text{ mm} = b$

The specific energy. $E = \frac{F_c}{b \times t}$

In orthogonal cutting $b \times t = d \times f$

$$F_c = E \times b \times t = E \times d \times f$$

$$= 2 \times 10^9 \times 2 \times 10^{-3} \times 0.2 \times 10^{-3} = 800 \text{ N}$$

Q. 57

GATE ME 2007
TWO MARK

A direct current welding machine with a linear power source characteristic provides open circuit voltage of 80 V and short circuit current of 800 A . During welding with the machine, the measured arc current is 500 A corresponding to an arc length of 5.0 mm and the measured arc current is 460 A corresponding to an arc length of 7.0 mm . The linear voltage (E) arc length (L) characteristic of the welding arc can be given as (where E is in volt and L in in mm)

- (A) $E = 20 + 2L$ (B) $E = 20 + 8L$
(C) $E = 80 + 2L$ (D) $E = 80 + 8L$

Sol. 57

Option (A) is correct.

Given : $\text{OCV} = 80 \text{ V}$, $\text{SCC} = 800 \text{ A}$

In Case (I) :

$I = 500 \text{ A}$ and $L = 5.0 \text{ mm}$

And in, Case (II) :

$$I = 460 \text{ A, and } L = 7.0 \text{ mm}$$

We know that, for welding arc,

$$E = a + bL \quad \dots(i)$$

And For power source,

$$E = \text{OCV} - \left(\frac{\text{OCV}}{\text{SCC}}\right)I = 80 - \left(\frac{80}{800}\right)I \quad \dots(ii)$$

Where : I = Arc current, E = Arc voltage

For stable arc,

$$\text{Welding arc} = \text{Power source}$$

$$80 - \left(\frac{80}{800}\right)I = a + bL \quad \dots(iii)$$

Find the value of a & b , from the case (I) & (II)

$$\text{For case (I), } I = 500 \text{ A, } L = 5 \text{ mm}$$

$$\text{So, } 80 - \left(\frac{80}{800}\right) \times 500 = a + 5b \quad \text{From equation (iii)}$$

$$80 - 50 = a + 5b$$

$$a + 5b = 30 \quad \dots(iv)$$

$$\text{For case II, } I = 460 \text{ A, } L = 7 \text{ mm}$$

$$\text{So, } 80 - \frac{80}{800} \times 460 = a + 7b \quad \text{From equation(iii)}$$

$$80 - 46 = a + 7b$$

$$a + 7b = 34 \quad \dots(v)$$

Subtracting equation (iv) from equation (v),

$$(a + 7b) - (a + 5b) = 34 - 30$$

$$2b = 4 \quad \Rightarrow b = 2$$

From equation (iv), put $b = 2$

$$a + 5 \times 2 = 30 \quad \Rightarrow a = 20$$

Substituting the value of a & b in equation (i), we get

$$E = 20 + 2L$$

Q. 58

GATE ME 2007
TWO MARK

A hole is specified as $40_{0.000}^{+0.050}$ mm. The mating shaft has a clearance fit with minimum clearance of 0.01 mm. The tolerance on the shaft is 0.04 mm. The maximum clearance in mm between the hole and the shaft is

- (A) 0.04 (B) 0.05
(C) 0.10 (D) 0.11

Sol. 58

Option (C) is correct.

Given :

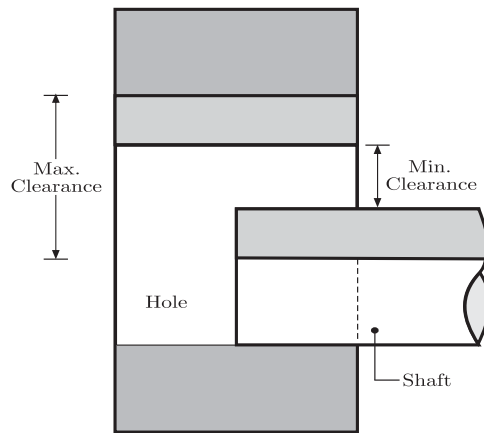
$$\text{Hole, } 40_{+0.000}^{+0.050} \text{ mm}$$

$$\text{Minimum hole size} = 40 \text{ mm}$$

$$\text{Minimum clearance} = 0.01 \text{ mm}$$

$$\text{Maximum size of hole} = 40 + 0.050 = 40.050 \text{ mm}$$

$$\text{Tolerance of shaft} = 0.04 \text{ mm}$$



Given that the mating shaft has a clearance fit with minimum clearance of 0.01 mm.

So,
$$\begin{aligned} \text{Maximum size of shaft} &= \text{Minimum hole size} - \text{Minimum clearance} \\ &= 40 - 0.01 = 39.99 \text{ mm} \end{aligned}$$

And
$$\begin{aligned} \text{Minimum size of shaft} &= \text{Maximum shaft size} - \text{Tolerance of shaft} \\ &= 39.99 - 0.04 = 39.95 \end{aligned}$$

Maximum clearance, (c)

$$= \text{Maximum size of hole} - \text{Minimum size of shaft}$$

$$c = 40.050 - 39.95 = 0.1 \text{ mm}$$

Q. 59

GATE ME 2007
TWO MARK

In orthogonal turning of low carbon steel pipe with principal cutting edge angle of 90° , the main cutting force is 1000 N and the feed force is 800 N. The shear angle is 25° and orthogonal rake angle is zero. Employing Merchant's theory, the ratio of friction force to normal force acting on the cutting tool is

- (A) 1.56 (B) 1.25
 (C) 0.80 (D) 0.64

Sol. 59

Option (C) is correct

Given : $\lambda = 90^\circ$, $F_c = 1000 \text{ N}$, $F_t = 800 \text{ N}$, $\phi = 25^\circ$, $\alpha = 0^\circ$

We know that, from the merchant's theory,

$$\frac{\text{Friction force } (F)}{\text{Normal force } (N)} = \mu = \frac{F_c \tan \alpha + F_t}{F_c - F_t \tan \alpha}$$

Substitute the values, we get

$$\frac{F}{N} = \frac{1000 \tan 0^\circ + 800}{1000 - 800 \tan 0^\circ} = \frac{800}{1000} = 0.80$$

Q. 60

GATE ME 2007
TWO MARK

Two metallic sheets, each of 2.0 mm thickness, are welded in a lap joint configuration by resistance spot welding at a welding current of 10 kA and welding time of 10 millisecond. A spherical fusion zone extending up to full thickness of each sheet is formed. The properties of the metallic sheets are given as :

Ambient temperature	= 293 K
Melting temperature	= 1793 K
Density	= 7000 kg/m ³
Latent heat of fusion	= 300 kJ/kg
Specific heat	= 800 J/kgK

Assume :

- (i) contact resistance along sheet interface is 500 micro-ohm and along electrode-sheet interface is zero;
(ii) no conductive heat loss through the bulk sheet materials ; and
(iii) the complete weld fusion zone is at the melting temperature.

