GATE SOLVED PAPER
Mechanical Engineering
2012

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GATE SOLVED PAPER - ME

2012

Q. 1
GATE ME 2012
ONE MARK

In abrasive jet machining, as the distance between the nozzle tip and the work surface increases, the material removal rate

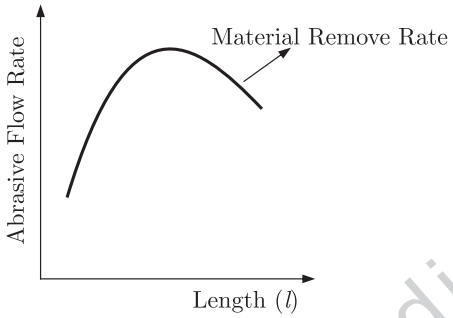
(A) increases continuously. (B) decreases continuously.

(C) decreases, becomes stable and then increases.

(D) increases, becomes stable and then decreases.

Sol. 1

Option (D) is correct.



Graph for abrasive jet machining for the distance between the nozzle tip and work surface (l) and abrasive flow rate is given in figure.

It is clear from the graph that the material removal rate is first increases because of area of jet increase than becomes stable and then decreases due to decrease in jet velocity.

Q. 2
GATE ME 2012
ONE MARK

Match the following metal forming processes with their associated stresses in the workpiece.

	Metal forming process		Types of stress
1.	Coining	P.	Tensile
2.	Wire Drawing	Q.	Shear
3.	Blanking	R.	Tensile and compressive
4.	Deep Drawing	S.	Compressive

(A) 1-S, 2-P, 3-Q, 4-R (B) 1-S, 2-P, 3-R, 4-Q

(C) 1-P, 2-Q, 3-S, 4-R (D) 1-P, 2-R, 3-Q, 4-S

Sol. 2

Option (A) is correct.

	Metal forming process		Types of stress
1.	Coining	S.	Compressive
2.	Wire Drawing	P.	Tensile
3.	Blanking	Q.	Shear
4.	Deep Drawing	R.	Tensile and compressive

Hence, correct match list is, 1-S, 2-P, 3-Q, 4-R

Q. 3
GATE ME 2012
ONE MARK

In an interchangeable assembly, shafts of size $25.000^{+0.040}_{-0.010}$ mm mate with holes of size $25.000^{+0.030}_{+0.020}$ mm. The maximum interference (in microns) in the assembly is

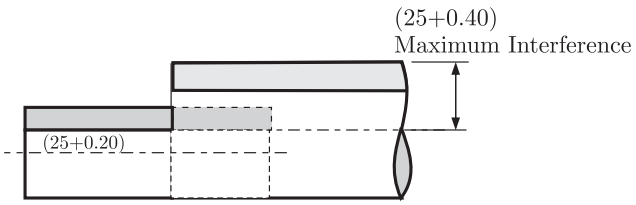
(A) 40 (B) 30

(C) 20 (D) 10

Sol. 3

Option (C) is correct.

An interference fit for shaft and hole is as given in figure below.



Maximum Interference

= Maximum limit of shat − Minimum limit of hole

$$= (25 + 0.040) - (25 + 0.020)$$

$$= 0.02 \text{ mm} = 20 \text{ microns}$$

Q. 4
GATE ME 2012
ONE MARK

During normalizing process of steel, the specimen is heated

(A) between the upper and lower critical temperature and cooled in still air.

(B) above the upper critical temperature and cooled in furnace.

(C) above the upper critical temperature and cooled in still air.

(D) between the upper and lower critical temperature and cooled in furnace

Sol. 4

Option (C) is correct

Normalizing involves prolonged heating just above the critical temperature to produce globular form of carbine and then cooling in air.

Q. 5
GATE ME 2012
ONE MARK

Oil flows through a 200 mm diameter horizontal cast iron pipe (friction factor, $f = 0.0225$) of length 500 m. The volumetric flow rate is $0.2\ \text{m}^3/\text{s}$. The head loss (in m) due to friction is (assume $g = 9.81\ \text{m/s}^2$)

(A) 116.18 (B) 0.116

(C) 18.22 (D) 232.36

Sol. 5

Option (A) is correct.

From Darcy Weischback equation head loss

$$h = f \times \frac{L}{D} \times \frac{V^2}{2g} \qquad ...(1)$$

Given that $h = 500$ m, $D = \frac{200}{1000} = 0.2$ m, $f = 0.0225$

Since volumetric flow rate

$$\dot{\nu} = \text{Area} \times \text{velocity of flow}\,(V)$$

$$V = \frac{\dot{\nu}}{\text{Area}} = \frac{0.2}{\frac{\pi}{4} \times (0.2)^2} = 6.37 \text{ m/s}$$

Hence,

$$h = 0.0225 \times \frac{500}{0.2} \times \frac{(6.37)^2}{2 \times 9.81}$$

$$h = 116.33 \text{ m} \simeq 116.18 \text{ m}$$

Q. 6
GATE ME 2012
ONE MARK

For an opaque surface, the absorptivity (α), transmissivity (τ) and reflectivity (ρ) are related by the equation :

(A) $\alpha + \rho = \tau$ (B) $\rho + \alpha + \tau = 0$

(C) $\alpha + \rho = 1$ (D) $\alpha + \rho = 0$

Sol. 6

Option (C) is correct.

The sum of the absorbed, reflected and transmitted radiation be equal to

$$\alpha + \rho + \tau = 1$$

$\alpha =$ Absorpivity, $\rho =$ Reflectivity, $\tau =$ Transmissivity

For an opaque surfaces such as solids and liquids

$$\tau = 0,$$

Thus, $$\alpha + \rho = 1$$

Q. 7
GATE ME 2012
ONE MARK

Steam enters an adiabatic turbine operating at steady state with an enthalpy of 3251.0 kJ/kg and leaves as a saturated mixture at 15 kPa with quality (dryness fraction) 0.9. The enthalpies of the saturated liquid and vapour at 15 kPa are $h_f = 225.94$ kJ/kg and $h_g = 2598.3$ kJ/kg respectively. The mass flow rate of steam is 10 kg/s. Kinetic and potential energy changes are negligible. The power output of the turbine in MW is

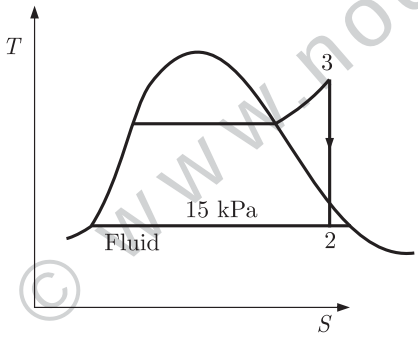
(A) 6.5 (B) 8.9

(C) 9.1 (D) 27.0

Sol. 7

Option (B) is correct.

For adiabatic expansion steam in turbine.



Given $h_1 = 3251.0$ kJ/kg, $m = 10$ kg/s, $x = 0.9$ (dryness fraction)

At 15 kPa

Enthalpy of liquid, $h_f = 225.94$ kJ/kg

Enthalpy of vapour, $h_g = 2598.3$ kJ/kg

Since Power output of turbine.

$$P = \dot{m}(h_1 - h_2)$$

(K.E and P.E are negligible) ...(i)

$$h_2 = h_f + xh_{fg} = h_f + x(h_g - h_f)$$
$$= 225.94 + 0.9(2598.3 - 225.94)$$
$$= 2361.064 \text{ kJ/kg}$$

From Eq. (i)

$$P = 10 \times (3251.0 - 2361.064)$$
$$= 8899 \text{ kW} = 8.9 \text{ MW}$$

Q. 8 GATE ME 2012 ONE MARK

The following are the data for two crossed helical gears used for speed reduction :

Gear I : Pitch circle diameter in the plane of rotation 80 mm and helix angle 30°.

Gear II : Pitch circle diameter in the plane of rotation 120 mm and helix angle 22.5°.

If the input speed is 1440 rpm, the output speed in rpm is

(A) 1200

(B) 900

(C) 875

(D) 720

Sol. 8

Option (B) is correct.

For helical gears, speed ratio is given by

$$\frac{N_1}{N_2} = \frac{D_2}{D_1} \times \frac{\cos\beta_2}{\cos\beta_1} \qquad ...(i)$$

$$N_1 = 1440 \text{ rpm},\ D_1 = 80 \text{ mm},\ D_2 = 120 \text{ mm}$$

$$\beta_1 = 30^\circ,\ \beta_2 = 22.5^\circ$$

Hence from Eq. (i)

$$N_2 = \frac{D_1}{D_2} \times \frac{\cos\beta_1}{\cos\beta_2} \times N_1$$

$$= \frac{80}{120} \times \frac{\cos 30^\circ}{\cos 22.5^\circ} \times 1440$$

$$N_2 = 899.88 \simeq 900 \text{ rpm}$$

Q. 9 GATE ME 2012 ONE MARK

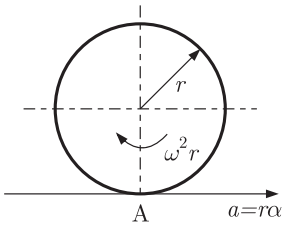
A solid disc of radius r rolls without slipping on a horizontal floor with angular velocity ω and angular acceleration α. The magnitude of the acceleration of the point of contact on the disc is

(A) zero (B) $r\alpha$

(C) $\sqrt{(r\alpha)^2 + (r\omega^2)^2}$ (D) $r\omega^2$

Sol. 9

Option (D) is correct.



For A solid disc of radius (r) as given in figure, rolls without slipping on a horizontal floor with angular velocity ω and angular acceleration α.

The magnitude of the acceleration of the point of contact (A) on the disc is only by centripetal acceleration because of no slip condition.

$$v = \omega r \qquad ...(i)$$

By differentiating Eq. (1) w.r.t. (t)

$$\frac{dv}{dt} = r\frac{d\omega}{dt} = r \cdot \alpha \qquad \left(\frac{d\omega}{dt} = \alpha, \frac{dv}{dt} = a\right)$$

or, $a = r \cdot \alpha$

Instantaneous velocity of point A is zero

So at point A, Instantaneous tangential acceleration = zero

Therefore only centripetal acceleration is there at point A.

Centripetal acceleration $= r\omega^2$

Q. 10 GATE ME 2012 ONE MARK

A thin walled spherical shell is subjected to an internal pressure. If the radius of the shell is increased by 1% and the thickness is reduced by 1%, with the internal pressure remaining the same, the percentage change in the circumferential (hoop) stress is

(A) 0 (B) 1

(C) 1.08 (D) 2.02

Sol. 10

Option (D) is correct.

