UNIT- II

TERMINOLOGY

Stiffness matrix	In the finite element method for the numerical solution of elliptic partial differential equations, the stiffness matrix represents the system of linear equations that must be solved in order to ascertain an approximate solution to the differential equation.
Shape function	The shape function is the function which interpolates the solution between the discrete values obtained at the mesh nodes. Therefore, appropriate functions have to be used and, as already mentioned, low order polynomials are typically chosen as shape functions.
Displacement function	Displacement function is the beginning point for the structural analysis by finite element method. On the basis of the problem to be solved, the displacement function needs to be approximated in the form of either linear or higher-order function. A convenient way to express it is by the use of polynomial expressions.
Natural coordinate system.	A relatively simple method is available for deriving shape functions for higher order triangular elements. And that method requires what is called spatial coordinate system, natural coordinate or area coordinate system.
Principle of Virtual Work	The principle of virtual work is a very useful approach for solving varieties of structural mechanics problem. When the force and displacement are unrelated to the cause and effect relation, the work is called virtual work.
Choice of Displacement Function	Displacement function is the beginning point for the structural analysis by finite element method. This function represents the variation of the displacement within the element. On the basis of the problem to be solved, the displacement function needs to be approximated in the form of either linear or higher-order function. A convenient way to express it is by the use of polynomial expressions.
Compatibility	Displacement should be compatible between adjacent elements. There should not be any discontinuity or overlapping while deformed. The adjacent elements must deform without causing openings, overlaps or discontinuous between the elements. Elements which satisfy all the three convergence requirements and compatibility condition are called Compatible or Conforming elements.

Geometric invariance	Displacement shape should not change with a change in local coordinate system. This can be achieved if polynomial is balanced in case all terms cannot be completed. This 'balanced' representation can be achieved with the help of Pascal triangle in case of two-dimensional polynomial.
Iso-parametric Elements	If the shape functions (Ni) used to represent the variation of geometry of the element are the same as the shape functions (N´i) used to represent the variation of the displacement then the elements are called isoparametric elements.
Global Stiffness Matrix	A structural system is an assemblage of number of elements. These elements are interconnected together to form the whole structure. Therefore, the element stiffness of all the elements are first need to be calculated and then assembled together in systematic manner. It may be noted that the stiffness at a joint is obtained by adding the stiffness of all elements meeting at that joint.

Concepts

Choice of Displacement Function

Displacement function is the beginning point for the structural analysis by finite element method. This function represents the variation of the displacement within the element. On the basis of the problem to be solved, the displacement function needs to be approximated in the form of either linear or higher-order function. A convenient way to express it is by the use of polynomial expressions.

Convergence criteria

The convergence of the finite element solution can be achieved if the following three conditions are fulfilled by the assumed displacement function.

a. The displacement function must be continuous within the elements. This can be ensured by choosing a suitable polynomial.

b. The displacement function must be capable of rigid body displacements of the element. The constant terms used in the polynomial ($\alpha 0$ to αn) ensure this condition.

c. The displacement function must include the constant strains states of the element. As element becomes infinitely small, strain should be constant in the element. Hence, the displacement function should include terms for representing constant strain states.

Compatibility

Displacement should be compatible between adjacent elements. There should not be any discontinuity or overlapping while deformed. The adjacent elements must deform without causing openings, overlaps or discontinuous between the elements. Elements which satisfy all the three convergence requirements and compatibility condition are called Compatible or Conforming elements.

Geometric invariance

Displacement shape should not change with a change in local coordinate system. This can be achieved if polynomial is balanced in case all terms cannot be completed. This 'balanced' representation can be achieved with the help of Pascal triangle in case of two-dimensional polynomial. For example, for a polynomial having four terms, the invariance can be obtained if the following expression is selected from the Pascal triangle.

$u = \alpha 0 + \alpha 1x + \alpha 2\ y + \alpha 3xy$

The geometric invariance can be ensured by the selection of the corresponding order of terms on either side of the axis of symmetry.

$$
\begin{array}{ccccccccc}
 & & & & 1 & & & & \\
 & & & X & & y & & & \\
 & & x^2 & & xy & & y^2 & & \\
 & x^3 & & x^2y & & xy^2 & & y^3 & \\
x^4 & & x^3y & & x^2y^2 & & xy^3 & & y^4
\end{array}
$$

Fig. Pascal's Triangle

Shape Function

In finite element analysis, the variations of displacement within an element are expressed by its nodal displacement (i i u =ΣN u) with the help of interpolation function since the true variation of displacement inside the element is not known. Here, u is the displacement at any point inside the element and ui are the nodal displacements. This interpolating function is generally a polynomial with n degree which automatically provides a single-valued and continuous field. In finite element literature, this interpolation function (Ni) is referred to "Shape function" as well. For linear interpolation will be 1 and for quadratic interpolation n will become 2 and so on. There are two types of interpolation functions namely (i) Lagrange interpolation and (ii) Hermitian interpolation. Lagrange interpolation function is widely used in practice. Here the assumed function takes on the same values as the given function at specified points. In case of Hermitian interpolation function, the slopes of the function also take the same values as the given function at specified points. The derivation of shape function for varieties of elements will be discussed in subsequent lectures.