The melting efficiency (in %) of the process is

- (A) 50.37 (B) 60.37
(C) 70.37 (D) 80.37

Sol. 60

Option (C) is correct.

Given : $w = 2 \text{ mm}$, $I = 10 \text{ kA} = 10^4 \text{ A}$, $t = 10 \text{ millisecond} = 10^{-2} \text{ sec}$.

$T_a = 293 \text{ K}$, $T_m = 1793 \text{ K}$, $\rho = 7000 \text{ kg/m}^3$, $L_f = 300 \text{ kJ/kg}$

$c = 800 \text{ J/kg K}$, $R = 500 \text{ micro-ohm} = 500 \times 10^{-6} \text{ ohm}$

Radius of sphere, $r = 2 \text{ mm} = 2 \times 10^{-3} \text{ m}$

Heat supplied at the contacting area of the element to be welded is

$$Q_s = I^2 R t$$

$$Q_s = (10^4)^2 \times 500 \times 10^{-6} \times 10^{-2} = 500 \text{ J}$$

As fusion zone is spherical in shape.

$$\begin{aligned} \text{Mass, } m &= \rho \times v = 7000 \times \frac{4}{3} \times 3.14 \times (2 \times 10^{-3})^3 \\ &= 2.344 \times 10^{-4} \text{ kg} \end{aligned}$$

Total heat for melting (heat input)

$$Q_i = mL_f + mc(T_m - T_a)$$

Where

$$mL_f = \text{Latent heat}$$

Substitute the values, we get

$$\begin{aligned} Q_i &= 2.344 \times 10^{-4} [300 \times 10^3 + 800 (1793 - 293)] \\ &= 2.344 \times 10^{-4} [300 \times 10^3 + 800 \times 1500] \end{aligned}$$

$$= 351.6 \text{ J}$$

$$\text{Efficiency } \eta = \frac{\text{Heat input } (Q_i)}{\text{Heat supplied } (Q_s)} \times 100$$

$$\eta = \frac{351.6}{500} \times 100 = 70.32\% \approx 70.37\%$$

Q. 61

GATE ME 2007
TWO MARK

Capacities of production of an item over 3 consecutive months in regular time are 100, 100 and 80 and in overtime are 20, 20 and 40. The demands over those 3 months are 90, 130 and 110. The cost of production in regular time and overtime are respectively Rs. 20 per item and Rs. 24 per item. Inventory carrying cost is Rs. 2 per item per month. The levels of starting and final inventory are nil. Backorder is not permitted. For minimum cost of plan, the level of planned production in overtime in the third month is

- (A) 40 (B) 30
(C) 20 (D) 0

Sol. 61

Option (B) is correct.

We have to make a table from the given data.

Month	Production (Pieces)		Demand	Excess or short form (pieces)	
	In regular time	In over time		Regular	Total
1	100	20	90	10	10 + 20 = 30
2	100	20	130	-30	-30 + 20 = -10
3	80	40	110	-30	-30 + 40 = 10

From the table,

For 1st month there is no need to overtime, because demand is 90 units and regular time production is 100 units, therefore 10 units are excess in amount. For 2nd month the demand is 130 unit and production capacity with overtime is $100 + 20 = 120$ units, therefore 10 units ($130 - 120 = 10$) are short in amount, which is fulfilled by 10 units excess of 1st month. So at the end of 2nd month there is no inventory.

Now for the 3rd month demand is 110 units and regular time production is 80 units. So remaining $110 - 80 = 30$ units are produced in overtime to fulfill the demand for minimum cost of plan.

Q. 62

GATE ME 2007
TWO MARK

In open-die forging, disc of diameter 200 mm and height 60 mm is compressed without any barreling effect. The final diameter of the disc is 400 mm. The true strain is

- (A) 1.986 (B) 1.686
(C) 1.386 (D) 0.602

Sol. 62

Option (C) is correct.

Given : $d_i = 200$ mm, $h_i = l_i = 60$ mm, $d_f = 400$ mm

Volume of disc remains unchanged during the whole compression process.

So, Initial volume = Final volume.

$$\frac{\pi}{4} d_i^2 \times l_i = \frac{\pi}{4} d_f^2 \times l_f$$

$$\frac{l_f}{l_i} = \frac{d_i^2}{d_f^2}$$

$$l_f = 60 \times \left(\frac{200}{400}\right)^2 = 60 \times \frac{1}{4} = 15 \text{ mm}$$

Strain,

$$\varepsilon = \frac{\Delta l}{l} = \frac{l_i - l_f}{l_f} = \frac{60 - 15}{15} = 3$$

True strain,

$$\varepsilon_0 = \ln(1 + \varepsilon) = \ln(1 + 3) = 1.386$$

Q. 63

GATE ME 2007
TWO MARK

The thickness of a metallic sheet is reduced from an initial value of 16 mm to a final value of 10 mm in one single pass rolling with a pair of cylindrical rollers each of diameter of 400 mm. The bite angle in degree will be.

- (A) 5.936 (B) 7.936
(C) 8.936 (D) 9.936

Sol. 63

Option (D) is correct.

Let, Bite angle = θ

$D = 400$ mm, $t_i = 16$ mm, $t_f = 10$ mm

$$\text{Bite angle, } \tan \theta = \sqrt{\frac{t_i - t_f}{R}} = \sqrt{\frac{16 - 10}{200}} = \sqrt{0.03}$$

$$\theta = \tan^{-1}(0.173) = 9.815^\circ \approx 9.936^\circ$$

Q. 64

Match the correct combination for following metal working processes.

GATE ME 2007
TWO MARK

	Processes		Associated state of stress
P:	Blanking	1.	Tension
Q:	Stretch Forming	2.	Compression
R:	Coining	3.	Shear
S:	Deep Drawing	4.	Tension and Compression
		5.	Tension and Shear

- (A) P - 2, Q - 1, R - 3, S - 4
 (B) P - 3, Q - 4, R - 1, S - 5
 (C) P - 5, Q - 4, R - 3, S - 1
 (D) P - 3, Q - 1, R - 2, S - 4

Sol. 64

Option (D) is correct.