For thin walled spherical shell circumferential (hoop) stress is

$$\sigma = \frac{pd}{4t} = \frac{pr}{2t}$$

For initial condition let radius r_1 and thickness t_1, then

$$\sigma_1 = \frac{pr_1}{2t_1} \quad \text{...(i)}$$

For final condition radius r_2 increased by 1%, then

$$r_2 = r_1 + \frac{r_1}{100} = 1.01\, r_1$$

Thickness t_2 decreased by 1% then

$$t_2 = t_1 - \frac{t_1}{100} = 0.99 t_1$$

and $$\sigma_2 = \frac{pr_2}{2t_2} = \frac{p \times 1.01 r_1}{1 \times 9.99 t_1} = 1.0202 \frac{pr_1}{2t_1}$$

From Eq. (i) $\sigma_2 = 1.0202 \times \sigma_1$

Change in hoop stress (%)

$$\sigma_c = \frac{\sigma_2 - \sigma_1}{\sigma_1} \times 100 = \frac{1.0202\sigma_1 - \sigma_1}{\sigma_1} \times 100$$

$$= 2.02\%$$

Q. 11 GATE ME 2012 ONE MARK

The area enclosed between the straight line $y = x$ and the parabola $y = x^2$ in the x-y plane is

(A) 1/6 (B) 1/4

(C) 1/3 (D) 1/2

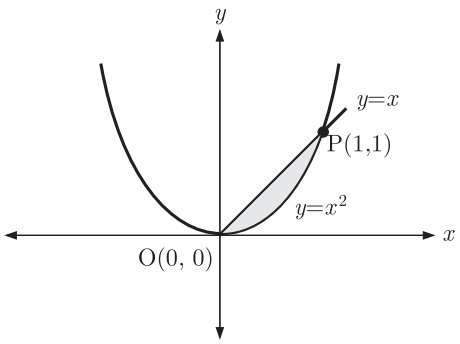
Sol. 11

Option (A) is correct.

For $y = x$ straight line and

$y = x^2$ parabola

Curve is as given. The shaded region is show the area, which is bounded by the both curves (common area).



On solving given equation, we get the intersection points as,

$$y = x^2 \text{ put } y = x$$
$$x = x^2$$
$$x^2 - x = 0$$
$$x(x-1) = 0$$
$$x = 0, 1$$

Then from $y = x$

For $x = 0 \Rightarrow y = 0$

& $x = 1 \Rightarrow y = 1$

We can see that curve $y = x^2$ and $y = x$ intersects at point $(0,0)$ and $(1,1)$

So, the area bounded by both the curves is

$$A = \int_{x=0}^{x=1} \int_{y=x}^{y=x^2} dy dx$$

$$= \int_{x=0}^{x=1} dx \int_{y=x}^{y=x^2} dy = \int_{x=0}^{x=1} dx [y]_x^{x^2}$$

After substituting the limit, we have

$$= \int_{x=0}^{x=1} (x^2 - x)\, dx$$

Integrating the equation, we get

$$= \left[\frac{x^3}{3} - \frac{x^2}{2}\right]_0^1 = \frac{1}{3} - \frac{1}{2} = -\frac{1}{6}$$

$$= \frac{1}{6}\text{unit}^2$$ Area is never negative

Q. 12

GATE ME 2012
ONE MARK

Consider the function $f(x) = |x|$ in the interval $-1 \le x \le 1$. At the point $x = 0$, $f(x)$ is

(A) continuous and differentiable

(B) non-continuous and differentiable

(C) continuous and non-differentiable

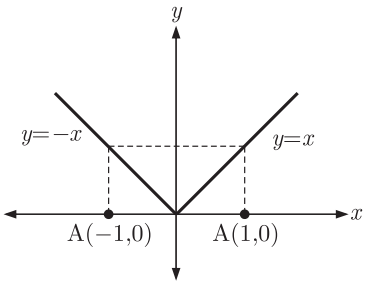
(D) neither continuous nor differentiable

Sol. 12

Option (C) is correct.

Given $f(x) = |x|$ (in $-1 \le x \le 1$)

For this function the plot is as given below.



At $x = 0$, function is continuous but not differentiable because.

For $x > 0$ and $x < 0$

$$f'(x) = 1 \text{ and } f'(x) = -1$$

$$\lim_{x \to 0^+} f'(x) = 1 \text{ and } \lim_{x \to 0^-} f'(x) = -1$$

R.H.S lim $= 1$ and L.H.S lim $= -1$

Therefore it is not differentiable

Q. 13 GATE ME 2012 ONE MARK

Which one of the following is NOT a decision taken during the aggregate production planning stage ?

(A) Scheduling of machines

(B) Amount of labour to be committed

(C) Rate at which production should happen

(D) Inventory to be carried forward

Sol. 13

Option (A) is correct.

Costs relevant to aggregate production planning is as given below.

(i) Basic production cost : Material costs, direct labour costs, and overhead cost.

(ii) Costs associated with changes in production rate : Costs involving in hiring, training and laying off personnel, as well as, overtime compensation.

(iii) Inventory related costs.

Hence, from above option (A) is not related to these costs. Therefore option (A) is not a decision taken during the APP.

Q. 14 GATE ME 2012 ONE MARK

$\lim_{x \to 0}\left(\frac{1 - \cos x}{x^2}\right)$ is

(A) 1/4 (B) 1/2

(C) 1 (D) 2

Sol. 14

Option (B) is correct.

Let $$y \lim_{x \to 0} \frac{(1 - \cos x)}{x^2}$$

It forms $\left[\frac{0}{0}\right]$ condition. Hence by L-Hospital rule

$$= \lim_{x \to 0} \frac{\frac{d}{dx}(1 - \cos x)}{\frac{d}{dx}(x^2)} = \lim_{x \to 0} \frac{\sin x}{2x}$$

Still these gives $\left[\frac{0}{0}\right]$ condition, so again applying L-Hospital rule

$$y = \lim_{x \to 0} \frac{\frac{d}{dx}(\sin x)}{2 \times \frac{d}{dx}(x)}$$

$$= \lim_{x \to 0} \frac{\cos x}{2}$$

$$= \frac{\cos 0}{2} = \frac{1}{2}$$

Q. 15 GATE ME 2012 ONE MARK

A CNC vertical milling machine has to cut a straight slot of 10 mm width and 2 mm depth by a cutter of 10 mm diameter between points $(0,0)$ and $(100,100)$ on the XY plane (dimensions in mm). The feed rate used for milling is 50 mm/ min . Milling time for the slot (in seconds) is

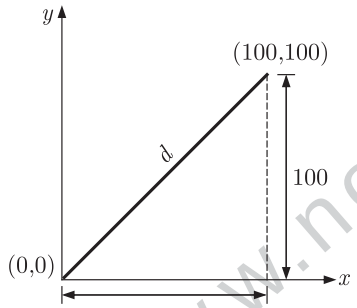
(A) 120 (B) 170

(C) 180 (D) 240

Sol. 15

Option (B) is correct.

Given : width $(b) = 10$ mm, depth $= 2$ mm



Distance travelled for cut between points $(0,0)$ and $(100,100)$

By Pythagoras theorem

$$d = \sqrt{100^2 + 100^2} = 141.42 \text{ mm}$$

$$\text{Feed rate } f = 50 \text{ mm/ min}$$

$$= \frac{50}{60} = 0.833 \text{ mm/ sec.}$$

Time required to cut distance (d)

$$t = \frac{d}{f} = \frac{141.42}{0.833} = 169.7 \simeq 170 \text{ sec.}$$

Q. 16 GATE ME 2012 ONE MARK

A solid cylinder of diameter 100 mm and height 50 mm is forged between two frictionless flat dies to a height of 25 mm. The percentage change in diameter is

(A) 0 (B) 2.07

(C) 20.7 (D) 41.4

Sol. 16

Option (D) is correct.

Since volume of cylinder remains same. Therefore

$$\text{Volume before forging} = \text{Volume after forging}$$

$$\pi \frac{d_1^2}{4} \times h_1 = \pi \frac{d_2^2}{4} \times h_2$$

$$\pi \times \frac{100^2}{4} \times 50 = \pi \times \frac{d_2^2}{4} \times 25$$

$$d_2^2 = (100)^2 \times 2$$

$$d_2 = 100 \times \sqrt{2} = 141.42$$

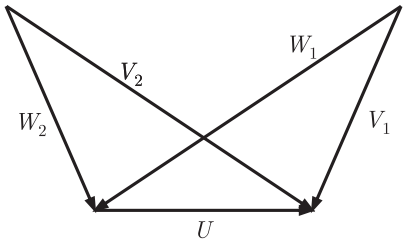
Percentage change in diameter

$$= \frac{d_2 - d_1}{d_1} \times 100 = \frac{141.42 - 100}{100} \times 100$$

$$\% \text{ change in } (d) = 41.42\%$$

Q. 17
GATE ME 2012
ONE MARK

The velocity triangles at the inlet and exit of the rotor of a turbomachine are shown. V denotes the absolute velocity of the fluid, W denotes the relative velocity of the fluid and U denotes the blade velocity. Subscripts 1 and 2 refer to inlet and outlet respectively. If $V_2 = W_1$ and $V_1 = W_2$, then the degree of reaction is



(A) 0 (B) 1

(C) 0.5 (D) 0.25

Sol. 17

Option (C) is correct.

Degree of reaction

$$R = 1 - \frac{(V_1^2 - V_2^2)}{(V_1^2 - V_2^2) + (U_1^2 - U_2^2) + (W_2^2 - W_1^2)}$$

where V_1 and V_2 are absolute velocities

W_1 and W_2 are relative velocities

U_1 and $U_2 = U$ for given figure

Given $W_2 = V_1$, $W_1 = V_2$

Hence

$$R = 1 - \frac{(V_1^2 - V_2^2)}{(V_1^2 - V_2^2) + (U^2 - U^2) + (V_1^2 - V_2^2)}$$

$$R = 1 - \frac{(V_1^2 - V_2^2)}{2(V_1^2 - V_2^2)}$$

$$R = 1 - \frac{1}{2}$$

$$R = 0.5$$

Q. 18
GATE ME 2012
ONE MARK

Which one of the following configurations has the highest fin effectiveness ?

(A) Thin, closely spaced fins (B) Thin, widely spaced fins

(C) Thick, widely spaced fins (D) Thick, closely spaced fins

Sol. 18

Option (A) is correct.

The performance of the fins is judged on the basis of the enhancement in heat transfer area relative to the no fin case. The fin effectiveness

$$\varepsilon_{fin} = \frac{\text{Heat transfer rate from the fin of base area}}{\text{Heat transfer rate from the surface area}}$$

When determining the rate of heat transfer from a finned surface, we must consider the unfinned portion of the surface as well as the fins and number of fins.

