Code : A1014 R18

II B.Tech I Semester (R18) Regular Examinations November 2019

LINEAR ALGEBRA AND COMPLEX VARIABLES

Time : 3 hours Max. Marks: 70

PART- A

(Compulsory Question, 10 x 2 = 20M)

1 a Define orthogonal matrix with one example

b State Cayley Hamilton Theorem.

c Show that $B(m,n)=B(m+1,n)+B(m,n+1)$.

d Obtain the symmetric matrix for the quadratic form $Q=x_1^2+2x_1x_2-4x_1x_3+6x_2x_3-5x_2^2+4x_3^2$

e Find the value of constants a,b,c,d such that the function $f(z)=x^2+axy+by^2+i(cx^2+dxy+y^2)$ is analytic.

f Find the fixed points of the Bilinear transformation $w=\frac{1-3iz}{z-i}$

g Evaluate $\int_C (x^2-2ixy)\,dz$ along the path $x=t, y=2t^2$ from the Point A(1,2) to B(2,8)

h Compute the residue at the singular points $z=-2$ of the function $f(z)=\frac{1+z+z^2}{(z-1)^2(z+2)}$

i State Cauchy Residue Theorem.

j Using Cauchy's residue theorem evaluate $\int_C \frac{z-3}{z^2+2z+5}dz$ given $C:|z|=1$

PART- B

(Each question carries 10 marks)

UNIT-1

2 a Reduce the matrix $\begin{bmatrix} 1 & 2 & -1 & 4 \\ 2 & 4 & 3 & 4 \\ 1 & 2 & 3 & 4 \\ -1 & -2 & 6 & -7 \end{bmatrix}$ to normal form and hence find its rank.

b Use Cayley-Hamilton theorem to find A^{-1} and A^3 given $A=\begin{bmatrix} 2 & 4 \\ 1 & 1 \end{bmatrix}$

(OR)

3 a Find the Eigen values and Eigen vectors of the Hermitian matrix $\begin{bmatrix} 2 & 3+4i \\ 3-4i & 2 \end{bmatrix}$

b Investigate for what values of λ, μ the simultaneous equations x+y+z=6, x+2y+3z=10, x+2y+λ z= μ have (i) No solution (ii)A unique solution (iii) An infinite number of solutions

UNIT-II

4 a Evaluate $\int_0^\infty \sqrt{x}e^{-x^2}dx$

b Express $\int_0^1 x^m(1-x^p)^n\,dx$ in terms of Beta function.

(OR)

5 Reduce the quadratic form $7x^2-6y^2+5z^2-4xy-4yz$ to canonical form and hence find nature, index and signature.

UNIT-III

6 a Find the analytic function whose real part is $y+e^x\cos y$

b Find the Bilinear Transformation which maps the points $z=0,1,\infty$ onto the points $w=-i,\infty,1$.

(OR)

7 Prove that $\left[\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right]|\text{Real } f(z)|^2=2|f'(z)|^2$, where $f(z)$ is analytic.

UNIT-IV

8 a Evaluate $\int_C \frac{z^3 - \sin 3z}{\left(z-\frac{\pi}{2}\right)^3} dz$ with $C: |z| = 2$ using Cauchy's integral formula.

b Expand $f(z) = \frac{z-1}{z+1}$ in Taylor's series about the point $z = 0$.

(OR)

9 a Using Cauchy's integral formula, Evaluate $\oint_C \frac{3z+5}{z^2+2z} dz$, given $C: |z| = 1$

b Find Laurent series expansion of $\frac{e^{2z}}{(z-1)^3}$ about $z = 1$

UNIT-V

10 a Evaluate the integral $\int_{-\infty}^{\infty} \frac{dz}{z^2+1}$

b Find the residue at each pole for the function $\frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)^2 (z-2)}$

(OR)

11 By integrating around a unit circle, evaluate $\int_0^{2\pi} \frac{\cos 3\theta}{5 - 4\cos\theta} d\theta$