	Processes		Associated state of stress
P.	Blanking	3.	Shear
Q.	Stretch Forming	1.	Tension
R.	Coining	2.	Compression
S.	Deep Drawing	4.	Tension and Compression

So, correct pairs are, P-3, Q-1, R-2, S-4

Q. 65

GATE ME 2007
TWO MARK

The force requirement in a blanking operation of low carbon steel sheet is 5.0 kN. The thickness of the sheet is 't' and diameter of the blanked part is 'd'. For the same work material, if the diameter of the blanked part is increased to 1.5d and thickness is reduced to 0.4t, the new blanking force in kN is

- (A) 3.0 (B) 4.5
 (C) 5.0 (D) 8.0

Sol. 65

Option (A) is correct.

Blanking force F_b is directly proportional to the thickness of the sheet 't' and diameter of the blanked part 'd'.

$$F_b \propto d \times t \quad F_b = \tau \times d \times t \dots (i)$$

For case (I) :

$$F_{b1} = 5.0 \text{ kN}, d_1 = d, t_1 = t$$

For case (II) :

$$d_2 = 1.5d, t_2 = 0.4t, F_{b2} = ?$$

From equation (i)

$$\frac{F_{b2}}{F_{b1}} = \frac{d_2 t_2}{d_1 t_1}$$

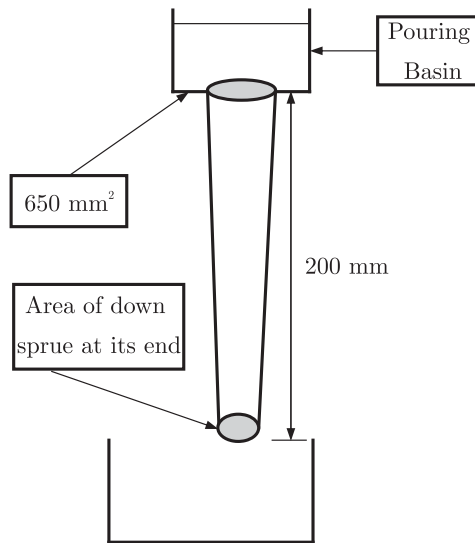
$$F_{b2} = 5 \times \frac{1.5d \times 0.4t}{d \times t} = 3 \text{ kN}$$

Q. 66

GATE ME 2007
TWO MARK

A 200 mm long down sprue has an area of cross-section of 650 mm² where the pouring basin meets the down sprue (i.e at the beginning of the down sprue). A constant head of molten metal is maintained by the pouring basin. The molten metal flow rate is 6.5×10^5 mm³/s. Considering the end of down sprue to be open to atmosphere and an acceleration due to gravity of 10⁴ mm/s², the area of the

down sprue in mm^2 at its end (avoiding aspiration effect) should be

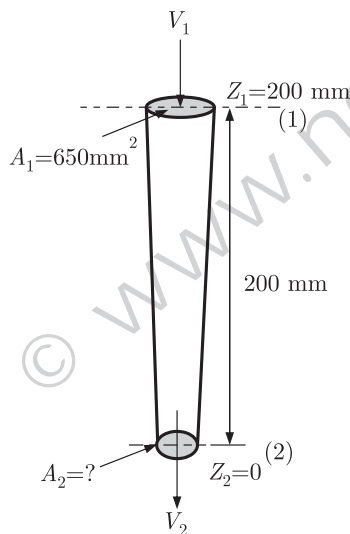


- (A) 650.0
- (B) 350.0
- (C) 290.7
- (D) 190.0

Sol. 66

Option (C) is correct.

Let molten metal enters at section 1st and leaves the object at section 2nd



Given : $A_1 = 650 \text{ mm}^2$, $Q = 6.5 \times 10^5 \text{ mm}^3/\text{sec}$, $g = 10^4 \text{ mm}/\text{sec}^2$

Now, for section 1st, flow rate

$$Q = A_1 V_1$$

$$V_1 = \frac{Q}{A_1} = \frac{6.5 \times 10^5}{650} = 1000 \text{ mm}/\text{sec}$$

Applying Bernoulli's equation at section 1st and 2nd.

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + Z_2$$

But

$$p_1 = p_2 = \text{atmosphere pressure}$$

So,

$$\frac{V_1^2}{2g} + Z_1 = \frac{V_2^2}{2g} + Z_2$$

$$\frac{(1000)^2}{2 \times 10^4} + 200 = \frac{V_2^2}{2 \times 10^4} + 0$$

$$(50 + 200) \times 2 \times 10^4 = V_2^2$$

$$V_2^2 = 500 \times 10^4 = 5 \times 10^6$$

$$V_2 = 2.236 \times 10^3 \text{ mm/sec} = 2236 \text{ mm/sec}$$

We know that, flow rate remains constant during the process (from continuity equation). So, for section 2nd

$$Q = A_2 V_2$$

$$A_2 = \frac{Q}{V_2} = \frac{6.5 \times 10^5}{2236} = 290.7 \text{ mm}^2$$

Q. 67

GATE ME 2007
TWO MARK

Match the most suitable manufacturing processes for the following parts.

	Parts		Manufacturing Process
P.	Computer chip	1.	Electrochemical Machining
Q.	Metal forming dies and molds	2.	Ultrasonic Machining
R.	Turbine blade	3.	Electrodischarge Machining
S.	Glass	4.	Photochemical Machining

(A) P - 4, Q - 3, R - 1, S - 2

(B) P - 4, Q - 3, R - 2, S - 1

(C) P - 3, Q - 1, R - 4, S - 2

(D) P - 1, Q - 2, R - 4, S - 3

Sol. 67

Option (A) is correct.

	Parts		Manufacturing Process
P.	Computer chip	4.	Photochemical Machining
Q.	Metal forming dies and molds	3.	Electrodischarge Machining
R.	Turbine blade	1.	Electrochemical Machining
S.	Glass	2.	Ultrasonic Machining

So, correct pairs are, P-4, Q-3, R-1, S-2

Q. 68

GATE ME 2007
TWO MARK

The maximum level of inventory of an item is 100 and it is achieved with infinite replenishment rate. The inventory becomes zero over one and half month due to consumption at a uniform rate. This cycle continues throughout the year. Ordering cost is Rs. 100 per order and inventory carrying cost is Rs. 10 per item per month. Annual cost (in Rs.) of the plan, neglecting material cost, is

(A) 800

(B) 2800

(C) 4800

(D) 6800

Sol. 68

Option (D) is correct.