Thin and closed spaced fin configuration, the unfinned portion of surface is reduced and number of fins is increased. Hence the fin effectiveness will be maximum for thin and closely spaced fins.

Q. 19
GATE ME 2012
ONE MARK

A ideal gas of mass m and temperature T_1 undergoes a reversible isothermal process from an initial pressure p_1 to final pressure p_2. The heat loss during the process is Q. The entropy change Δs of the gas is

(A) $mR\ln\left(\frac{p_2}{p_1}\right)$ (B) $mR\ln\left(\frac{p_1}{p_2}\right)$

(C) $mR\ln\left(\frac{p_2}{p_1}\right)-\frac{Q}{T_1}$ (D) zero

Sol. 19

Option (B) is correct.

We know that

$$Tds = du + Pd\nu \qquad ...(i)$$

For ideal gas

$$p\nu = mRT$$

For isothermal process

$$T = \text{constant}$$

For reversible process

$$du = 0$$

Then from equation (i)

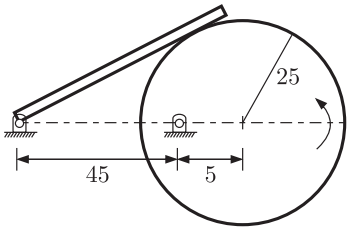
$$ds = \frac{pd\nu}{T} = \frac{mRT}{T}\frac{d\nu}{\nu} = mR\frac{d\nu}{\nu}$$

$$\int ds = \Delta s = mR\int_{\nu_1}^{\nu_2}\frac{d\nu}{\nu} = mR\ln\frac{\nu_2}{\nu_1}$$

$$\Delta s = mR\ln\frac{p_1}{p_2} \qquad \left[\frac{p_1}{p_2}=\frac{\nu_2}{\nu_1}\right]$$

Q. 20
GATE ME 2012
ONE MARK

In the mechanism given below, if the angular velocity of the eccentric circular disc is 1 rad/s, the angular velocity (rad/s) of the follower link for the instant shown in the figure is (Note. All dimensions are in mm).



(A) 0.05

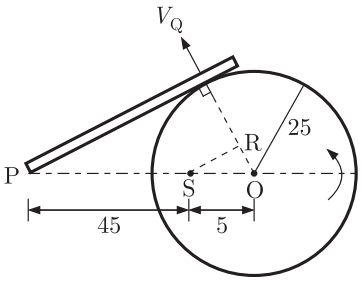
(B) 0.1

(C) 5.0

(D) 10.0

Sol. 20

Option (B) is correct.



From similar ΔPQO and ΔSRO

$$\frac{PQ}{SR} = \frac{PO}{SO} \qquad \text{...(i)}$$

$$PQ = \sqrt{(50)^2 - (25)^2} = 43.3\,\text{mm}$$

From Eq. (i)

$$\frac{43.3}{SR} = \frac{50}{5}$$

$$SR = \frac{43.5 \times 5}{50} = 4.33\,\text{mm}$$

Velocity of Q = Velocity of R (situated at the same link)

$$V_Q = V_R = SR \times \omega$$
$$= 4.33 \times 1 = 4.33\,\text{m/s}$$

Angular velocity of PQ

$$\omega_{PQ} = \frac{V_Q}{PQ} = \frac{4.33}{43.3} = 0.1\,\text{rad/s}$$

Q. 21 GATE ME 2012 ONE MARK

A circular solid disc of uniform thickness 20 mm, radius 200 mm and mass 20 kg, is used as a flywheel. If it rotates at 600 rpm, the kinetic energy of the flywheel, in Joules is

(A) 395 (B) 790

(C) 1580 (D) 3160

Sol. 21

Option (B) is correct.

For flywheel

$$K.E = \frac{1}{2} I\omega^2$$

$$\omega = \frac{2\pi N}{60} = \frac{2 \times \pi \times 600}{60} = 62.83\,\text{rad/s}$$

$$I \text{ (for solid circular disk)} = \frac{1}{2} mR^2$$

$$= \frac{1}{2} \times 20 \times (0.2)^2 = 0.4\,\text{kg-m}^2$$

Hence,

$$K.E = \frac{1}{2} \times (0.4) \times (62.83)^2$$

$$= 789.6 \simeq 790\ \text{Joules}$$

Q. 22
GATE ME 2012
ONE MARK

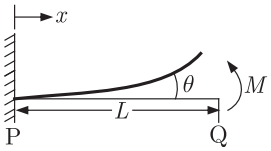
A cantilever beam of length L is subjected to a moment M at the free end. The moment of inertia of the beam cross section about the neutral axis is I and the Young's modulus is E. The magnitude of the maximum deflection is

(A) $\frac{ML^2}{2EI}$ (B) $\frac{ML^2}{EI}$

(C) $\frac{2ML^2}{EI}$ (D) $\frac{4ML^2}{EI}$

Sol. 22

Option (A) is correct.



Since $EI\frac{d^2y}{dx^2} = M$

By integrating

$$EI\frac{dy}{dx} = mx + C_1 \quad \text{...(i)}$$

At $x = 0$, $\frac{dy}{dx} = 0$

So $EI(0) = M(0) + C_1$

$C_1 = 0$

Hence Eq.(i) becomes

$$EI\frac{dy}{dx} = mx$$

Again integrate

$$EIy = \frac{mx^2}{2} + C_2 \quad \text{...(ii)}$$

At $x = 0$, $y = 0$

$$EI(0) = \frac{m(0)^2}{2} + C_2$$

$C_2 = 0$

Then Eq. (ii) becomes

$$EIy = \frac{Mx^2}{2}$$

$$y = \frac{Mx^2}{2EI} \qquad \Rightarrow y_{max} = \frac{ML^2}{2EI}$$

$(\text{At } x = L, y = y_{max})$

Q. 23
GATE ME 2012
ONE MARK

For a long slender column of uniform cross section, the ratio of critical buckling load for the case with both ends clamped to the case with both the ends hinged is

(A) 1 (B) 2

(C) 4 (D) 8

Sol. 23

Option (C) is correct.

Critical buckling load

$$= \frac{\pi EI}{L^2} \quad \text{...(i)}$$

For both ends clamped $L = \frac{L}{2}$

For both ends hinged $L = L$

Hence, Ratio for both ends clamped to both ends hinged

$$= \frac{\frac{\pi EI}{\left(\frac{L}{2}\right)^2}}{\frac{\pi EI}{L^2}} = \frac{4}{L^2} \times \frac{L^2}{1} = 4$$

Q. 24 GATE ME 2012 ONE MARK

At $x = 0$, the function $f(x) = x^3 + 1$ has

(A) a maximum value (B) a minimum value

(C) a singularity (D) a point of inflection

Sol. 24

Option (D) is correct.

We have $f(x) = x^3 + 1$

$f'(x) = 3x^2 + 0$

By putting $f'(x)$ equal to zero

$f'(x) = 0$

$3x^2 + 0 = 0$

$x = 0$

Now $f''(x) = 6x$

At $x = 0$, $f''(0) = 6 \times 0 = 0$

Hence $x = 0$ is the point of inflection.

Q. 25 GATE ME 2012 ONE MARK

For the spherical surface $x^2 + y^2 + z^2 = 1$, the unit outward normal vector at the point $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right)$ is given by

(A) $\frac{1}{\sqrt{2}}i + \frac{1}{\sqrt{2}}j$ (B) $\frac{1}{\sqrt{2}}i - \frac{1}{\sqrt{2}}j$

(C) k (D) $\frac{1}{\sqrt{3}}i + \frac{1}{\sqrt{3}}j + \frac{1}{\sqrt{3}}k$

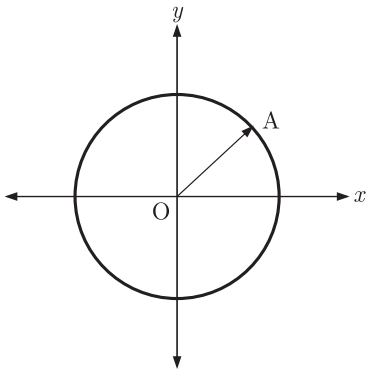
Sol. 25

Option (A) is correct.

Given :

$$x^2 + y^2 + z^2 = 1$$

This is a equation of sphere with radius $r = 1$



The unit normal vector at point $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right)$ is OA

Hence

$$OA = \left(\frac{1}{\sqrt{2}} - 0\right)i + \left(\frac{1}{\sqrt{2}} - 0\right)j + (0-0)k$$

$$= \frac{1}{\sqrt{2}}i + \frac{1}{\sqrt{2}}j$$

Q. 26
GATE ME 2012 TWO MARK

The homogeneous state of stress for a metal part undergoing plastic deformation is

$$T = \begin{bmatrix} 10 & 5 & 0 \\ 5 & 20 & 0 \\ 0 & 0 & -10 \end{bmatrix}$$

where the stress component values are in MPa. Using Von Mises Yield criterion, the value of estimated shear yield stress, in MPa is

(A) 9.50 (B) 16.07

(C) 28.52 (D) 49.41

Sol. 26

Option (B) is correct.

According to Von Mises Yield criterion

$$\sigma_Y^2 = \frac{1}{2}\left[(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + 6(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2)\right]$$

Given, $T = \begin{bmatrix} 10 & 5 & 0 \\ 5 & 20 & 0 \\ 0 & 0 & -10 \end{bmatrix}$

From given Matrix

$$\sigma_x = 10 \qquad \tau_{xy} = 5$$
$$\sigma_y = 20 \qquad \tau_{yz} = 0$$
$$\sigma_z = -10 \qquad \tau_{zx} = 0$$

So, $\sigma_Y^2 = \frac{1}{2}\left[(10-20)^2 + (20+10)^2 + (-10-10)^2 + 6(5^2 + 0^2 + 0^2)\right]$

$$= \frac{1}{2} \times [100 + 900 + 400 + (6 \times 25)]$$

$$\sigma_Y = 27.83 \text{ MPa}$$

Shear yield stress

$$\tau_Y = \frac{\sigma_Y}{\sqrt{3}} = \frac{27.83}{\sqrt{3}} = 16.06 \text{ MPa}$$

Q. 27
GATE ME 2012 TWO MARK

Detail pertaining to an orthogonal metal cutting process are given below

Chip thickness ratio	0.4
Undeformed thickness	0.6 mm
Rake angle	$+10^\circ$
Cutting speed	2.5 m/s
Mean thickness of primary shear zone	25 microns

The shear strain rate in s^{-1} during the process is

(A) 0.1781×10^5 (B) 0.7754×10^5

(C) 1.0104×10^5 (D) 4.397×10^5

Sol. 27

Option (C) is correct.