Degree of Continuity

Let consider ϕ as an interpolation function in a piecewise fashion over finite element mesh. While such interpolation function ϕ can be ensured to vary smoothly within the element, the transition between adjacent elements may not be smooth. The term Cm is considered to define the continuity of a piecewise displacement. A function Cm is continuous if its derivative up to and including degree m are inter-element continuous. For example, for one dimensional problem, ϕ=ϕ(x) is C0 continuous if ϕ is continuous, but ϕ,x is not. Similarly, ϕ=ϕ(x) is C1 continuous if ϕ and ϕ,x are continuous, but ϕ,xx is not. In general, C0 element is used to model plane and solid body and C1 element is used to model.

Isoparametric Elements

If the shape functions (Ni) used to represent the variation of geometry of the element are the same as the shape functions (N´i) used to represent the variation of the displacement then the elements are called isoparametric elements.

Element Stiffness Matrix

The stiffness matrix of a structural system can be derived by various methods like Variational principle, Galerkin method etc. The derivation of an element stiffness matrix has already been discussed in earlier lecture. The stiffness matrix is an inherent property of the structure. Element stiffness is obtained with respect to its axes and then transformed this stiffness to structure axes. The properties of stiffness matrix are as follows:

1. Stiffness matrix is symmetric and square.
2. In stiffness matrix, all diagonal elements are positive.
3. Stiffness matrix is positive definite

Global Stiffness Matrix

A structural system is an assemblage of number of elements. These elements are interconnected together to form the whole structure. Therefore, the element stiffness of all the elements are first need to be calculated and then assembled together in systematic manner. It may be noted that the stiffness at a joint is obtained by adding the stiffness of all elements meeting at that joint. To start with, the degrees of freedom of the structure are numbered first. This numbering will start from 1 to n where n is the total degrees of freedom. These numberings are referred to as degrees of freedom corresponding to global degrees of freedom. The element stiffness matrix of each element should be placed in their proper position in the overall stiffness matrix. The following steps may be performed to calculate the global stiffness matrix of the whole structure.

a. Initialize global stiffness matrix [K] as zero. The size of global stiffness matrix will be equal to the total degrees of freedom of the structure.

b. Compute individual element properties and calculate local stiffness matrix [k] of that element.

c. Add local stiffness matrix [k] to global stiffness matrix [K] using proper locations

d. Repeat the Step b. and c. till all local stiffness matrices are placed globally.

Boundary Conditions

Under this section, procedure to include the effect of boundary condition in the stiffness matrix for the finite element analysis will be discussed. The solution cannot be obtained unless support conditions are included in the stiffness matrix. This is because, if all the nodes of the structure are included in displacement vector, the stiffness matrix becomes singular and cannot be solved if the structure is not supported amply, and it cannot resist the applied loads. A solution cannot be achieved until the boundary conditions i.e., the known displacements are introduced.

In finite element analysis, the partitioning of the global matrix is carried out in a systematic way for the hand calculation as well as for the development of computer codes. In partitioning, normally the equilibrium equations can be partitioned by rearranging corresponding rows and columns, so that prescribed displacements are grouped together.

Natural coordinate system

Natural coordinate system is basically a local coordinate system which allows the specification of a point within the element by a set of dimensionless numbers whose magnitude never exceeds unity. This coordinate system is found to be very effective in formulating the element properties in finite element formulation. This system is defined in such that the magnitude at nodal points will have unity or zero or a convenient set of fractions. It also facilitates the integration to calculate element stiffness.

One Dimensional Line Elements

The line elements are used to represent spring, truss, beam like members for the finite element analysis purpose. Such elements are quite useful in analyzing truss, cable and frame structures. Such structures tend to be well defined in terms of the number and type of elements used. For example, to represent a truss member, a two node linear element is sufficient to get accurate results. However, three node line elements will be more suitable in case of analysis of cable structure to capture the nonlinear effects.

Important Questions

1. Derive Stiffness matrix for bar element.
2. Describe shape functions for one dimensional element.
3. Describe shape functions for Two Dimensional Elements.
4. Explain Different types of elements for plane stress and plane strain analysis.
5. Write about Displacement models, generalized coordinates & shape functions.
6. Explain about convergent and compatibility requirements.
7. Derive area coordinates for Natural coordinate system.
8. Derive volume coordinates for Natural coordinate system.