$$\begin{aligned} & \text{Total annual cost} \\ &= \text{Annual holding cost} + \text{Annual ordering cost} \end{aligned}$$

$$\text{Maximum level of inventory } N = 100$$

$$\text{So, Average inventory} = \frac{N}{2} = 50$$

$$\text{Inventory carrying cost } C_h = \text{Rs. 10 per item per month}$$

$$= \text{Rs. } 10 \times 12 \text{ per item per year}$$

$$= \text{Rs. } 120 \text{ per item per year}$$

$$\text{So, Annual holding cost} = \frac{N}{2} \times C_h$$

$$C_{hA} = 50 \times 120$$

$$= \text{Rs. } 6000 \text{ item per year}$$

And, Ordering cost $C_o = 100$ per order

$$\text{Number of orders in a year} = \frac{12}{1.5} \text{ order}$$

$$= 8 \text{ order}$$

So, Annual ordering cost C_{oA}

= ordering cost per order \times no. of orders

$$= 100 \times 8$$

$$= \text{Rs. } 800 \text{ per order}$$

Hence,

$$\text{Total Annual cost} = 6000 + 800$$

$$= \text{Rs. } 6800$$

Q. 69

GATE ME 2007
TWO MARK

In a machine shop, pins of 15 mm diameter are produced at a rate of 1000 per month and the same is consumed at a rate of 500 per month. The production and consumption continue simultaneously till the maximum inventory is reached. Then inventory is allowed to be reduced to zero due to consumption. The lot size of production is 1000. If backlog is not allowed, the maximum inventory level is

(A) 400

(B) 500

(C) 600

(D) 700

Sol. 69

Option (B) is correct.

Given :

Number of items produced per month

$$K = 1000 \text{ per month}$$

Number of items required per month

$$R = 500 \text{ per month}$$

$$\text{Lot size } q_0 = 1000$$

When backlog is not allowed, the maximum inventory level is given by,

$$I_m = \frac{K - R}{K} \times q_0$$

$$= \frac{1000 - 500}{1000} \times 1000$$

$$= 500$$

Q. 70

GATE ME 2007
TWO MARK

The net requirements of an item over 5 consecutive weeks are 50-0-15-20-20. The inventory carrying cost and ordering cost are Rs. 1 per item per week and Rs. 100 per order respectively. Starting inventory is zero. Use "Least Unit Cost Technique" for developing the plan. The cost of the plan (in Rs.) is

(A) 200

(B) 250

(C) 225

(D) 260

Sol. 70

Option (B) is correct.

Given :

$$C_h = \text{Rs. 1 per item per week}$$

$$C_o = \text{Rs. 100 per order}$$

$$\text{Requirements} = 50 - 0 - 15 - 20 - 20$$

Total cost is the cost of carrying inventory and cost of placing order.

Case (I) Only one order of 105 units is placed at starting.

Weeks	Quantity			Cost		
	Inventory	Used	Carried forward	Order	Carrying	Total
1.	105 (ordered)	50	55	100	55	155
2.	55	0	55	0	55	55
3.	55	15	40	0	40	40
4.	40	20	20	0	20	20
5.	20	20	0	0	0	0

$$\begin{aligned} \text{Total cost of plan} &= 155 + 55 + 40 + 20 \\ &= 270 \text{ Rs.} \end{aligned}$$

Case (II) Now order is placed two times, 50 units at starting and 55 units after 2nd week.

Weeks	Quantity			Cost		
	Inventory	Used	Carried forward	Ordering Rs.	Carrying Rs.	Total Rs.
1.	50 (ordered)	50	0	100	0	100
2.	0	0	0	0	0	0
3.	55 (ordered)	15	40	100	40	140
4.	40	20	20	0	20	20
5.	20	20	0	0	0	0

$$\text{Total cost of plan} = 100 + 140 + 20 = 260 \text{ Rs.}$$

Case (III) The order is placed two times, 65 units at starting and 40 units after 3rd week.

Weeks	Quantity			Cost		
	Inventory	Used	Carried forward	Ordering Rs.	Carrying Rs.	Total Rs.
1.	65 (ordered)	50	15	100	15	115
2.	15	0	15	0	15	15
3.	15	15	0	0	0	0
4.	40 (ordered)	20	20	100	20	120
5.	20	20	0	0	0	0

$$\text{Total cost of plan} = 115 + 15 + 120 = 250 \text{ Rs.}$$

Case (IV) Now again order is placed two times, 85 units at starting and 20 units after 4th week.

Weeks	Quantity			Cost		
	Inventory	Used	Carried forward	Order	Carrying	Total
1.	85 (ordered)	50	35	100	35	135
2.	35	0	35	0	35	35
3.	35	15	20	0	20	20
4.	20	20	0	0	0	0
5.	20 (ordered)	20	0	100	0	100

Total cost of plan = 135 + 35 + 20 + 100 = 290 Rs.

So, The cost of plan is least in case (III) & it is 250 Rs.

Statement for Linked Answer Questions 71 to 73 :

A gear set has a pinion with 20 teeth and a gear with 40 teeth. The pinion runs at 30 rev/s and transmits a power of 20 kW. The teeth are on the 20° full-depth system and have a module of 5 mm. The length of the line of action is 19 mm.

Q. 71

GATE ME 2007
TWO MARK

The center distance for the above gear set in mm is
(A) 140 (B) 150
(C) 160 (D) 170

Sol. 71

Option (B) is correct.

Given : $Z_P = 20$, $Z_G = 40$, $N_P = 30$ rev/sec, $P = 20$ kW = 20×10^3 W, $m = 5$ mm

Module,

$$m = \frac{D}{Z} = \frac{D_P}{Z_P} = \frac{D_G}{Z_G}$$

$$D_P = m \times Z_P = 5 \times 20 = 100 \text{ mm}$$

or,

$$D_G = m \times Z_G = 5 \times 40 = 200 \text{ mm}$$

Centre distance for the gear set,

$$L = \frac{D_P + D_G}{2} = \frac{100 + 200}{2} = 150 \text{ mm}$$

Q. 72

GATE ME 2007
TWO MARK

The contact ratio of the contacting tooth is

(A) 1.21 (B) 1.25
(C) 1.29 (D) 1.33

Sol. 72

Option (C) is correct.

Given :

Length of line of action, $L = 19$ mm

Pressure angle, $\phi = 20^\circ$

$$\text{Length of arc of contact} = \frac{\text{Length of path of contact (L)}}{\cos \phi}$$

$$= \frac{19}{\cos 20^\circ} = 20.21 \text{ mm}$$

Contact ratio or number of pairs of teeth in contact,

$$= \frac{\text{Length of arc of contact}}{\text{circular pitch}}$$

$$= \frac{20.21}{\pi m} = \frac{20.21}{3.14 \times 5} = 1.29$$

Q. 73

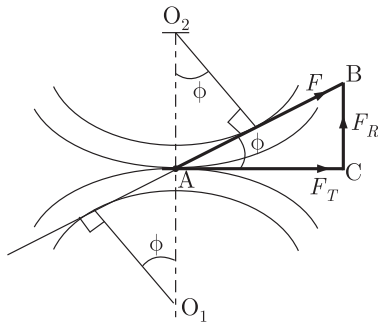
GATE ME 2007
TWO MARK

The resultant force on the contacting gear tooth in N is

- (A) 77.23
- (B) 212.20
- (C) 2258.1
- (D) 289.43

Sol. 73

Option (C) is correct.