Shear strain rate $= \frac{\cos\alpha}{\cos(\phi - \alpha)} \times \frac{V}{\Delta y}$

Where α = Rake angle = 10°

V = cutting speed = 2.5 m/s

Δy = Mean thickness of primary shear zone

= 25 microns = 25×10^{-6} m

ϕ = shear angle

Shear angle, $\tan\phi = \dfrac{r\cos\alpha}{1 - r\sin\alpha}$

where r = chip thickness ratio = 0.4

$$\tan\phi = \frac{0.4 \times \cos 10^\circ}{1 - 0.4\sin 10^\circ} = 0.4233$$

$$\phi = \tan^{-1}(0.4233) \cong 23^\circ$$

Shear Strain rate $= \dfrac{\cos 10^\circ}{\cos(23 - 10)} \times \dfrac{2.5}{25 \times 10^{-6}} = 1.0104 \times 10^5\, s^{-1}$

Q. 28

GATE ME 2012 TWO MARK

In a single pass drilling operation, a through hole of 15 mm diameter is to be drilled in a steel plate of 50 mm thickness. Drill spindle speed is 500 rpm, feed is 0.2 mm/rev and drill point angle is 118°. Assuming 2 mm clearance at approach and exit, the total drill time (in seconds) is

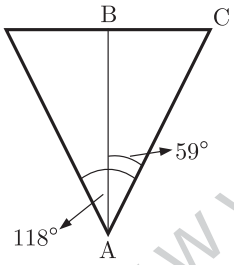
(A) 35.1 (B) 32.4

(C) 31.2 (D) 30.1

Sol. 28

Option (A) is correct.

Drill bit tip is shown as below.



BC = radius of hole or drill bit $(R) = \dfrac{15}{2} = 7.5$ mm

From ΔABC $\tan 59^\circ = \dfrac{BC}{AB} = \dfrac{7.5}{AB}$

$$AB = \frac{7.5}{\tan 59^\circ} = 4.506 \text{ mm}$$

Travel distance of drill bit

l = thickness of steel plate (t) + clearance at approach + clearance at exit + AB

$= 50 \text{ mm} + 2 + 2 + 4.506 = 58.506$ mm

$$\text{Total drill time} = \frac{\text{distance}}{\text{feed rate}}$$

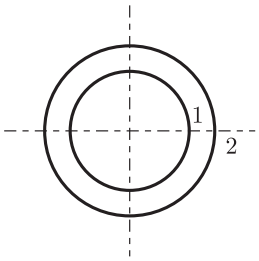
$f = 0.2$ mm/rev

$$= \frac{0.2 \times \text{rpm}}{60} = \frac{0.2 \times 500}{60} = 1.66 \text{ mm/s}$$

Hence drill time, $t = \dfrac{58.506}{1.60} = 35.1$ sec.

Q. 29
GATE ME 2012
TWO MARK

Consider two infinitely long thin concentric tubes of circular cross section as shown in the figure. If D_1 and D_2 are the diameters of the inner and outer tubes respectively, then the view factor F_{22} is give by



(A) $\left(\frac{D_2}{D_1}\right)-1$ (B) zero

(C) $\left(\frac{D_1}{D_2}\right)$ (D) $1-\left(\frac{D_1}{D_2}\right)$

Sol. 29

Option (D) is correct.

According to the reciprocity relation.

$$A_1F_{12} = A_2F_{21}$$

Which yields

$$F_{21} = \frac{A_1}{A_2}\times F_{12} = \frac{\pi D_1 L}{\pi D_2 L}\times 1 = \left(\frac{D_1}{D_2}\right)$$

$F_{11} = 0$ since no radiation leaving surface 1 and strikes 1

$F_{12} = 1$, since all radiation leaving surface 1 and strikes 2

The view factor F_{22} is determined by applying summation rule to surface 2,

$$F_{21}+F_{22} = 1$$

Thus

$$F_{22} = 1-F_{21}$$

$$= 1-\left(\frac{D_1}{D_2}\right)$$

Q. 30
GATE ME 2012
TWO MARK

An incompressible fluid flows over a flat plate with zero pressure gradient. The boundary layer thickness is $1\,\text{mm}$ at a location where the Reynolds number is 1000. If the velocity of the fluid alone is increased by a factor of 4, then the boundary layer thickness at the same location, in mm will be

(A) 4 (B) 2

(C) 0.5 (D) 0.25

Sol. 30

Option (C) is correct.

For flat plate with zero pressure gradient and $\text{Re} = 1000$ (laminar flow).

Boundary layer thickness

$$\delta(x) = \frac{4.91x}{\sqrt{\text{Re}_x}} = \frac{4.91x}{\sqrt{\frac{Vx}{\upsilon}}} = \frac{4.91x^{1/2}}{\sqrt{\frac{V}{\upsilon}}}$$

$$\Rightarrow \quad \delta \propto \frac{x^{1/2}}{V^{1/2}}$$ For a same location $(x = 1)$

$$\delta \propto (V)^{-1/2}$$

where V = velocity of fluid

$$\frac{\delta_1}{\delta_2} = \left(\frac{V_1}{V_2}\right)^{-1/2}$$

$$\delta_2 = \left(\frac{V_1}{V_2}\right)^{1/2} \times \delta_1 = \left(\frac{V_1}{4V_1}\right)^{1/2} \times 1\, V_2 = 4V_1 \text{ (Given)}$$

$$\delta_2 = \left(\frac{1}{4}\right)^{1/2} \times 1 = \frac{1}{2} = 0.5$$

Q. 31 GATE ME 2012 TWO MARK

A room contains 35 kg of dry air and 0.5 kg of water vapor. The total pressure and temperature of air in the room are 100 kPa and 25°C respectively. Given that the saturation pressure for water at 25°C is 3.17 kPa, the relative humidity of the air in the room is

(A) 67% (B) 55%

(C) 83% (D) 71%

Sol. 31

Option (D) is correct.

We have $m_a = 35\,\text{kg}$, $m_v = 0.5\,\text{kg}$, $p_t = 100\,\text{kPa}$ and $p_{vs} = 3.17\,\text{kPa}$.

Specific humidity $$W = \frac{m_v}{m_a} = \frac{0.5}{35} = 0.01428$$

Also, $$W = 0.612\frac{p_v}{p_a} = 0.612\frac{p_v}{p_t - p_v}$$

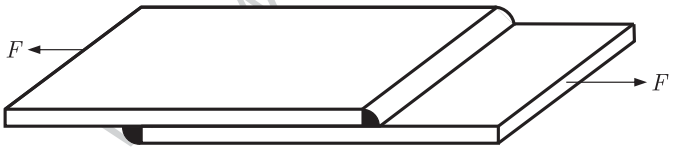
$$0.01428 = 0.612\frac{p_v}{100 - p_v}$$

$$p_v = 2.28\,\text{kPa}$$

Relative humidity $$\phi = \frac{p_v}{p_{vs}} = \frac{2.28}{3.17} \times 100 = 71.9\%$$

Q. 32 GATE ME 2012 TWO MARK

A fillet welded joint is subjected to transverse loading F as shown in the figure. Both legs of the fillets are of 10 mm size and the weld length is 30 mm. If the allowable shear stress of the weld is 94 MPa, considering the minimum throat area of the weld, the maximum allowable transverse load in kN is



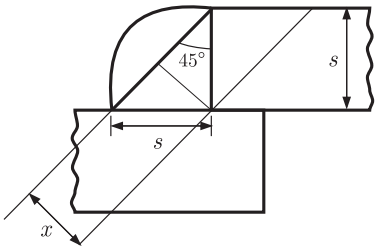
(A) 14.44 (B) 17.92

(C) 19.93 (D) 22.16

Sol. 32

Option (C) is correct.

Given : Width of fillets $s = 10\,\text{mm}$, $l = 30\,\text{mm}$, $\tau = 94\,\text{MPa}$



The shear strength of the joint for single parallel fillet weld is,

$$P = \text{Throat Area} \times \text{Allowable stress}$$

$$= t \times l \times \tau$$

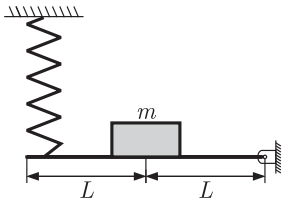
From figure $$t = s\sin 45^\circ = 0.707\,s$$

$$P = 0.707 \times s \times l \times \tau$$
$$= 0.707 \times (0.01) \times (0.03) \times (94 \times 10^6)$$
$$= 19937 \text{ N or } 19.93 \text{ kN}$$

Q. 33

GATE ME 2012 TWO MARK

A concentrated mass m is attached at the centre of a rod of length $2L$ as shown in the figure. The rod is kept in a horizontal equilibrium position by a spring of stiffness k. For very small amplitude of vibration, neglecting the weights of the rod and spring, the undamped natural frequency of the system is

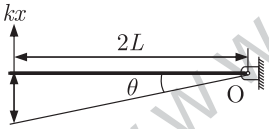


(A) $\sqrt{\frac{k}{m}}$ (B) $\sqrt{\frac{2k}{m}}$

(C) $\sqrt{\frac{k}{2m}}$ (D) $\sqrt{\frac{4k}{m}}$

Sol. 33

Option (D) is correct.
For a very small amplitude of vibration.



From above figure change in length of spring

$$x = 2L \sin\theta = 2L\theta \quad \text{(is very small so } \sin\theta \simeq \theta\text{)}$$

Mass moment of inertia of mass (m) about O is

$$I = mL^2$$

As no internal force acting on the system. So governing equation of motion from Newton's law of motion is,

$$I\ddot{\theta} + kx \times 2L = 0$$

or,
$$mL^2\ddot{\theta} + k2L\theta \times 2L = 0$$
$$\ddot{\theta} + \frac{4kL^2\theta}{mL^2} = 0$$

or
$$\ddot{\theta} + \frac{4k\theta}{m} = 0$$

By comparing general equation

$$\ddot{\theta} + \omega_n^2\theta = 0$$
$$\omega_n^2 = \frac{4k}{m}$$
$$\omega_n = \sqrt{\frac{4k}{m}}$$

Q. 34
GATE ME 2012
TWO MARK

The state of stress at a point under plane stress condition is

$$\sigma_{xx} = 40\,\text{MPa},\ \sigma_{yy} = 100\,\text{MPa} \text{ and } \tau_{xy} = 40\,\text{MPa}$$

The radius of the Mohr's circle representing the given state of stress in MPa is

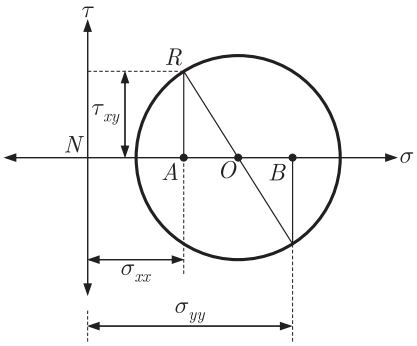
(A) 40 (B) 50

(C) 60 (D) 100

Sol. 34

Option (B) is correct.

Diagram for Moh's circle



Given, $\sigma_{xx} = 40\,\text{MPa} = AN$, $\sigma_{yy} = 100\,\text{MPa} = BN$, $\tau_{xy} = 40\,\text{MPa} = AR$

Radius of Mohr's circle

$$OR = \sqrt{(AR)^2 + (AO)^2}$$

$$AO = \frac{AB}{2} = \frac{BN - AN}{2} = \frac{100 - 40}{2} = 30$$

Therefore,

$$OR = \sqrt{(40)^2 + (30)^2} = 50\,\text{MPa}$$

Q. 35
GATE ME 2012
TWO MARK

The inverse Laplace transform of the function $F(s) = \dfrac{1}{s(s+1)}$ is given by

(A) $f(t) = \sin t$ (B) $f(t) = e^{-t}\sin t$

(C) $f(t) = e^{-t}$ (D) $f(t) = 1 - e^{-t}$

Sol. 35

Option (D) is correct.

First using the partial fraction to break the function.

$$F(s) = \frac{1}{s(s+1)} = \frac{A}{s} + \frac{B}{s+1}$$

$$= \frac{A(s+1) + Bs}{s(s+1)}$$

$$\frac{1}{s(s+1)} = \frac{(A+B)s}{s(s+1)} + \frac{A}{s(s+1)}$$

By comparing the coefficients both the sides,

$$(A + B) = 0 \text{ and } A = 1$$

$$B = -1$$

So

$$\frac{1}{s(s+1)} = \frac{1}{s} - \frac{1}{s+1}$$

$$F(t) = L^{-1}[F(s)]$$

$$= L^{-1}\left[\frac{1}{s(s+1)}\right] = L^{-1}\left[\frac{1}{s} - \frac{1}{s+1}\right]$$

$$= L^{-1}\left[\frac{1}{s}\right] - L^{-1}\left[\frac{1}{s+1}\right]$$

$$= 1 - e^{-t}$$

Q. 36

GATE ME 2012
TWO MARK

For the matrix $A = \begin{bmatrix} 5 & 3 \\ 1 & 3 \end{bmatrix}$, ONE of the normalized eigen vectors given as

(A) $\begin{pmatrix} \frac{1}{2} \\ \frac{\sqrt{3}}{2} \end{pmatrix}$ (B) $\begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} \end{pmatrix}$

(C) $\begin{pmatrix} \frac{3}{\sqrt{10}} \\ \frac{-1}{\sqrt{10}} \end{pmatrix}$ (D) $\begin{pmatrix} \frac{1}{\sqrt{5}} \\ \frac{2}{\sqrt{5}} \end{pmatrix}$

Sol. 36

Option (B) is correct.

Given $A = \begin{bmatrix} 5 & 3 \\ 1 & 3 \end{bmatrix}$

For finding eigen values, the characteristic equation is

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} 5-\lambda & 3 \\ 1 & 3-\lambda \end{vmatrix} = 0$$

$$\Rightarrow (5-\lambda)(3-\lambda) - 3 = 0$$

$$\lambda^2 - 8\lambda + 12 = 0$$

$$(\lambda - 6)(\lambda - 2) = 0$$

$$\lambda = 2, 6$$

Now from characteristic equation for eigen vector.

$$[A - \lambda I]\{x\} = [0]$$

For $\lambda = 2$

$$\begin{bmatrix} 5-2 & 3 \\ 1 & 3-2 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 3 & 3 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$X_1 + X_2 = 0 \qquad \Rightarrow \; X_1 = -X_2$$

So,

$$\text{eigen vector} = \begin{Bmatrix} 1 \\ -1 \end{Bmatrix}$$

$$\text{Magnitude of eigen vector} = \sqrt{(1)^2 + (1)^2} = \sqrt{2}$$

$$\text{Normalized eigen vector} = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} \end{bmatrix}$$

Q. 37
GATE ME 2012
TWO MARK

Calculate the punch size in mm, for a circular blanking operation for which details are given below.

Size of the blank	25 mm
Thickness of the sheet	2 mm
Radial clearance between punch and die	0.06 mm
Die allowance	0.05 mm

(A) 24.83 (B) 24.89

(C) 25.01 (D) 25.17

Sol. 37

Option (A) is correct.

Punch diameter, $d = D - 2c - a$

where D = Blank diameter = 25 mm

c = Clearance = 0.06 mm

a = Die allowance = 0.05 mm

Hence, $d = 25 - 2 \times 0.06 - 0.05 = 24.83$ mm

Q. 38
GATE ME 2012
TWO MARK

In a single pass rolling process using 410 mm diameter steel rollers, a strip of width 140 mm and thickness 8 mm undergoes 10% reduction of thickness. The angle of bite in radians is

(A) 0.006 (B) 0.031

(C) 0.062 (D) 0.600

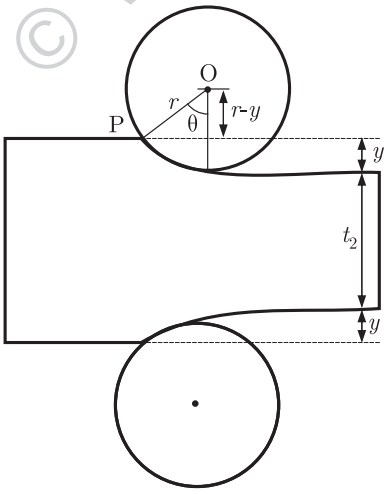
Sol. 38

Option (C) is correct.

Given : $t_1 = 8$ mm, $d = 410$ mm, $r = 205$ mm

Reduction of thickness, $\Delta t = 10\%$ of t_1

$= \frac{10}{100} \times 8 = 0.8$ mm



$y = \frac{\Delta t}{2} = 0.4$ mm

From ΔOPQ, $\cos\theta = \left(\frac{r - y}{r}\right)$

$$= \left[\frac{205 - 0.4}{205}\right] = 0.99804$$

$$\theta = \cos^{-1}(0.99804) = 3.58^\circ$$

Angle of bite in radians is

$$\theta = 3.58 \times \frac{\pi}{180} \text{ rad} = 0.062 \text{ rad.}$$

Alternate Method.

Angle of bite, $\theta = \tan^{-1}\left[\sqrt{\frac{t_i - t_f}{r}}\right]$

Where, t_i = Initial thickness = 8 mm

t_f

= Final reduced thickness $= 8 - 8 \times \frac{10}{100} = 7.2$ mm

r = radius of roller $= \frac{410}{2} = 205$ mm

$$\theta = \tan^{-1}\left[\sqrt{\frac{8 - 7.2}{205}}\right] = 3.5798^\circ$$

And in radians, $\theta = 3.5798 \times \frac{\pi}{180} = 0.0624$ rad.

Q. 39
GATE ME 2012
TWO MARK

In a DC are welding operation, the voltage-arc length characteristic was obtained as $V_{arc} = 20 + 5l$ where the arc length l was varied between 5 mm and 7 mm. Here V_{arc} denotes the arc voltage in Volts. The arc current was varied from 400 A to 500 A. Assuming linear power source characteristic, the open circuit voltage and short circuit current for the welding operation are

(A) 45 V, 450 A (B) 75 V, 750 A

(C) 95 V, 950 A (D) 150 V, 1500 A

Sol. 39

Option (C) is correct.

From power source characteristic,

$$\frac{V}{OCV} + \frac{I}{SCC} = 1 \qquad ...(i)$$

Where, V = Voltage

OCV = Open circuit voltage

SCC = Short circuit current

I = Current.

From voltage arc length characteristic

$$V_{arc} = 20 + 5l$$

For $l_1 = 5$ mm, $V_1 = 20 + 5 \times 5 = 45$ V

For $l_2 = 7$ mm, $V_2 = 20 + 5 \times 7 = 55$ V

and $I_1 = 500$ Amp. and $I_2 = 400$ Amp.

Substituting these value in Eq. (i)

$$\frac{V_1}{OCV} + \frac{I_1}{SCC} = 1$$

$$\frac{45}{OCV} + \frac{500}{SCC} = 1 \qquad ...(ii)$$

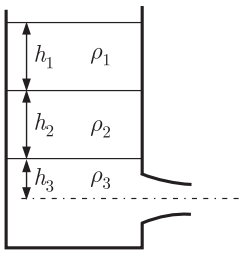
$$\frac{V_2}{OCV} + \frac{I_2}{SCC} = 1 \quad \Rightarrow \quad \frac{55}{OCV} + \frac{400}{SCC} = 1 \qquad ...(iii)$$

By solving Eq. (ii) and (iii), we get

$$OCV = 95\text{ V}$$
$$SCC = 950\text{ Amp.}$$

Q. 40
GATE ME 2012
TWO MARK

A large tank with a nozzle attached contains three immiscible, inviscide fluids as shown. Assuming that the change in h_1, h_2 and h_3 are negligible, the instantaneous discharge velocity is



(A) $\sqrt{2gh_3\left(1+\frac{\rho_1}{\rho_3}\frac{h_1}{h_3}+\frac{\rho_2}{\rho_3}\frac{h_2}{h_3}\right)}$ (B) $\sqrt{2g(h_1+h_2+h_3)}$

(C) $\sqrt{2g\left(\frac{\rho_1h_1+\rho_2h_2+\rho_3h_3}{\rho_1+\rho_2+\rho_3}\right)}$ (D) $\sqrt{2g\frac{\rho_1h_2h_3+\rho_2h_3h_1+\rho_3h_1h_2}{\rho_1h_1+\rho_2h_2+\rho_3h_3}}$

Sol. 40

Option (A) is correct.