Let, $T \rightarrow$ Torque transmitted in N-m

We know that power transmitted is,

$$P = T\omega = T \times \frac{2\pi N}{60}$$

$$T = \frac{60P}{2\pi N}$$

$$= \frac{60 \times 20 \times 10^3}{2 \times 3.14 \times 1800} = 106.157 \text{ N-m}$$

$$F_T = \frac{T}{R_P} \quad \text{Tangential load on the pinion}$$

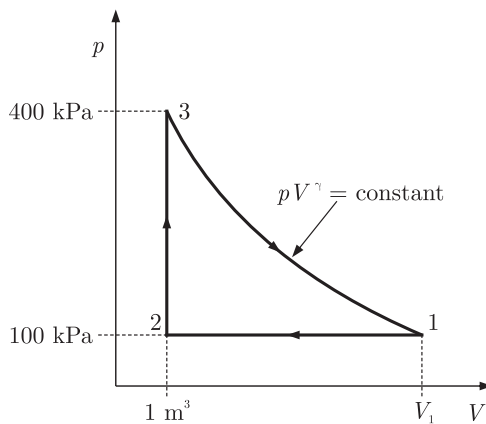
$$= \frac{106.157}{0.05} = 2123.14 \text{ N}$$

From the geometry, total load due to power transmitted,

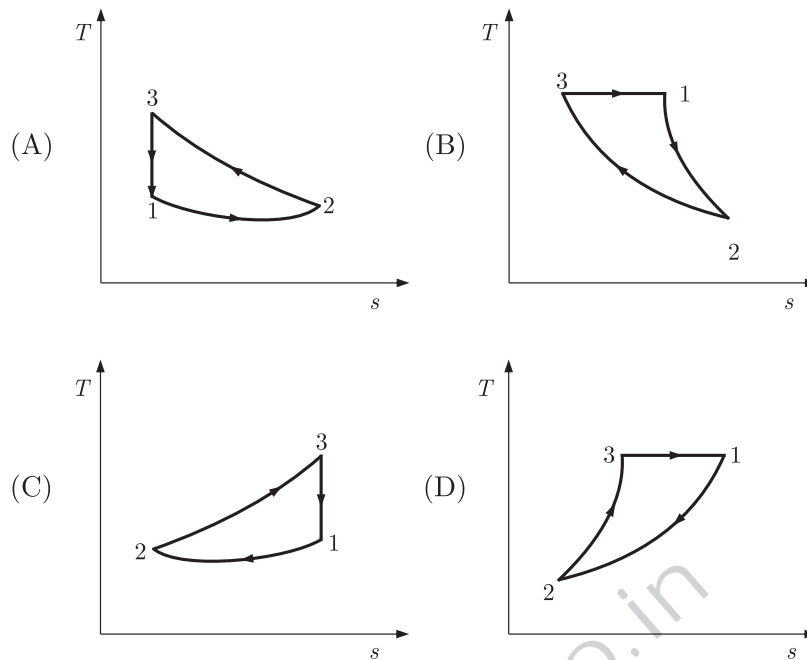
$$F = \frac{F_T}{\cos \phi} = \frac{2123.14}{\cos 20^\circ} \approx 2258.1 \text{ N}$$

Common Data for Questions 74 & 75 :

A thermodynamic cycle with an ideal gas as working fluid is shown below.



Q. 74

GATE ME 2007
TWO MARKThe above cycle is represented on $T-s$ plane by

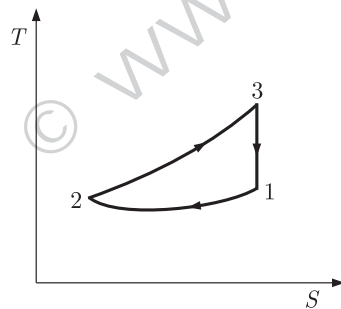
Sol. 74

Option (C) is correct.

In the given $p-v$ diagram, three processes are occurred.

- (i) Constant pressure (Process 1 – 2)
- (ii) Constant Volume (Process 2 – 3)
- (iii) Adiabatic (Process 3 – 1)

We know that, Constant pressure & constant volume lines are inclined curves in the $T-s$ curve, and adiabatic process is drawn by a vertical line on a $T-s$ curve.



Given $p-v$ curve is clock wise. So $T-s$ curve must be clockwise.

Q. 75

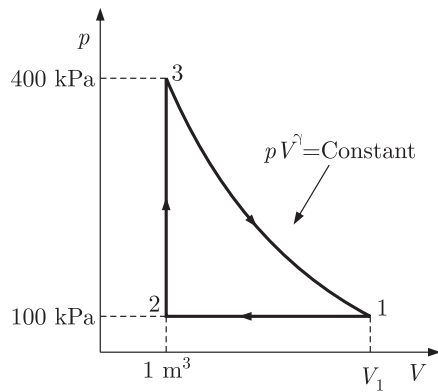
GATE ME 2007
TWO MARK

If the specific heats of the working fluid are constant and the value of specific heat ratio is 1.4, the thermal efficiency (%) of the cycle is

- (A) 21
- (B) 40.9
- (C) 42.6
- (D) 59.7

Sol. 75

Option (A) is correct.



This cycle shows the Lenoir cycle.

For Lenoir cycle efficiency is given by

$$\eta_L = 1 - \gamma \left(\frac{r_p^{\frac{1}{\gamma}} - 1}{r_p - 1} \right)$$

Where, $r_p = \frac{p_2}{p_1} = \frac{400}{100} = 4$

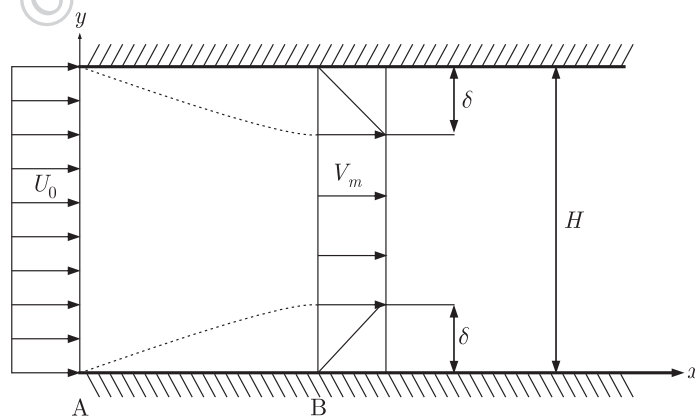
And $\gamma = \frac{C_p}{C_v} = 1.4$ (Given)

So, $\eta_L = 1 - 1.4 \left[\frac{(4)^{\frac{1}{1.4}} - 1}{4 - 1} \right] = 1 - 0.789 = 0.211$

$\eta_L = 21.1\% \approx 21\%$

Statement for Linked Answer Questions 76 & 77 :

Consider a steady incompressible flow through a channel as shown below.



The velocity profile is uniform with a value of U_0 at the inlet section A. The velocity profile at section B downstream is

$$u = \begin{cases} V_m \frac{y}{\delta}, & 0 \leq y \leq \delta \\ V_m, & \delta \leq y \leq H - \delta \\ V_m \frac{H - y}{\delta}, & H - \delta \leq y \leq H \end{cases}$$

Q. 76

The ratio V_m/U_0 isGATE ME 2007
TWO MARK

(A) $\frac{1}{1 - 2(\delta/H)}$

(B) 1

(C) $\frac{1}{1 - (\delta/H)}$

(D) $\frac{1}{1 + (\delta/H)}$

Sol. 76

Option (C) is correct.

Let width of the channel = b

From mass conservation

Flow rate at section A = flow rate at B

or Velocity A \times Area of A = Velocity at B \times Area of B

$$U_0 \times (H \times b) = \text{Velocity for } (0 \leq y \leq \delta) \times dy \times b$$

+ velocity for $(\delta \leq y \leq H - \delta) \times dy \times b$

+ velocity for $(H - \delta \leq y \leq H) \times dy \times b$

or

$$U_0 \times H = V_m \int_0^\delta \frac{y}{\delta} dy + V_m \int_\delta^{H-\delta} dy + V_m \int_{H-\delta}^H \frac{H-y}{\delta} dy$$

or

$$U_0 \times H = V_m \frac{\delta}{2} + V_m(H - 2\delta) + \frac{V_m \delta}{2}$$

$$U_0 \times H = V_m \delta + V_m(H - 2\delta) = V_m(\delta + H - 2\delta)$$

or

$$\frac{V_m}{U_0} = \frac{H}{\delta + H - 2\delta}$$
$$= \frac{H}{H - \delta} = \frac{1}{1 - \frac{\delta}{H}}$$

Q. 77

The ratio $\frac{p_A - p_B}{\frac{1}{2}\rho U_0^2}$ (where p_A and p_B are the pressures at section A and B)GATE ME 2007
TWO MARKrespectively, and ρ is the density of the fluid) is

(A) $\frac{1}{[1 - (\delta/H)]^2} - 1$

(B) $\frac{1}{[1 - (\delta/H)]^2}$

(C) $\frac{1}{[1 - (2\delta/H)]^2} - 1$

(D) $\frac{1}{1 + (\delta/H)}$

Sol. 77

Option (A) is correct.

Applying Bernoulli's Equation at the section A and B.

$$\frac{p_A}{\rho g} + \frac{V_A^2}{2g} + z_A = \frac{p_B}{\rho g} + \frac{V_B^2}{2g} + z_B$$

Here, $z_A = z_B = 0$

So,

$$\frac{p_A - p_B}{\rho g} = \frac{V_B^2 - V_A^2}{2g}$$

$$\frac{p_A - p_B}{\rho} = \frac{V_B^2 - V_A^2}{2} = \frac{V_m^2 - U_0^2}{2} \quad V_B = V_m \text{ \& } V_A = U_0$$

$$= \frac{U_0^2 \left[\frac{V_m^2}{U_0^2} - 1 \right]}{2}$$

$$\frac{p_A - p_B}{\frac{1}{2} \rho U_0^2} = \frac{V_m^2}{U_0^2} - 1 = \left(\frac{V_m}{U_0} \right)^2 - 1$$

Substitute,

$$\frac{V_m}{U_0} = \frac{1}{1 - \frac{\delta}{H}} \quad \text{From previous part of question}$$

$$\frac{p_A - p_B}{\frac{1}{2} \rho U_0^2} = \frac{1}{\left[1 - \frac{\delta}{H} \right]^2} - 1$$

Statement for linked Answer Questions 78 and 79 :

Consider steady one-dimensional heat flow in a plate of 20 mm thickness with a uniform heat generation of 80 MW/m³. The left and right faces are kept at constant temperatures of 160°C and 120°C respectively. The plate has a constant thermal conductivity of 200 W/mK.

Q. 78

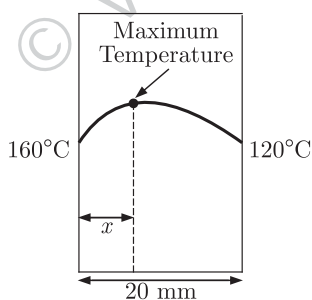
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TWO MARK

The location of maximum temperature within the plate from its left face is

- (A) 15 mm
- (B) 10 mm
- (C) 5 mm
- (D) 0 mm

Sol. 78

Option (C) is correct.



Let the location of maximum temperature occurs at the distance x from the left face. We know that steady state heat flow equation in one dimension with a uniform heat generation is given by,

$$\frac{\partial^2 T}{\partial x^2} + \frac{q_g}{k} = 0 \quad \dots(i)$$

Here q_g = Heat generated per unit volume & per unit time,

Given : $q_g = 80 \text{ MW/m}^3 = 80 \times 10^6 \text{ W/m}^3$, $k = 200 \text{ W/m K}$

Substituting the value of q_g & k in equation (i), we get

$$\frac{\partial^2 T}{\partial x^2} + \frac{80 \times 10^6}{200} = 0$$

$$\frac{\partial^2 T}{\partial x^2} + 4 \times 10^5 = 0$$

On integrating the above equation,

$$\frac{\partial T}{\partial x} + 4 \times 10^5 \times x + c_1 = 0 \quad \dots \text{(ii)}$$

Again integrating, we get

$$T + 4 \times 10^5 \times \frac{x^2}{2} + c_1 x + c_2 = 0 \quad \dots \text{(iii)}$$