Takes point (1) at top and point (2) at bottom

By Bernoulli equation between (1) and (2)

$$p_1+\rho_1gh_1+\rho_2gh_2+\rho_3gh_3+\frac{V_1^2(p_1+p_2+p_3)}{2g}=p_{atm.}+\frac{V_2^2}{2g}$$

At Reference level (2) $z_2=0$ and $V_1=0$ at point (1)

Therefore

$$\Rightarrow \quad p_1+\rho_1gh_1+\rho_1gh_2+\rho_3gh_3=p_{atm.}+\frac{V_2^2}{2g} \quad ...(1)$$

Since $p_1 = \text{atmospheric pressure}$ (because tank is open)

Hence $p_1 = p_{\text{atm.}}$

Therefore

$$V_2=\sqrt{2g\times[\rho_1gh_1+\rho_2gh_2+\rho_3gh_3]}$$

By Rearranging

$$V_2=\sqrt{2g\times\left[\frac{\rho_1gh_1}{\rho_3g}+\frac{\rho_2gh_2}{\rho_3g}+h_3\right]}$$

$$=\sqrt{2g\times\left[\frac{\rho_1h_1}{\rho_3}+\frac{\rho_2h_2}{\rho_3}+h_3\right]}$$

$$=\sqrt{2gh_3\times\left[1+\frac{\rho_1h_1}{\rho_3h_3}+\frac{\rho_2h_2}{\rho_3h_3}\right]}$$

Q. 41
GATE ME 2012
TWO MARK

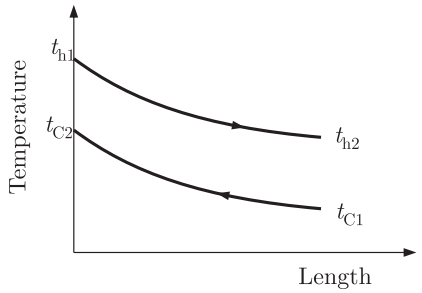
Water ($c_p = 4.18\text{ kJ/kgK}$) at 80°C enters a counter flow heat exchanger with a mass flow rate of 0.5 kg/s. Air ($c_p = 1\text{ kJ/kgK}$) enters at 30°C with a mass flow rate of 2.09 kg/s. If the effectiveness of the heat exchanger is 0.8, the LMTD (in $^\circ$C) is

(A) 40 (B) 20

(C) 10 (D) 5

Sol. 41 Option (C) is correct.

Given : $t_{h1} = 80^\circ C$, $t_{c1} = 30^\circ C$, $\dot{m}_h = 0.5\,kg/sec$, $\dot{m}_c = 2.09\,kg/sec.$, $\varepsilon = 0.8$



Capacity rate for hot fluid

$$C_h = 4.18 \times 0.5 = 2.09\,kJ/K sec.$$

$$C_c = 1 \times 2.09 = 2.09\,kJ/K\,sec.$$

So, $C_h = C_c$

$$\text{Effectiveness } \varepsilon = \frac{\dot{Q}}{\dot{Q}_{max}} = \frac{(t_{h1} - t_{h1})\,C_h}{(t_{h1} - t_{c1})\,C_c}$$

$$0.8 = \frac{80 - t_{h2}}{80 - 30}$$

or, $80 - t_{h2} = 40$

$$t_{h2} = 40^\circ C$$

From energy balance,

$$C_h(t_{h1} - t_{h1}) = C_c(t_{c2} - t_{c1})$$

$$80 - 40 = t_{c2} - 30$$

$$t_{c2} = 70^\circ C$$

Now LMTD

$$\theta_m = \frac{\theta_1 - \theta_2}{\ln\frac{\theta_1}{\theta_2}} \quad ...(i)$$

$$\theta_1 = t_{h1} - t_{c2} = 80 - 70 = 10^\circ C$$

$$\theta_2 = t_{h2} - t_{c1} = 40 - 30 = 10^\circ C$$

$$\theta_1 = \theta_2 \quad ...(ii)$$

So LMTD is undefined

Let $\frac{\theta_1}{\theta_2} = x \;\Rightarrow\; \theta_1 = x\theta_2$

Put in equation (i), so

$$\theta_m = \lim_{x \to 1} \frac{x\theta_2 - \theta_2}{\ln\frac{x\theta_2}{\theta_2}} = \lim_{x \to 1} \frac{\theta_2(x-1)}{\ln x}$$

It is a $\left[\frac{0}{0}\right]$ form, applying L-Hospital rule

$$\theta_m = \lim_{x \to 1} \frac{\theta_2(1-0)}{\frac{1}{x}} = \lim_{x \to 1} x\theta_2$$

$$\theta_m = \theta_2 = \theta_1 \quad \text{From equation (ii)}$$

$$\theta_m = \theta_1 = t_{h1} - t_{c2} = 80 - 70 = 10^\circ C$$

Q. 42
GATE ME 2012
TWO MARK

A solid steel cube constrained on all six faces is heated so that the temperature rises uniformly by ΔT. If the thermal coefficient of the material is α, Young's modulus is E and the Poisson's ratio is υ, the thermal stress developed in the cube due to heating is

(A) $-\dfrac{\alpha(\Delta T)E}{(1-2\upsilon)}$ (B) $-\dfrac{2\alpha(\Delta T)E}{(1-2\upsilon)}$

(C) $-\dfrac{3\alpha(\Delta T)E}{(1-2\upsilon)}$ (D) $-\dfrac{\alpha(\Delta T)E}{3(1-2\upsilon)}$

Sol. 42

Option (A) is correct.

For a solid cube strain in x, y and z axis are

$$\varepsilon_x = \frac{\sigma_x}{E} - \frac{\upsilon(\sigma_y + \sigma_z)}{E}$$

$$\varepsilon_y = \frac{\sigma_y}{E} - \frac{\upsilon(\sigma_x + \sigma_z)}{E}$$

$$\varepsilon_z = \frac{\sigma_z}{E} - \frac{\upsilon(\sigma_x + \sigma_y)}{E}$$

From symmetry of cube

$$\varepsilon_x = \varepsilon_y = \varepsilon_z = \varepsilon$$

and $$\sigma_x = \sigma_y = \sigma_z = \sigma$$

So $$\varepsilon = \frac{(1-2\upsilon)}{E} \times \sigma$$

Where $\varepsilon = -\alpha\Delta T$ (Thermal compression stress)

Therefore, $$\sigma = \frac{\varepsilon \times E}{(1-2\upsilon)} = -\frac{-\alpha\Delta TE}{(1-2\upsilon)} = -\frac{\alpha\Delta TE}{(1-2\upsilon)}$$

Q. 43
GATE ME 2012
TWO MARK

A solid circular shaft needs to be designed to transmit a torque of 50 Nm. If the allowable shear stress of the material is 140 MPa, assuming a factor of safety of 2, the minimum allowable design diameter is mm is

(A) 8 (B) 16

(C) 24 (D) 32

Sol. 43

Option (B) is correct.

$$F.O.S = \frac{\text{Allowable shear stress}}{\text{Design shear stress}}$$

Design shear stress for solid circular shaft

$$\tau = \frac{16T}{\pi d^3} = \frac{16 \times 50 \times 10^3}{\pi d^3} \qquad \text{From } \frac{T}{J} = \frac{\tau}{r}$$

Therefore

$$F.O.S = \frac{140 \times \pi d^3}{16 \times 50 \times 10^3}$$

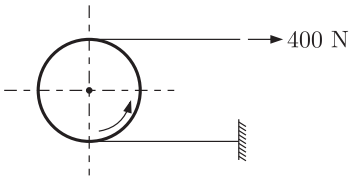
or, $$2 = \frac{140 \times \pi d^3}{16 \times 50 \times 10^3}$$

$$d^3 = \frac{2 \times 16 \times 50 \times 10^3}{140 \times \pi}$$

$$d = 15.38 \text{ mm} \cong 16 \text{ mm}$$

Q. 44
GATE ME 2012
TWO MARK

A force of 400 N is applied to the brake drum of 0.5 m diameter in a band-brake system as shown in the figure, where the wrapping angle is 180°. If the coefficient of friction between the drum and the band is 0.25, the braking torque applied, in Nm is



(A) 100.6 (B) 54.4
(C) 22.1 (D) 15.7

Sol. 44

Option (B) is correct.

Given : $T_1 = 400\,\text{N},\ \mu = 0.25,\ \theta = 180^\circ = 180^\circ \times \frac{\pi}{180^\circ} = \pi\,\text{rad}.$

$D = 0.5\,\text{m},\ r = \frac{D}{2} = 0.25\,\text{m}$

For the band brake, the limiting ratio of the tension is given by the relation,

$$\frac{T_1}{T_2} = e^{\mu\theta}$$

$$\frac{400}{T_2} = e^{0.25 \times \pi} = 2.19$$

$$T_2 = \frac{400}{2.19} = 182.68\,\text{N}$$

For Band-drum brake, Braking Torque is

$$T_B = (T_1 - T_2) \times r$$
$$= (400 - 182.68) \times 0.25 = 54.33\,\text{Nm} \cong 54.4\,\text{Nm}$$

Q. 45
GATE ME 2012
TWO MARK

A box contains 4 red balls and 6 black balls. Three balls are selected randomly from the box one after another, without replacement. The probability that the selected set contains one red ball and two black balls is

(A) 1/20 (B) 1/12
(C) 3/10 (D) 1/2

Sol. 45

Option (D) is correct.

Given : No. of Red balls = 4

No. of Black ball = 6

3 balls are selected randomly one after another, without replacement.
1 red and 2 black balls are will be selected as following

Manners	Probability for these sequence
$R\ B\ B$	$\frac{4}{10} \times \frac{6}{9} \times \frac{5}{8} = \frac{1}{6}$
$B\ R\ B$	$\frac{6}{10} \times \frac{4}{9} \times \frac{5}{8} = \frac{1}{6}$
$B\ B\ R$	$\frac{6}{10} \times \frac{5}{9} \times \frac{4}{8} = \frac{1}{6}$

Hence Total probability of selecting 1 red and 2 black ball is

$$P = \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{3}{6} = \frac{1}{2}$$

Q. 46
GATE ME 2012
TWO MARK

Consider the differential equation $x^2(d^2y/dx^2)+x(dy/dx)-4y=0$ with the boundary conditions of $y(0)=0$ and $y(1)=1$. The complete solution of the differential equation is

(A) x^2 (B) $\sin\left(\frac{\pi x}{2}\right)$

(C) $e^x\sin\left(\frac{\pi x}{2}\right)$ (D) $e^{-x}\sin\left(\frac{\pi x}{2}\right)$

Sol. 46

Option (A) is correct.