Applying boundary conditions on equation (iii), we get

(1) At $x = 0$, $T = 160^\circ \text{C}$

$$160 + c_2 = 0$$

$$c_2 = -160 \quad \dots \text{(iv)}$$

(2) At $x = 20 \text{ mm} = 0.020 \text{ m}$, $T = 120^\circ \text{C}$

$$120 + 4 \times 10^5 \times \frac{(0.020)^2}{2} + c_1 \times 0.020 + (-160) = 0 \quad c_2 = -160$$

$$120 + 80 + 0.020c_1 - 160 = 0$$

$$0.020c_1 + 40 = 0$$

$$c_1 = -\frac{40}{0.020} = -2000 \dots \text{(v)}$$

To obtain the location of maximum temperature, applying maxima-minima principle and put $\frac{dT}{dx} = 0$ in equation (ii), we get

$$0 + 4 \times 10^5 x + (-2000) = 0 \quad c_1 = -2000$$

= 5 mm

$$x = \frac{2000}{4 \times 10^5} = 500 \times 10^{-5} = 5 \times 10^{-3} \text{ m}$$

Q. 79

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TWO MARK

The maximum temperature within the plate in $^\circ \text{C}$ is

- (A) 160 (B) 165
(C) 200 (D) 250

Sol. 79

Option (B) is correct.

From the previous part of the question, at $x = 5 \text{ mm}$ temperature is maximum.

So, put $x = 5 \text{ mm} = 5 \times 10^{-3} \text{ m}$ in equation (iii), we get

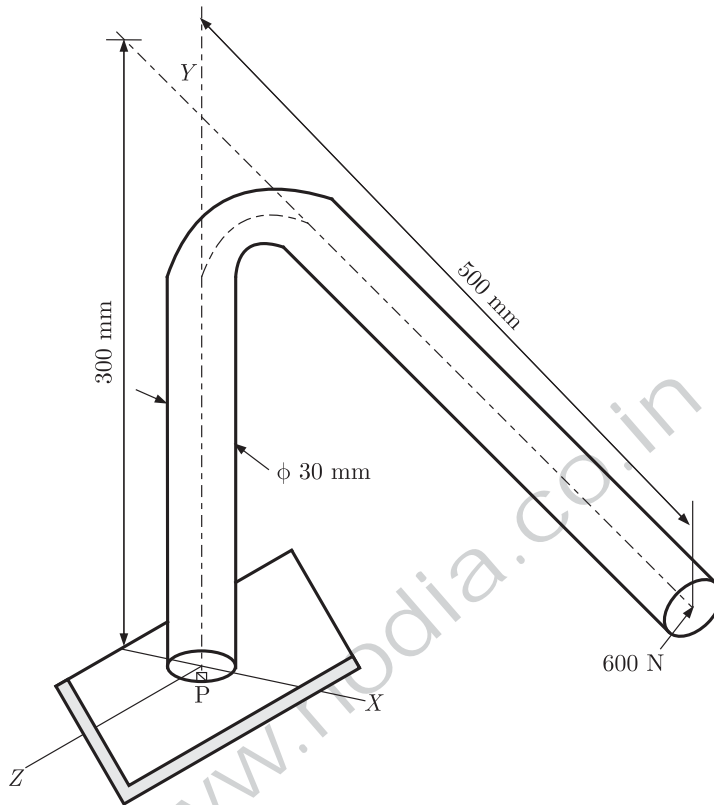
$$T + 4 \times 10^5 \times \frac{(5 \times 10^{-3})^2}{2} + (-2000) \times 5 \times 10^{-3} + (-160) = 0$$

$$T + 5 \times 10^6 \times 10^{-6} - 10 - 160 = 0$$

$$T + 5 - 170 = 0 \quad \Rightarrow \quad T = 165^\circ \text{C}$$

Statement for Linked Answer Questions 80 and 81 :

A machine frame shown in the figure below is subjected to a horizontal force of 600 N parallel to Z-direction.



Q. 80

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TWO MARK

The normal and shear stresses in MPa at point P are respectively

- (A) 67.9 and 56.6
- (B) 56.6 and 67.9
- (C) 67.9 and 0.0
- (D) 0.0 and 56.6

Sol. 80

Option (A) is correct.

Given : $F = 600 \text{ N}$ (Parallel to Z-direction), $d = 30 \text{ mm}$

Normal stress at point P, from bending equation

$$\begin{aligned} \sigma &= \frac{M}{I} \times y \quad \text{Here } M = \text{bending moment} \\ &= \frac{600 \times 300}{\frac{\pi}{64} d^4} \times \frac{d}{2} \\ \sigma &= \frac{18 \times 10^4 \times 32}{\pi d^3} = \frac{18 \times 10^4 \times 32}{3.14 (30)^3} = 67.9 \text{ MPa} \end{aligned}$$

And from Torsional equation, shear stress,

$$\begin{aligned} \frac{T}{J} &= \frac{\tau}{r} \\ \tau &= \frac{T}{J} \times r = \frac{600 \times 500}{\frac{\pi}{32} d^4} \times \frac{d}{2} = \text{Force} \times \text{Area length} \\ &= \frac{16 \times 600 \times 500}{3.14 \times (30)^3} = 56.61 \text{ MPa} \end{aligned}$$

Q. 81

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TWO MARK

The maximum principal stress in MPa and the orientation of the corresponding principal plane in degrees are respectively

- (A) -32.0 and -29.52
 (B) 100.0 and 60.48
 (C) -32.0 and 60.48
 (D) 100.0 and -29.52

Sol. 81

Option (D) is correct.

Here : $\sigma_x = 0$, $\sigma_y = 67.9$ MPa, $\tau_{xy} = 56.6$ MPa

Maximum principal stress,

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \frac{1}{2} \sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2} \quad \sigma_x = ?$$

Substitute the values, we get

$$\begin{aligned} \sigma_1 &= \frac{0 + 67.9}{2} + \frac{1}{2} \sqrt{(-67.9)^2 + 4 \times (56.6)^2} \\ &= 33.95 + \frac{1}{2} \sqrt{17424.65} = 33.95 + 66 \\ &= 99.95 \approx 100 \text{ MPa} \end{aligned}$$

And $\tan 2\theta = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$

Substitute the values, we get

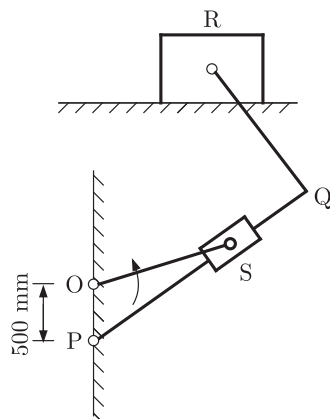
$$\tan 2\theta = \frac{2 \times 56.6}{0 - 67.9} = -1.667$$

$$2\theta = -59.04$$

$$\theta = -\frac{59.04}{2} = -29.52^\circ$$

Statement for Linked Answer Questions 82-83 :

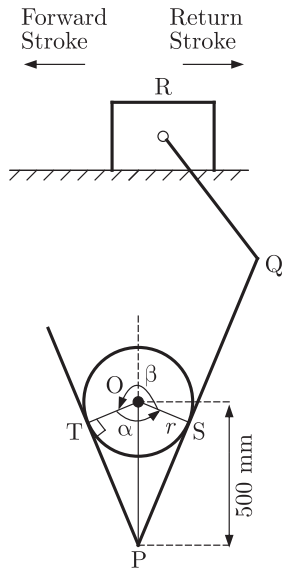
A quick return mechanism is shown below. The crank OS is driven at 2 rev/s in counter-clockwise direction.