We have

$$x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}-4y=0 \qquad ...(1)$$

Let $x=e^z$ then

$$z=\log x$$
$$\frac{dz}{dx}=\frac{1}{x}$$

So, we get

$$\frac{dy}{dx}=\left(\frac{dy}{dz}\right)\left(\frac{dz}{dx}\right)=\frac{1}{x}\frac{dy}{dz}$$
$$x\frac{dy}{dx}=Dy \qquad \text{where } \frac{d}{dz}=D$$

Again

$$\frac{d^2y}{dx^2}=\frac{d}{dx}\left(\frac{dy}{dx}\right)=\frac{d}{dx}\left(\frac{1}{x}\frac{dy}{dz}\right)$$
$$=\frac{-1}{x^2}\frac{dy}{dz}+\frac{1}{x}\frac{d}{dz}\left(\frac{dy}{dz}\right)\frac{dz}{dx}$$
$$=\frac{-1}{x^2}\frac{dy}{dz}+\frac{1}{x}\frac{d^2y}{dz^2}\frac{dz}{dx}$$
$$=\frac{1}{x^2}\left(\frac{d^2y}{dz^2}-\frac{dy}{dz}\right)$$
$$\frac{x^2d^2y}{dx^2}=(D^2-D)y=D(D-1)y$$

Now substitute in equation (i)

$$[D(D-1)+D-4]y=0$$
$$(D^2-4)y=0$$
$$D=\pm 2$$

So the required solution is

$$y=C_1x^2+C_2x^{-2} \qquad ...(ii)$$

From the given limits

$y(0)=0$, equation (ii) gives

$$0=C_1\times 0+C_2$$
$$C_2=0$$

And from $y(1)=1$, equation (ii) gives

$$1=C_1+C_2$$
$$C_1=1$$

Substitute C_1 & C_2 in equation (ii), the required solution be

$$y=x^2$$

Q. 47

GATE ME 2012
TWO MARK

$$\begin{aligned} x+2y+z &= 4 \\ 2x+y+2z &= 5 \\ x-y+z &= 1 \end{aligned}$$

The system of algebraic equations given above has

(A) a unique solution of $x=1, y=1$ and $z=1$.

(B) only the two solutions of $(x=1, y=1, z=1)$ and $(x=2, y=1, z=0)$

(C) infinite number of solutions

(D) no feasible solution

Sol. 47

Option (C) is correct.

For given equation matrix form is as follows

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 2 \\ 1 & -1 & 1 \end{bmatrix},\ B = \begin{bmatrix} 4 \\ 5 \\ 1 \end{bmatrix}$$

The augmented matrix is

$$\left[A : B\right] = \begin{bmatrix} 1 & 2 & 1 & : & 4 \\ 2 & 1 & 2 & : & 5 \\ 1 & -1 & 1 & : & 1 \end{bmatrix}$$

Applying row operations $R_2 \to R_2 - 2R_1,\ R_3 \to R_3 - R_1$

$$= \begin{bmatrix} 1 & 2 & 1 & : & 4 \\ 0 & -3 & 0 & : & -3 \\ 0 & -3 & 0 & : & -3 \end{bmatrix}$$

$R_3 \to R_3 - R_2$

$$= \begin{bmatrix} 1 & 2 & 1 & : & 4 \\ 0 & -3 & 0 & : & -3 \\ 0 & 0 & 0 & : & 0 \end{bmatrix}$$

$R_2 \to R_2 / -3$

$$= \begin{bmatrix} 1 & 2 & 1 & : & 4 \\ 0 & 1 & 0 & : & 1 \\ 0 & 0 & 0 & : & 0 \end{bmatrix}$$

This gives rank of A

$$\rho(A) = 2 \text{ and Rank of } \left[A : B\right] = \rho\left[A : B\right] = 2$$

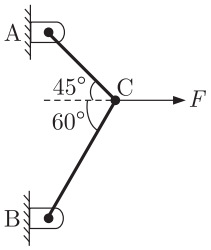
Which is less than the number of unknowns (3)

$$\rho[A] = \rho\left[A : B\right] = 2 < 3$$

Hence, this gives infinite No. of solutions.

Common Data for Questions 48 and 49.

Two steel truss members, AC and BC, each having cross sectional area of $100\ \text{mm}^2$, are subjected to a horizontal force F as shown in figure. All the joints are hinged.



Q. 48
GATE ME 2012
TWO MARK

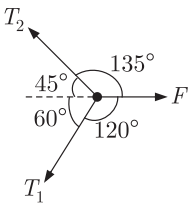
If $F = 1\,\text{kN}$, the magnitude of the vertical reaction force developed at the point B in kN is

(A) 0.63 (B) 0.32

(C) 1.26 (D) 1.46

Sol. 48

Option (A) is correct.



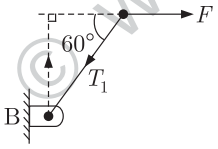
From above figure. Three forces are acting on a common point. Hence by Lami's Theorem.

$$\frac{F}{\sin(105^\circ)} = \frac{T_2}{\sin 120^\circ} = \frac{T_1}{\sin 135^\circ}$$

$$\Rightarrow \quad \frac{T_1}{\sin 135^\circ} = \frac{F}{\sin 105^\circ} = \frac{1}{\sin 105^\circ}$$

$$T_1 = 0.7320\,\text{kN}$$

Hence vertical reaction at B



$$R_{NT_1} = T_1 \cos 30^\circ$$
$$R_{NT_1} = 0.73205 \times \cos 30^\circ$$
$$R_{NT_1} = 0.634\,\text{kN}$$

Q. 49
GATE ME 2012
TWO MARK

The maximum force F is kN that can be applied at C such that the axial stress in any of the truss members DOES NOT exceed $100\,\text{MPa}$ is

(A) 8.17 (B) 11.15

(C) 14.14 (D) 22.30

Sol. 49

Option (B) is correct.

From Previous question

$$\frac{F}{\sin 105^\circ} = \frac{T_2}{\sin 120^\circ}$$

$$T_2 = \frac{\sin 120^\circ}{\sin 135} \times F = 0.8965F$$

and
$$T_1 = (0.73205)F$$
$$T_2 > T_1$$
$$\sigma = 100\,\text{MPa (given)}$$

As we know
$$F = \sigma \times A_1$$
$$\Rightarrow \quad F_{max} = \sigma_{max} \times A_1$$
$$T_2 = 100 \times 100$$
$$0.8965F = 100 \times 100$$
$$F = \frac{100 \times 100}{0.8965} = 11154.5\,\text{N}$$
$$F = 11.15\,\text{kN}$$

Common Data for Questions 50 and 51 :

A refrigerator operates between 120 kPa and 800 kPa in an ideal vapour compression cycle with R-134a as the refrigerant. The refrigerant enters the compressor as saturated vapour and leaves the condenser as saturated liquid. The mass flow rate of the refrigerant is 0.2 kg/s. Properties for R134a are as follows :

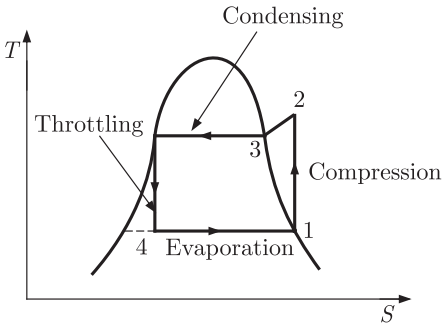
Saturated R-134a					
p(kPa)	T(°C)	h_f(kJ/kg)	h_g(kJ/kg)	s_f(kJ/kgK)	s_g(kJ/kgK)
120	−22.32	22.5	237	0.093	0.95
800	31.31	95.5	267.3	0.354	0.918
Superheated R-134a					
p(kPa)		T(°C)	h(kJ/kg)		s(kJ/kgK)
800		40	276.45		0.95

Q. 50 GATE ME 2012 TWO MARK

The rate at which heat is extracted, in kJ/s from the refrigerated space is
(A) 28.3 (B) 42.9
(C) 34.4 (D) 14.6

Sol. 50

Option (A) is correct.



T-s diagram for given Refrigeration cycle is given above
Since Heat is extracted in evaporation process.
So rate of heat extracted

$$= \dot{m}(h_1 - h_4)$$

From above diagram $(h_3 = h_4)$ for throttling process, so

$$\text{Heat extracted} = \dot{m}(h_1 - h_3)$$

From given table

$h_1 = h_g$ at 120 kPa, $h_g = 237\ \text{kJ/kg}$

$h_3 = h_f$ at 120 kPa, $h_f = 95.5\ \text{kJ/kg}$

Hence

$$\text{Heat extracted} = \dot{m}(h_g - h_f)$$
$$= 0.2 \times (237 - 95.5)$$
$$= 28.3\ \text{kJ/s}$$

Q. 51 GATE ME 2012 TWO MARK

The power required for the compressor in kW is

(A) 5.94 (B) 1.83

(C) 7.9 (D) 39.5

Sol. 51

Option (C) is correct.

Since power is required for compressor in refrigeration is in compression cycle (1-2)

Hence

$$\text{Power required} = \dot{m}(h_2 - h_1)$$
$$= \dot{m}(h_2 - h_f)$$

Since for isentropic compression process.

$$s_1 = s_2 \text{ from figure.} = 0.95$$

For entropy $s = 0.95$ the enthalpy $h = 276.45\ \text{kJ/kg}$

$$h = h_2 = 276.45 \text{ (From table)}$$

Hence

$$\text{Power} = 0.2(276.45 - 237)$$
$$= 7.89 \simeq 7.9\ \text{kW}$$

Statement for Linked Answer Question 52 and 53 :

Air enters an adiabatic nozzle at 300 kPa, 500 K with a velocity of 10 m/s. It leaves the nozzle at 100 kPa with a velocity of 180 m/s. The inlet area is 80 cm^2. The specific heat of air c_p is 1008 J/kgK.

Q. 52 GATE ME 2012 TWO MARK

The exit temperature of the air is

(A) 516 K (B) 532 K

(C) 484 K (D) 468 K

Sol. 52

Option (C) is correct.

From energy balance for steady flow system.

$$E_{in} = E_{out}$$

$$\dot{m}\left(h_1 + \frac{V_1^2}{2}\right) = \dot{m}\left(h_2 + \frac{V_2^2}{2}\right) \quad \text{...(i)}$$

As

$$h = c_p T$$

Equation (i) becomes

$$c_p T_1 + \frac{V_1^2}{2} = c_p T_2 + \frac{V_2^2}{2}$$

$$T_2 = \left(\frac{V_1^2 - V_2^2}{2 \times c_p}\right) + T_1$$

$$= \frac{10^2 - 180^2}{2 \times 1008} + 500 = -16.02 + 500$$

$$T_2 = 483.98 \simeq 484 \text{ K}$$

Q. 53 GATE ME 2012 TWO MARK

The exit area of the nozzle in cm^2 is

(A) 90.1 (B) 56.3

(C) 4.4 (D) 12.9

Sol. 53

Option (D) is correct.

From Mass conservation.

$$\dot{m}_{in} = \dot{m}_{out}$$

$$\frac{V_1 A_1}{\nu_1} = \frac{V_2 A_2}{\nu_2} \quad \text{...(i)}$$

where ν = specific volume of air $= \frac{RT}{p}$

Therefore Eq. (1) becomes

$$\frac{p_1 V_1 A_1}{R T_1} = \frac{p_2 V_2 A_2}{R T_2}$$

$$A_2 = \frac{p_1 \times V_1 \times A_1 \times T_2}{p_2 \times V_2 \times T_1}$$

$$= \frac{300 \times 10 \times 80 \times 484}{100 \times 180 \times 500}$$

$$= 12.9 \text{ cm}^2$$

Statement for Linked Answer Questions 54 and 55 :

For a particular project, eight activities are to be carried out. Their relationships with other activities and expected durations are mentioned in the table below.