- Q. 82 If the quick return ratio is 1 : 2, then the length of the crank in mm is
 (A) 250 (B) $250\sqrt{3}$
 (C) 500 (D) $500\sqrt{3}$

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 TWO MARK

Sol. 82 Option (A) is correct.



Given Quick return ratio = 1:2, $OP = 500$ mm
 Here $OT =$ Length of the crank. We see that the angle β made by the forward stroke is greater than the angle α described by the return stroke.
 Since the crank has uniform angular speed, therefore

$$\text{Quick return ratio} = \frac{\text{Time of return stroke}}{\text{Time of cutting stroke}}$$

$$\frac{1}{2} = \frac{\alpha}{\beta} = \frac{\alpha}{360 - \alpha}$$

$$360 - \alpha = 2\alpha$$

$$3\alpha = 360$$

$$\alpha = 120^\circ$$

& Angle $\angle TOP = \frac{\alpha}{2} = \frac{120}{2} = 60^\circ$

From the $\triangle TOP$, $\cos \frac{\alpha}{2} = \frac{OT}{OP} = \frac{r}{500}$ $OT = r$

$$\cos 60^\circ = \frac{r}{500}$$

$$r = 500 \times \frac{1}{2} = 250 \text{ mm}$$

- Q. 83 The angular speed of PQ in rev/s when the block R attains maximum speed during forward stroke (stroke with slower speed) is

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 TWO MARK

- (A) $\frac{1}{3}$ (B) $\frac{2}{3}$
 (C) 2 (D) 3

Sol. 83 Option (B) is correct.

We know that maximum speed during forward stroke occur when QR & QP are perpendicular.

$$\begin{aligned} \text{So, } V &= OS \times \omega_{OS} = PQ \times \omega_{PQ} & V &= r\omega \\ 250 \times 2 &= 750 \times \omega_{PQ} \\ \omega_{PQ} &= \frac{500}{750} = \frac{2}{3} \text{ rad/sec} \end{aligned}$$

Statement for Linked Answer Questions 84 & 85 :

A low carbon steel bar of 147 mm diameter with a length of 630 mm is being turned with uncoated carbide insert. The observed tool lives are 24 min and 12 min for cutting velocities of 90 m/min and 120 m/min. respectively. The feed and depth of cut are 0.2 mm/rev and 2 mm respectively. Use the unmachined diameter to calculate the cutting velocity.

Q. 84

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TWO MARK

When tool life is 20 min, the cutting velocity in m/min is

- (A) 87 (B) 97
(C) 107 (D) 114

Sol. 84

Option (B) is correct.

Given : $T_1 = 24$ min, $T_2 = 12$ min, $V_1 = 90$ m/min, $V_2 = 120$ m/min

We have to calculate velocity, when tool life is 20 minute.

First of all we calculate the values of n , From the Taylor's tool life equation.

$$VT^n = C$$

For case 1st and 2nd, we can write

$$V_1 T_1^n = V_2 T_2^n$$

$$\left(\frac{T_1}{T_2}\right)^n = \frac{V_2}{V_1}$$

$$\left(\frac{24}{12}\right)^n = \frac{120}{90}$$

$$(2)^n = 1.33$$

Taking log both the sides,

$$n \log 2 = \log 1.33$$

$$n \times 0.301 = 0.124$$

$$n = 0.412$$

For V_3 , we can write from tool life equation,

$$V_1 T_1^n = V_3 T_3^n$$

$$90 \times (24)^{0.412} = V_3 (20)^{0.412}$$

$$333.34 = V_3 \times 3.435$$

$$V_3 = 97 \text{ m/min}$$

Q. 85

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TWO MARK

Neglect over-travel or approach of the tool. When tool life is 20 min., the machining time in min for a single pass is

- (A) 5 (B) 10
(C) 15 (D) 20

Sol. 85

Option (C) is correct.

Given : $D = 147$ mm, $l = 630$ mm, $f = 0.2$ mm/rev.

$d = 2$ mm, $V_3 = 97$ m/min

$$\text{Machining time } t = \frac{l}{fN}$$

$$V = \pi DN \text{ m/min}$$

So,

$$t = \frac{l \times \pi \times D}{fV}$$

$$t = \frac{0.63 \times 3.14 \times 0.147}{0.2 \times 10^{-3} \times 97}$$

$$V = V_3$$

$$t = 15 \text{ min}$$

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Answer Sheet									
1.	(B)	18.	(B)	35.	(B)	52.	(A)	69.	(B)
2.	(A)	19.	(C)	36.	(C)	53.	(A)	70.	(B)
3.	(B)	20.	(D)	37.	(D)	54.	(C)	71.	(B)
4.	(A)	21.	(B)	38.	(B)	55.	(A)	72.	(C)
5.	(D)	22.	(B)	39.	(A)	56.	(D)	73.	(C)
6.	(A)	23.	(C)	40.	(B)	57.	(A)	74.	(C)
7.	(D)	24.	(A)	41.	(A)	58.	(C)	75.	(A)
8.	(B)	25.	(A)	42.	(A)	59.	(C)	76.	(C)
9.	(A)	26.	(D)	43.	(A)	60.	(C)	77.	(A)
10.	(B)	27.	(B)	44.	(B)	61.	(B)	78.	(C)
11.	(D)	28.	(B)	45.	(C)	62.	(C)	79.	(B)
12.	(A)	29.	(C)	46.	(B)	63.	(D)	80.	(A)
13.	(C)	30.	(B)	47.	(B)	64.	(D)	81.	(D)
14.	(C)	31.	(B)	48.	(A)	65.	(A)	82.	(A)
15.	(B)	32.	(D)	49.	(C)	66.	(C)	83.	(B)
16.	(D)	33.	(C)	50.	(C)	67.	(A)	84.	(B)
17.	(C)	34.	(C)	51.	(A)	68.	(D)	85.	(C)

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