Activity	Predecessors	Durations (days)
a	-	3
b	*a*	4
c	*a*	5
d	*a*	4
e	*b*	2
f	*d*	9
g	*c*, *e*	6
h	*f*, *g*	2

Q. 54 GATE ME 2012 TWO MARK

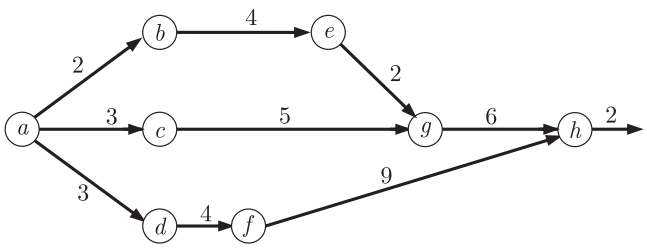
The critical path for the project is

(A) *a* - *b* - *e* - *g* - *h* (B) *a* - *c* - *g* - *h*

(C) *a* - *d* - *f* - *h* (D) *a* - *b* - *c* - *f* - *h*

Sol. 54

Option (C) is correct.



For path	Duration
a - b - e - g - h	$= 3 + 4 + 2 + 6 + 2 = 17$ days
a - c - g - h	$= 3 + 5 + 6 + 2 = 16$ days
a - d - f - h	$= 3 + 4 + 9 + 2 = 18$ days

The critical path is one that takes longest path.
Hence, path a - d - f - $h = 18$ days is critical path

Q. 55
GATE ME 2012
TWO MARK

If the duration of activity f alone is changed from 9 to 10 days, then the

(A) critical path remains the same and the total duration to complete the project changes to 19 days.

(B) critical path and the total duration to complete the project remains the same.

(C) critical path changes but the total duration to complete the project remains the same.

(D) critical path changes and the total duration to complete the project changes to 17 days.

Sol. 55

Option (A) is correct.
From previous question
For critical path
a-d-f-$h = 18$ days, the duration of activity f alone is changed from 9 to 10 days, then

$$a - d - f - h = 3 + 4 + 10 + 2 = 19 \text{ days}$$

Hence critical path remains same and the total duration to complete the project changes to 19 days.

Q. 56
GATE ME 2012
ONE MARK

Choose the most appropriate alternative from the options given below to complete the following sentence :
Suresh's dog is the one .....was hurt in the stampede.

(A) that (B) which

(C) who (D) whom

Sol. 56

Option (A) is correct.
"Which" is used in a sentence when the person is unknown. But here the person means Suresh's dog is known and "that" is used in a sentence, when the person is known.
So, that will be used in this sentence.

Q. 57
GATE ME 2012
ONE MARK

The cost function for a product in a firm is given by $5q^2$, where q is the amount of production. The firm can sell the product at a market price of Rs. 50 per unit. The number of units to be produced by the firm such that the profit maximized is

(A) 5 (B) 10

(C) 15 (D) 25

Sol. 57

Option (A) is correct.

Profit is given by,

$$P = \text{Selling price} - \text{Total cost of production}$$
$$= 50q - 5q^2$$

Using the principle of maxima – minima,

$$\frac{dP}{dq} = 50 - 10q = 0$$

$$q = \frac{50}{10} = 5$$

and $$\frac{d^2P}{dq^2} = -10 \text{ (maxima)}$$

So, for 5 units the profit is maximum.

Q. 58
GATE ME 2012
ONE MARK

Choose the most appropriate alternative from the options given below to complete the following sentence.

Despite several .....the mission succeeded in its attempt to resolve the conflict.

(A) attempts (B) setbacks

(C) meetings (D) delegations

Sol. 58

Option (B) is correct.

Despite several setbacks the mission succeeded in its attempt to resolve the conflict.

Q. 59
GATE ME 2012
ONE MARK

Which one of the following options is the closest in meaning to the word given below ?

Mitigate

(A) Diminish (B) Divulge

(C) Dedicate (D) Denote

Sol. 59

Option (A) is correct.

From the following options Diminish is the closest meaning to the Mitigate.

Q. 60
GATE ME 2012
ONE MARK

Choose the grammatically INCORRECT sentence :

(A) They gave us the money back less the service charges of Three Hundred Rupees.

(B) This country's expenditure is not less than that of Bangladesh.

(C) The committee initially asked for a funding of Fifty Lakh rupees, but later settled for a lesser sum.

(D) This country's expenditure on educational reforms is very less.

Sol. 60

Option (A) is correct.

The grammatically incorrect sentence is :

(A) They gave us the money back less the service charges of three hundred rupees.

Q. 61
GATE ME 2012
TWO MARK

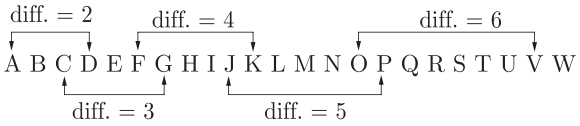
Given the sequence of terms, $AD\ CG\ FK\ JP$, the next term is

(A) OV (B) OW

(C) PV (D) PW

Sol. 61

Option (A) is correct.



So, the next term is OV.

Q. 62
GATE ME 2012
TWO MARK

Wanted Temporary, Part-time persons for the post of Field Interviewer to conduct personal interviews to collect and collate economic data. Requirements : High School-pass, must be available for Day, Evening and Saturday work. Transportation paid, expenses reimbursed.

Which one of the following is the best inference from the above advertisement ?

(A) Gender-discriminatory (B) Xenophobic

(C) Not designed to make the post attractive

(D)Not gender-discriminatory

Sol. 62

Option (D) is correct.

Not gender-discriminatory

Discriminatory involves the actual behaviors towards groups such as excluding or restricting members of one group from opportunities that are available to another group.

This given advertisement is not exclude or restrict Male or Female members from one another. Hence this is Not-gender discriminatory.

Q. 63
GATE ME 2012
TWO MARK

A political party order an arch for the entrance to the ground in which the annual convention is being held. The profile of the arch follows the equations $y = 2x - 0.1x^2$ where y is the height of the arch in meters. The maximum possible height of the arch is

(A) 8 meters (B) 10 meters

(C) 12 meters (D) 14 meters

Sol. 63

Option (B) is correct.

We have $y = 2x - 0.1x^2$ ...(i)

Using the principle of $\text{maxima} - \text{minima}$,

$$\frac{dy}{dx} = 2 - 0.2x = 0$$

$$x = \frac{2}{0.2} = 10$$

And $$\frac{d^2y}{dx^2} = -0.2 \text{ (maxima)}$$

So, for maximum possible height, substitute $x = 10$ in equation (i),

$$y = 2 \times 10 - 0.1 \times (10)^2$$
$$= 20 - 10 = 10 \text{ meter}$$

Q. 64
GATE ME 2012
TWO MARK

An automobile plant contracted to buy shock absorbers from two suppliers X and Y. X supplies 60% and Y supplies 40% of the shock absorbers. All shock absorbers are subjected to a quality test. The ones that pass the quality test are considered reliable. Of X's shock absorbers, 96% are reliable. Of Y's shock absorbers, 72% are reliable.

The probability that a randomly chosen shock absorber, which is found to be reliable, is made by Y is

(A) 0.288 (B) 0.334

(C) 0.667 (D) 0.720

Sol. 64

Option (B) is correct.

Supplier X supplies 60% of shock absorbers, out of which 96% are reliable. So overall reliable fraction of shock absorbers from supplier X,

$$= 0.6 \times 0.96$$

$$= 0.576$$

And for supplier Y, suppliers 40% of shock absorbers, out of which 72% are reliable. So fraction of reliability $= 0.4 \times 0.72 = 0.288$.

$$\text{Total fraction of reliability} = 0.576 + 0.288 = 0.864$$

Hence the probability that is found to be reliable, is made by Y is,

$$= \frac{0.288}{0.288 + 0.576} = 0.334$$

Q. 65
GATE ME 2012
TWO MARK

Which of the following assertions are CORRECT ?

P : Adding 7 to each entry in a list adds 7 to the mean of the list

Q : Adding 7 to each entry in a list adds 7 to the standard deviation of the list

R : Doubling each entry in a list doubles the mean of the list

S : Doubling each entry in a list leaves the standard deviation of the list unchanged

(A) P, Q (B) Q, R

(C) P, R (D) R, S

Sol. 65

Option (C) is correct.

For statement P, take three variables a, b, c

$$\text{Mean } (m) = \frac{a+b+c}{3}$$

Adding 7 to each entry,

$$m_1 = \frac{(a+7)+(b+7)(c+7)}{3}$$

$$m_1 = \frac{a+b+c}{3} + \frac{21}{3} = m + 7$$

So, it is correct.

(Q) Standard deviation

$$\sigma = \sqrt{\frac{(a-m)^2 + (b-m)^2 + (c-m)^2}{3}}$$

Adding 7 to each entry,

$$\sigma_1 = \sqrt{(a-m+7)^2 + (b-m+7)^2 + (c-m+7)^2} \neq (\sigma + 7)$$

It is wrong.

(R) By doubling each entry.

$$m_1 = \frac{2a + 2b + 2c}{3} = 2m \text{ (it is correct)}$$

(S) doubling each entry

$$\sigma_1 = \sqrt{\frac{(m-2a)^2 + (m-2b)^2 + (m-2c)^2}{3}} \neq (2\sigma)$$

Hence it is wrong.

Answer Sheet									
1.	(D)	14.	(B)	27.	(C)	40.	(A)	53.	(D)
2.	(A)	15.	(B)	28.	(A)	41.	(C)	54.	(C)
3.	(C)	16.	(D)	29.	(D)	42.	(A)	55.	(A)
4.	(C)	17.	(C)	30.	(C)	43.	(B)	56.	(A)
5.	(A)	18.	(A)	31.	(D)	44.	(B)	57.	(A)
6.	(C)	19.	(B)	32.	(C)	45.	(D)	58.	(B)
7.	(B)	20.	(B)	33.	(D)	46.	(A)	59.	(A)
8.	(B)	21.	(B)	34.	(B)	47.	(C)	60.	(A)
9.	(D)	22.	(A)	35.	(D)	48.	(A)	61.	(A)
10.	(D)	23.	(C)	36.	(B)	49.	(B)	62.	(D)
11.	(A)	24.	(D)	37.	(A)	50.	(A)	63.	(B)
12.	(C)	25.	(A)	38.	(C)	51.	(C)	64.	(B)
13.	(A)	26.	(B)	39.	(C)	52.	(C)	65.	(C)